RISK MANAGEMENT STRATEGIES
FOR GRAIN TRADERS AND PROCESSORS

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ABSTRACT


This study reviewed the theory and current techniques of risk management for firms operating in the agribusiness industry. Various motivations and tools of hedging were examined along with statistical techniques for determining optimal hedge ratios. A new theoretical approach was developed explicitly modeling the operations of a firm, including both procurement and marketing functions. The concept of strategic demand for hedging was developed in the context of an analytical model, representing the adjustment in hedge ratios in relation to the hedge horizon and input-output price correlations. Three business cases were developed for illustrating and evaluating the theoretical model. Suggestions for additional research were offered.
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I. INTRODUCTION

Risk management is an integral function in the operations of agribusiness firms, whether they are involved in production, processing, or trading activities. Market price risk of commodities is one of the obvious and perhaps the most frequently studied areas in risk management. In the case of end-users and importers, the presence of multiple sources of uncertainty (including input and output prices) complicates hedging decisions by adding several dimensions to the problem.

Optimal hedge ratio calculations (i.e., the size of offsetting positions, usually as in futures or options markets) have been the subject of a substantial amount of research in the past, and the theory of hedging and price risk management is well documented in the literature. Numerous techniques are available for estimating hedge ratios, based on assumptions of risk minimization and utility maximization. The focus of this thesis is to enhance existing models by incorporating both input (procurement) and output (marketing) decisions into a single framework. In addition, to demonstrate the impact of industry structure and competition on risk management strategies, several business cases were developed and analyzed, covering successive stages in the agribusiness industry.

Within the context of business cases, hedging and procurement strategies are analyzed for traders and processors, considering the key sources of risks they encounter. Beyond the goal of price risk management, other crucial points, such as the optimal hedge horizon, dynamic correlations between input and output prices, and the effects of industry structure on hedging strategies, are considered. As noted above, the risk management decision-making framework is extended to cover not only input or output functions, but rather to encompass both procurement and marketing operations.
Statement of Problem

Effective procurement strategies can give end-users a competitive advantage over rivals and increase the firm’s profitability. Due to the importance of risk management issues for most agribusiness entities, much research effort has been dedicated to this area. This research, however, has focused on producers, traders, and handlers, often considering commodity prices as the single source of price risk.

Additional sources of uncertainty for end-users necessitate the study of several additional risk factors in the formulation of risk management strategies. A major source of uncertainty is the price at which outputs will be sold after processing takes place and the distribution of product demand in the marketplace, which has some similarities to yield risk for producers. Given the uncertainty in demand (or the length of processing period), the size of cash positions in ingredient input markets is not considered a known parameter in the analysis.

Uncertainty about the size of cash positions is further complicated by the choice of the hedge horizon. The hedge horizon is the time period for which the hedge strategy is implemented. Simply put, it shows how far forward a firm should cover its positions. Given a firm’s production capacity, the hedge horizon implicitly refers to the length of time the hedging strategy will cover. Since the size of the cash input position is uncertain (the longer the hedge horizon, the higher the uncertainty), the decision of an optimal hedge horizon becomes a crucial part of the analysis. As an example, a bread manufacturer may be thought of as having an infinite short flour position, yet decisions have to be made about how much of that position should be hedged in futures markets on an ongoing basis. In
other words, the company will have to make a decision whether to lock in prices for inputs over the next month, six months, or some other hedge horizon.

The relationship between input and output prices differs by industry. In the case of grain merchandisers, input and output prices typically move closely together within a short time-period. Therefore, hedging strategies can be significantly different from those of a processor that adjusts output prices on a quarterly or semiannual basis.

The correlation between input and output prices can have a significant effect on procurement decisions. The importance of correlations is illustrated by a recent example from the baking industry, as reported by *Milling and Baking News*. During July 2000, white bread prices experienced a large increase nationwide. At the same time, flour prices remained relatively stable, but “substantial increases in the cost of diesel and gasoline fuel and natural gas, as well as in labor and labor-related expenses” resulted in a sharp rise of input costs (“White bread price gains,” 2000). When input and output prices move together over time, the profit margin is naturally protected by this dynamic correlation. However, when the time lag is increased, the degree of correlation may change, which could affect hedging strategies.

The correlation of prices may also be affected by the relative distance of processed goods from the base commodity in the vertical integrated industry structure. In addition, the nature of competition within the given industry may determine the way input cost changes are translated into output price changes. Higher *value-added* in a product tends to reduce the correlation between the price of a raw commodity and that of processed goods (Blank et. al, 1991).
In addition, the competitive characteristics of a particular industry affect risk management strategies by influencing the way end-users react to input price changes. Depending on the development and competitiveness of a particular industry, rivals may react differently to the actions of particular competitors (Porter, 1980). The reaction of the baking industry to rising input costs during the summer of 2000 is illustrated by the following quotation: “Leading wholesalers indicated that price increases (bread prices) had been initiated to compensate for rising costs” (Milling & Baking News, August 29, 2000). Depending on the structure of the industry, output prices may or may not be closely related to input prices, and this relationship can have an important effect on procurement strategies.

In the case when an importer only performs a trading function, the firm essentially buys and sells the same commodity with little processing, which leads to high correlations of input and output prices over short periods of time. On the other hand, when the importer purchases the commodity as an input to some value-added processing activity (i.e., flour milling or bread baking), correlations of input and output prices may be lower, depending on the significance of the particular commodity’s price relative to other input prices and the degree of other factors such as exchange rate risk (FX risk).

According to Porter (1980, Pg. 3), “The essence of formulating a competitive strategy is relating a company to its environment.” The structure of an industry has an important effect on the potential strategies firms can pursue, whether the industry is best described as an oligopoly or a fragmented industry. In commodity-based industries, as well as mature product markets such as most processed foods where buyers are familiar
with product characteristics, costs become an important source of strategic advantage, and procurement strategies are aimed at minimizing input costs (Porter, 1980).

The general trend in agriculture-related industries is a consolidation towards fewer and larger companies. This trend is observed in several sub-sectors, including farm production, “farm input, food processing, wholesaling, service and retailing industries, and grain merchandising” (Hayenga and Wisner, 2000). Changes in industry structure and increased consolidation may result in pressures for firms to adjust their procurement strategies to these new conditions.

For companies involved in international transactions, additional risks may include fluctuations in ocean freight rates (or other transportation costs) and foreign currency exchange rates (FX risk). Procurement strategies must consider all relevant sources of uncertainty and use a systematic approach to achieve desired organizational results. Most companies are not directly interested in minimizing their risk but rather managing risks to ensure that the maximum potential level of returns are generated for the amount of risk they are willing to take.

Several instruments are available for agribusiness firms to manage and reduce their exposure to the sources of price risk described above. In addition to forward contracts, futures and options on futures for a wide variety of agricultural commodities, foreign currencies, and transportation costs have been created and are widely used today. Other arrangements, including various forms of insurance, are also available and could be considered as viable risk management alternatives. Also, in formulating risk management strategies, depending on the structure of the industry, firms may have to consider the reaction of rivals to market price fluctuations and to their own hedging efforts. These
factors are best described in a game-theory context, where competitors’ reaction functions depend on the competitive structure of the industry (Porter, 1980).

The multiple sources of risk faced by grain traders and processors have a cumulative and interactive effect on profitability. In order to reduce the possibility of large ”swings” in margins, end-users and importers have to make decisions about when and how to purchase and sell input commodities, and what instruments to use for manage their exposures.

An additional issue is that the goal of a firm is not simply to reduce its exposure to risk, but to simultaneously reduce risk and enhance profits. In other words, management of risk is often preferable to risk minimization. An optimal strategy may be one which maximizes profit subject to a risk constraint or which otherwise takes account of risk preferences. The costs of managing risk (brokerage fees, potential margin calls, and option premiums) have to be weighed with the potential benefits. While, in some cases, hedging is clearly necessary to reduce risks, narrow profit margins and the actions of competitors may limit the application of several strategies which are too expensive and, in some cases, too risky to implement.

**Need for the Study**

Hedging can be viewed as a routine operating function in the agricultural sector as farmers, processors, and end-users of commodities attempt to find the most advantageous prices for inputs and outputs. Prices can be locked in for a given transaction through a variety of risk management tools. The more important issues relate to the choice of hedge horizon, and the correlation between input and output prices. For this reason, in many
firms, procurement and risk management functions are considered of strategic importance and are closely monitored by senior management.

Risk-management decisions of producers and trading entities have been researched extensively in the past, but little attention was given to end-users. Current models usually consider prices of underlying commodities as the only source of risk in their analysis. Since end-users are exposed to different sources of risks than producers, it is necessary to develop a model to accommodate these additional parameters.

This study will provide a risk-management framework for businesses with elaborate procurement strategies. Grain importers, food processors, and many other types of businesses are included in these categories.

Many countries recently began experimenting with formally organized futures and options markets, and technical knowledge of risk management strategies of individual firm managers may be limited. In these markets, trading in grains and processed goods tends to be conducted through spot transactions, often without taking advantage of the benefits of well-designed hedging strategies.

**Description of the Study**

The study includes an analysis and comparison of alternative hedging strategies among several stages of the vertical agribusiness industry. Risk management techniques are evaluated with a focus on futures contracts and, in some cases, considering forward contracting arrangements. Since options on futures contracts are an important alternative available to traders and processors, they are included in the literature review chapter. The method of evaluation for optimal strategies will be the familiar mean-variance framework.
The analytical model is developed to address questions of the hedge horizon and to account for the correlation between input and output prices. In addition, a stochastic simulation model using @RISK™ is built to illustrate hedging with multiple futures contracts. The model is extended to analyze business cases within the agribusiness industry.

An important aspect of this project is to develop a model which would be useful for companies to evaluate their risk-management process. The model will include risk factors applicable to importers and processors of grain, and evaluate the effects of different hedging strategies on their profitability.

**Study Objectives**

There is a need to identify, quantify, and evaluate a firm’s risk exposure and to choose appropriate procurement strategies. The general objective of this study is to incorporate procurement and marketing decisions into a single hedging model, considering risk factors typically faced by firms in the agribusiness industry. There are three specific objectives.

The first objective is to develop an analytical model for deriving hedge ratios to reflect the needs of end-users, with a focus on hedge horizon and input-output price correlations. The second objective is to build a stochastic simulation model which includes multiple risks on the input side as well as multiple output products. Finally, several case studies are conducted to examine and contrast hedging strategies between sectors of the agribusiness industry.
Thesis Outline

Chapter II presents a background and review of previous studies in the areas of risk management; and the use of futures, options, and forward contracts. A review of previously developed hedging models is also included. Chapter III presents the theoretical model framework, and describes the key assumptions and analytical derivations as well as a numerical sensitivity analysis. In Chapter IV, the empirical cases and data are presented. Chapter V presents the analytical results of empirical tests for the grain trading, flour milling, and bread baking cases. Chapter VI presents the results of a stochastic simulation model for the case of a Mexican flour milling operation. Chapter VII summarizes the findings, and lists some of the implications and potential areas for further research.
II. BACKGROUND AND PREVIOUS STUDIES

Risk management and hedging have been the focus of much research during the past fifty years, and these areas are well documented in the agricultural, economics, and finance literature. One of the important questions in risk management and procurement strategies relates to estimating the optimal hedge ratio, which refers to the proportion of the underlying cash position that is hedged in futures markets.

There are several approaches to the problem of finding optimal hedge ratios, including risk-minimizing and utility-maximizing techniques. It is important to recognize that estimating hedge ratios is an empirical, as well as a theoretical, issue. The analysis is further complicated by the decision between using price levels, differences, or log returns as the basis for estimating sample statistics of parameters (Blank, 1989). In each of these areas, models have been developed to accommodate single sources of risk as well as more complicated models to handle multiple sources of uncertainty. A description of these methods can be found in Schroeder and Mintert (1988). Regression analysis; analytical techniques; and, more recently, stochastic simulation have all been used to estimate model parameters (Collins, 1997).

Risk affects the revenues and the profitability of importers and end-users. Most companies employ some form of risk management strategies in order to maximize their profits and protect themselves from adverse market movements. Some strategies are simple and follow the policy of taking equal and opposite positions in futures markets. In this case, the hedge ratio equals 1.0. Others are based on sophisticated technical and quantitative analysis, and computer-based trading models to arrive at optimal hedge positions.
According to Baker and Gloy (1999), risk management involves the identification, evaluation, and implementation of strategies to reduce uncertainty in the revenue flow. Implementing risk-reducing strategies comes at a cost, and these costs have to be weighed against potential benefits. Spahr and Sawaya (1981) point out that, while hedging generally results in reduced risk, it also often significantly reduces profits, which reflects a tradeoff between risk and return.

As stated in Collins (1997), many of the current hedging models fail to explain observed behavior of actual hedgers, including producers and processors. Specifically, he stated that “processors do routinely hedge, but the observed hedge ratios vary considerably from year to year and in general are inconsistent with any existing model of hedging behavior” (Collins, 1997, Pg. 489). Therefore, a model specifically fitting and explaining the hedging behavior of processors and end-users needs to be developed.

This chapter begins with a review of commonly used hedging instruments, such as forward, futures, and options contracts. Alternative risk-management strategies are reviewed as well for select instruments. Next, previous studies in the areas of risk-minimizing and utility-maximizing hedge ratios are summarized, followed by a review of the frequently used mean-variance framework. Concluding the chapter, the idea of quantity uncertainty is introduced.

**Hedging Instruments**

The most frequently used hedging instruments include futures, forward, and options contracts. The choice of contract depends on overall costs and benefits, and expectations of future price movements (Elam, 1988). The hedger uses a futures contract, which exhibits the highest price correlation to the underlying cash market. As a general rule, the
choice of futures contracts is the delivery month closest, but after, the date of the cash position (Spahr and Sawaya, 1981).

Futures contracts are popular risk-management instruments, widely used by hedgers in numerous industries. Seidel and Ginsberg (1983) attribute their popularity to liquidity and small margin requirements. Hedging with options reduces the risk of loss from rising input prices, and may also be used as a way to insure availability of raw material supplies (Seidel and Ginsberg, 1983). Economic agents interested in reducing their risk can engage in risk transference to speculators via futures or options markets (Rolfo, 1980). Instruments such as futures and options differ in their characteristics, and the costs and benefits they offer.

Since optimal hedge ratios often result in recommendations that use fractions of futures and options contracts, it is important to remember that contracts cannot be partially used. Futures and options must be traded in integer units, which may result in importers or processors not being able to cover their entire position without being either over-hedged or under-hedged. The lack of fractional contracts is especially problematic for relatively smaller importers and processors with limited underlying positions.

Forward contracts

Forward contracts are individually tailored arrangements between two parties for the delivery of a particular commodity or financial asset. These contracts are typically used by large operations to accommodate their specific risk-management needs. They represent an obligation to both parties, and performance on the contract is mandatory. A forward contract calls for the exchange of a predetermined quantity of the commodity at a given price on a future date.
Once the contract has been entered, the parties must offset their positions on the settlement date. The flexibility of negotiating the terms of the contract have to be weighed against the potential negative effects arising from low liquidity. Forward contracts are also available to cover foreign currency risk exposure. These arrangements can be made between the importer and large commercial banks, but the volume of these contracts is normally quite large. One of the benefits of forward contracts is the possibility to accommodate particular needs of individual businesses in terms of volume, price, and contract maturity (Hull, 1998).

**Futures contracts**

Futures contracts allow for the purchase or sale of a set quantity of a commodity at a future point in time for a given price. Futures are standardized instruments, traded on organized exchanges, offering a larger degree of flexibility and liquidity since the contracting parties can offset (i.e., trade in and out of) their position instead of taking or making delivery. Users of futures contracts lock in the purchase or selling price of the commodity, reducing their exposure to the risk of price fluctuations. While the use of futures to cover existing cash positions is considered hedging, buying or selling futures without an underlying cash position is regarded a speculative strategy (Blank et al., 1991).

Hedgers can trade in and out of their futures positions any time before the settlement period. Besides brokerage fees, futures trading also involves potential margin calls, and the hedger has to have a certain level of risk capital reserved to meet these payments. Futures are highly standardized and are used routinely for some grains; foreign currencies; and, in some cases, transportation services (Blank et al., 1991).
Holt and McKenzie (1998) investigated the efficiency of futures markets and evaluated the convergence of futures and spot prices. The study found that most futures markets are efficient in the long run, meaning that current futures prices reflect all available information about future spot prices. In the short run, some market inefficiencies were found, which may have implications for processors and importers with a short hedge horizon. Whether futures prices are a good indicator of future cash prices depends on the efficiency of the market and the risk premium realized by market participants. The paper concludes that futures markets are efficient and offer good hedging opportunities for market participants (Holt and McKenzie, 1998).

While the availability of futures has been extended to include a wide range of commodities and financial assets, many processed or semi-processed goods do not have exchange traded futures contracts. This problem leads to the idea of “cross-hedging,” whereby the hedger identifies a related contract with a preferably high positive correlation to the underlying (Blake and Catlett, 1984). Flour mills or a large bakery firm may be a good example of companies with positions in the underlying without a matching futures contract. The determination of hedge ratios for a cross hedge is the same as for a regular hedge (Anderson and Danthine, 1981).

Futures contracts always trade for a specified number of units in the underlying physical commodity. Since the contracts tend to represent a relatively large amount (i.e., 5,000 bushels of corn), there may be some limitations with respect to the ability of the hedger to enter a futures position that exactly meets his or her needs (Anderson and Danthine, 1981).
Options on futures contracts

Options on futures contracts are also risk-management tools that end-users and processors may incorporate into their hedging strategies. The two basic types are call and put options, and a hedger may assume a short (sell) or long (purchase) position in each of these derivatives. Puts confer to the buyer a right, but not an obligation, to sell the underlying (which may be a future) at a pre-determined price, the strike price.

American-style options can be exercised any time before or on the maturity date while European options can only be exercised on the contract maturity date. Exchanges in the United States trade both American and European style options. The buyer of a call option has the right, but not the obligation, to purchase a given underlying (i.e., futures) at a pre-determined price, the strike price, any time before the maturity date, similarly to put options (Blank et al., 1991).

The buyers of options are not subject to margin calls, and their investment at risk is limited to the premium paid for the contracts. The profit potential (upside) of the sellers of either a put or a call option is limited to the amount of the premium received, yet their potential losses (downside) could be significant. An advantage of options is that they allow the holder to participate in anticipated price movements while establishing either a price floor or a price ceiling. Put options could essentially be viewed as an insurance policy against adverse price movements, and as with any other sort of insurance, a premium is associated with these arrangements (Blank et al., 1991; Hull, 1998).
Alternative Risk Management Strategies

Diagrams can be used to visualize payoff (or profit) functions of alternative hedging strategies. The functions here do not include commission costs and interest expenses. Figure 2.1 illustrates the payoff functions for holding open cash positions.

Figure 2.1. Payoff Functions from Long or Short Cash Positions.

A long (short) position in a cash commodity results in gains (losses) as the market price increases. Holding large open cash positions can expose a firm to significant price risk, especially in highly volatile markets and in industries with low profit margins.

Payoff functions from holding forward contracts are similar to those of open cash positions. When combined with a cash position, the resulting payoff function at maturity of a forward contract is a horizontal line since basis risk is nil. Figure 2.2 shows the payoff from holding positions in forward contracts as a result of changes in the price of the underlying asset.
An advantage of forward contracting is that firms know the exact price they will pay and/or receive when the transaction is completed. In essence, when using forward contracts to offset an open cash position and holding the forward position until maturity, the firm eliminates basis risk but also foregoes any potential gains from favorable price movements.

A short hedge is the combination of long cash and short futures, or long cash and short forward positions. Grain buyers and/or processors have a risk exposure after buying or processing their products, and will want to reduce price risk. A long hedge is composed of short cash and long futures positions. Processors and buyers with the short cash input position may use this strategy to reduce their risk exposure to adverse price movements. Both of the above strategies can involve basis risk, the risk that futures and cash prices will change by different amounts and/or in opposite directions.

Using either short or long hedge strategies would not reduce the firm’s risk exposure to zero, as illustrated by Figure 2.3. Changes in the basis result in potential losses or gains unless futures and cash prices are perfectly correlated and the minimum-variance
hedge ratio is used. However, the risk of loss from a hedged position is substantially lower than the risks of holding an open position. This loss reduction is due to the observation that the volatility of the basis tends to be significantly smaller than the volatility of the price of underlying cash positions.

Figure 2.3. Payoff Functions from Long and Short Hedges.

Payoff functions of options contracts (long and short puts and calls) are shown in Figures 2.4 and 2.5. A long position in a call option limits the downside potential while allowing the holder to take advantage of favorable price movements.

Figure 2.4. Profit Functions for Put Options.
Payoffs for the writers of call options are limited to the premium they receive while their potential dollar losses are unlimited. An advantage of using options when compared with futures is that the buyer is not subject to margin calls.

![Figure 2.5. Profit Functions for Call Options.](image)

Risk-management strategies that combine put or call options with a position in the underlying commodity result in payoff functions similar in shape to simple payoffs of option contracts. Buyers and producers with long cash positions can either buy put options, or sell call options, depending on their attitude toward risk and their expectations of future price movements. Short cash positions can be hedged by buying call options or writing (selling) put options.

**Optimal Hedge Ratios**

Hedgers are firms and individuals with positions in the cash market, including producers, merchandisers, and end-users, who use commodity futures markets to transfer part of their risk of loss to speculators. Having a cash position separates hedgers from speculators who do not have a position in the underlying physical commodity. According
to Anderson and Danthine (1981), speculators take on the price level risk that hedgers are not willing to carry. In turn, hedgers are still left with basis risk, or price difference (spread) risk. The source of risk is most often attributed to market price fluctuations of the underlying commodities but may include the risk of supply shortages (Seidel and Ginsberg, 1983).

The traditional view of hedging assumed that the optimal strategy is to hold a position in the futures market which is equal and opposite of the position in the underlying commodity (Rolfo, 1980). While this approach has intuitive appeal and ease of execution, there has been considerable research in the literature focused on improving the estimation of hedge ratios. Two main categories of hedge ratio estimation include the risk-minimizing and utility-maximizing approaches. In the literature review, both of these categories are discussed and summarized; then, the points of departure for this thesis are outlined.

Most of the approaches used to find optimal hedge ratios follow the same general procedure, whereby the revenue or profit function is maximized in terms of the choice variables. Examples can be found in Lapan and Moschini (1994), Martinez and Zering (1992), Rolfo (1980), Lence and Hayes (1994), Moschini and Lapan (1995), Blank et al. (1991), Vukina et al. (1996), and Nayak and Turvey (2000). As the number of choice variables and possible risk-management instruments increases and correlations are also considered, finding analytical formulas for the variance of the profit functions becomes more complex.

Hedging with futures contracts reduces the total amount of risk by substituting the risk of absolute price movements with the risk of movements in the basis. The basis tends
to be less volatile than actual prices; therefore, the amount of risk assumed by the hedger can be substantially reduced (Seidel and Ginsberg, 1983).

Risk minimization

The risk-minimizing approach to finding hedge ratios uses the coefficient of regression, also given by the formula $H = -\left(\frac{\sigma_{sf}}{\sigma_f^2}\right)$. Here, $H$ is the risk-minimizing hedge ratio, which is the ratio of the covariance between cash and futures markets to the variance of the futures. Since, traditionally, the reason for hedging is risk reduction, this hedge ratio is calculated with the objective of minimizing the variance of income once the position in the cash market has been determined. Derivation of this formula is documented in Lence and Hayes (1994), Johnson (1960), Blank et al. (1991), and Rolfo (1980).

Collins notes that the risk-minimizing hedge ratio is inappropriate for processors as it “does not match the behavior of processors and farmers who frequently hedge only part or none of their commitments” (Collins, 1997, Pg. 491). He concludes that the risk-minimizing model tends to be best at predicting the behavior of arbitrage traders who take close to equal and opposite positions in futures markets.

Lence and Hayes (1994) used the risk-minimizing approach in calculating hedge ratios but also allowed the estimation of uncertainty for the random variables in the model. The minimum-variance hedge ratio (MVH) is the optimal hedge position if futures prices are unbiased or if the agent is infinitely risk-averse. According to Lence and Hayes (1994), different authors estimated different MVHs for the same commodity. The estimation results imply that “MVH estimation risk is important” (Lence and Hayes, 1994, Pg. 97). The MVH is sensitive to the estimated parameters because the statistical characteristics of cash and futures markets (such as volatilities and correlations) change over time.
Lence (1996) reexamines the performance of minimum-variance hedge ratios and finds that it is only consistent with maximizing utility under certain conditions. When the assumptions of MVHs are relaxed, such as allowing for production uncertainty, alternative investment opportunities, and brokerage fees, the optimal hedge ratios are substantially lower than those suggested by MVHs. His analysis suggests that MVHs are only optimal under very specific situations with a limited number of stochastic variables (Lence, 1996).

Vukina et al. (1996) developed a risk-minimizing model where both price and yield risk are hedged in futures markets. Their optimal hedge ratio calculation is similar to that in Blank et al. (1991) but includes two sources of uncertainty. An analytical model is also developed for measuring hedging effectiveness in terms of the reduction in the variance of outcomes (Vukina et al., 1996).

Hedging strategies of firms with both production and price risk are further complicated by including foreign exchange risk, as in the case of companies involved in international transactions. Nayak and Turvey (2000) developed a risk-minimizing hedge ratio model to address all of the above questions in a simultaneous decision-making problem. They solved a system of three equations simultaneously for price, currency, and yield futures hedge ratios. The effectiveness of minimum-variance hedges is measured by the amount of risk reduction gained from a particular strategy. One of the approaches to measure risk reduction is comparing the risk of unhedged positions to a variety of hedging strategies. As expected, the authors conclude: “The magnitude of risk reduction depends upon the correlation and covariance between the random outcomes” (Nayak and Turvey, 2000, Pg. 131).

Utility maximization
Utility-maximizing models are used by Haigh and Holt (1995), Sakong et al. (1993), Lapan et al. (1991), Collins (1997), Garcia et al. (1994), and Rolfo (1980), among others. The models make explicit assumptions about the utility function of the decision-maker. These models typically include a risk-aversion parameter in the hedge ratio formula as well as the agent’s expectations of futures price movements. Derivation of a representative model is described in Blank et al. (1991) and is also found in numerous articles in the literature such as Lapan et al. (1991), Sakong et al. (1993), and Collins (1997). The mean-variance model with a single source of uncertainty yields the following solution:

\[ H^* = \frac{E(f_t) - f_0}{2\lambda \sigma_f^2} - \frac{\sigma_{sf}}{\sigma_f^2}, \]

where the second component is the same as the risk-minimizing hedge ratio. The first component includes the risk aversion parameter, \( \lambda \), and the bias term in the numerator. The first term is often referred to as the speculative component of the hedge ratio (Blank et al., 1991; Vukina et al., 1996). The bias term, \( E(f_t) - f_0 \), refers to the hedger’s point of view regarding futures prices. When current futures prices \( (f_0) \) are believed to be the best estimate of “future” futures prices \( (f_t) \), the bias is zero. However, if the hedger believes he or she possesses some unique knowledge which allows him or her to anticipate price changes, the utility-maximizing hedge ratio can include these expectations. Of course, the level of risk aversion \( (\lambda) \) will determine the degree to which the hedge position reflects anticipation of price movements.
Lapan et al. (1991) extend the scope of the hedging decision by allowing the use of options. Their model includes production uncertainty, which is also referred to as yield risk for producers. Speculative positions can be taken as part of the hedging strategy when current prices are considered biased. Utility is maximized by the decision-maker through his or her choice of optimal production levels and hedge ratios in both futures and options contracts. The mean-variance framework is relaxed since options result in non-linear payoff functions. The results show that, in the absence of bias, only futures would be used for hedging and the futures position equals the minimum-variance hedge ratio (Lapan et al., 1991).

Sakong et al. (1993) found that options may be optimal to use for risk management, combined with futures hedges when both price risk and production uncertainty are present. Their model is an expansion of the model developed by Lapan et al. (1991) except for allowing production uncertainty. The utility-maximizing solution is given by a combination of a futures position equal to the size of the minimum expected yield, and the additional production volume was hedged by put options. Hedging strategies using options and futures are often considered separately, but in this case, both instruments were part of the same risk management program (Sakong et al., 1993).

Rolfo (1980) incorporates price and production uncertainty into a risk-minimizing hedge ratio model, and applies it to four producer countries. The results indicate that, in the presence of yield risk, producers are made better off by selling less than the equal and opposite volume in the futures markets as opposed to entering a full short hedge. Two models, one of risk minimization under a mean-variance framework and a utility-maximizing model under a logarithmic utility function, are considered (Rolfo, 1980).
Martinez and Zering (1992) also consider price and yield uncertainty in a dynamic hedging model for grain producers. Regression analysis is used to estimate parameters of forecasting models, which are used to estimate the key parameters and their statistical relationships. The model was expressed as a linear exponential Gaussian optimal control problem. The mean-variance framework is used to examine the effect of hedging strategies on average returns and risk. The results indicate that, while dynamic hedging strategies may increase expected profits, the complexity and effort needed in their implementation often outweigh the benefits.

A similar model was developed by Lapan and Moschini (1994) for a risk-averse producer facing price, basis, and production risk. The producer’s utility function was assumed to be of the Constant Absolute Risk Aversion (CARA) form, where utility is expressed as \( U = -\exp(-\lambda \Pi) \) with \( \lambda \) representing the risk aversion factor and \( \Pi \) the profit function. The suggested hedge ratio is very similar to that in Blank et al. (1991) with a pure hedge component and a speculative component which is affected by the bias and the risk aversion parameter. However, the inclusion of yield uncertainty results in continually updated futures positions as production was being realized. Assuming no production uncertainty and no bias, the hedge ratio is reduced to the familiar regression coefficient. One of the key findings is the negative relationship between production risk and the hedge ratio.

Haigh and Holt (1995) extended traditional hedge ratio models to include risks other than volatility in commodity prices. They moved beyond a domestic company and modeled the hedging decision of an international trading entity facing multiple sources of risk. They recognized the need to include both foreign currency exchange rates and
transportation costs as a significant source of uncertainty which have to be taken into account. Results show the importance of correlation among closely related markets, especially when each market is the source of significant uncertainty. Due to large volumes of grain being transferred in international transactions, even small increases in transportation costs can potentially diminish profits.

Three common approaches to estimate hedge ratios are reviewed by Schroeder and Mintert (1988) and Blank (1989). The three methods include price-level models, where cash price is regressed on the nearby futures price; price-change models, where the change in cash price over the hedging period is regressed on the change in the respective futures price; and percentage-change models, where the percentage change in cash price is regressed on the percentage change in respective futures price. “Price difference models of hedge ratios vary depending upon the decision maker’s goal” (Blank, 1989, Pg. 132). In addition, logarithmic returns are also often used, which represent the continuously compounded rate of return. Nevertheless, there is no consensus in the literature as to which model produces superior results.

Many of the models, whether the objective is risk minimization or utility maximization, fail to explain the observed behavior of a wide range of agents. Collins (1997) constructs his positive model of hedging behavior as a financial decision where the firm’s objective is to maximize terminal equity by making choices about current operations. The model explains hedging as avoidance of financial failure, as apposed to an approach to reduce price uncertainty. It explains why some agents, such as most farmers, choose not to hedge at all while others, such as arbitrages, typically hedge most of their positions (Collins, 1997).
Mean-Variance Framework

Portfolio analysis and the mean-variance framework are often used to evaluate optimal positions in a group of assets with particular levels of risk and return. Decision-makers “consider expected return a desirable thing and variance of return an undesirable thing” (Markowitz, 1952, Pg. 77). The objectives can be characterized either by maximizing portfolio returns for a given level of risk or by minimizing risk for a particular level of expected returns. Positions in cash and futures markets can be viewed as a portfolio of assets, which the hedger takes in order to achieve the desired risk and return objectives (Seidel and Ginsberg, 1983; Markowitz, 1952).

The mean-variance technique is a useful tool in identifying optimal hedge ratios and exploring the effects of including options in risk management strategies. According to Markowitz, “The reason for performing a mean-variance analysis in fact, rather than a theoretically correct expected utility analysis, is convenience, cost or feasibility” (Markowitz, 1991, Pg. 475). In addition, while recognizing the limitations of the mean-variance model, empirical analysis has showed that the results of mean-variance approximation are very good for some utility functions (Levy and Markowitz, 1979).

There are problems with the mean-variance approach that arise from the assumptions about the distribution of returns and the form of the utility function (Kroll et al., 1984). According to Pulley, “The mean-variance model is precisely consistent with the expected utility hypothesis only in the special cases of normally distributed security returns or quadratic utility functions” (Pulley, 1981, Pg. 361). However, Pulley shows that the important factor is the decision-maker’s Pratt-Arrow risk class and that “the particular functional form of an investor’s preferences is shown to be essentially irrelevant in
developing a predictive or descriptive model of his (local) decision behavior” (Pulley, 1981, Pg. 376).

Garcia et al. (1994) found the mean-variance framework to be the most appropriate method for identifying appropriate risk management strategies. Chavas and Pope (1982) included production uncertainty, as well as transactions and other hedging costs, and used a mean-variance utility function to model hedging decisions.

Tsiang (1972) re-examined the familiar mean-variance model to find that the first two moments are adequate in making portfolio choices for small risk takers. Large risk takers, however, are more likely to consider higher moments also, such as skewness in their portfolio decisions (Tsiang, 1972). Lapan et al. (1991) show that the assumptions of the mean-variance framework have to be relaxed in the presence of options since the price distributions will be truncated.

Anderson and Danthine (1981) also used the mean-variance utility specification in their model of hedging behavior, recognizing that cash and futures positions are determined simultaneously by the hedger. They also point out a general problem with the agent’s assumption on the probability distribution of uncertain variables. This suggests that the statistical characteristics of stochastic variables hold constant over time. In fact, this assumption is “fundamental to our results, as indeed it is to the notion of hedging” (Anderson and Danthine, 1981, Pg. 1196).

In Rolfo (1980), the mean-variance framework is used to maximize expected income for a particular level of variance of income. The producer’s utility function is assumed to follow the form of a constant absolute risk aversion function. Expected utility is increasing in income and decreasing in the variance of income. The article also includes
the effects of correlation between output price and yields into finding optimal hedge strategies (Rolfo, 1980).

Vukina et al. (1996) also use the mean-variance framework to find risk-minimizing hedge ratios in the presence of multiple risks. Risk is defined as the variation in profits, and risk-aversion is considered explicitly in the model. The effectiveness of alternative hedge ratios is evaluated according to the amount of risk reduction. Their finding indicated that “the mean-variance approach is identical to minimizing the variance of profit in cases where futures markets are efficient, or when producers are infinitely risk averse” (Vukina et. al, 1996, Pg. 1019).

In addition to futures, Garcia et al. (1994) incorporate the use of option contracts into the hedging decision within the mean-variance framework. They recognize the problems with truncated and skewed return distributions when using options, and examine the effects of considering these in optimal hedging decisions. Results confirm that the mean-variance framework is useful in evaluating alternative risk management strategies, even when options contracts are used (Garcia et al., 1994).

**Quantity Uncertainty**

Traditional views of hedging considered the positions in underlying commodities a known value. More recent models have included production uncertainty which, in the context of producer hedges, is also referred to as yield risk. Production risk typically results in a reduced hedge ratio for most producers. In the presence of quantity uncertainty, even if price risk is shifted to speculators, overall risk is not necessarily eliminated (Seidel and Ginsberg, 1983).
A crucial question to processors relates to the size of their cash input position, in other words, the period of production for which inputs should be procured at any one time. These questions are not only complicated by the nature of inputs (perishable versus storable commodities), but also the distribution of demands for output goods. These questions are explored in the following chapters.

**Summary**

In the literature review, an overview of past studies in the areas of risk management and hedge ratio estimation was presented. While the literature has been well developed for the case of producers and also includes models for intermediaries and processors, several key factors have not been addressed in the past. These factors include the effects of demand uncertainty, the correlations of input and output prices, the competitive nature of an industry, and the choice of optimal hedge horizon. In Chapter III, the theoretical model is developed to incorporate these additional factors.
III. THEORETICAL MODEL OF HEDGING

In the previous chapter, traditional hedging models were reviewed, organized by the objective of the hedger. One of the methods of finding the optimal hedge position (the hedge ratio) is to minimize the variance of profits by selecting optimal positions in futures markets. An alternative approach is maximizing the utility of profits for a particular level of risk aversion. An agent’s utility can be modeled by assuming various utility functions. Some examples include the quadratic, constant absolute risk aversion (CARA), and the constant relative risk aversion (CRRA) functions.

The analysis in this chapter builds on and extends traditional models by considering two additional features which are important to end-users. The first feature is the relationship between product input and output markets. Conventional hedging models are typically concerned with managing the risk of one activity, either marketing (output) or procurement (input) processes.

An optimal hedging model is developed, recognizing that the firm’s procurement decisions are an integral part of overall strategy, as apposed to being viewed as an isolated function. Therefore, decisions like whether to hedge and how much to hedge are related to the joint risks that arise from the firm’s presence in both input and output markets. The second important feature is the hedge-horizon which refers to the length of coverage whereby the firm determines how far forward to purchase inputs in futures markets.

As described in Markowitz (1991, Pg. 469), “In the real world, there is a delay between the decision to produce, the time of production and the time of sale. The price of the product at the time of sale may differ from that which was expected when the production decision was made.” Taking the time lag into consideration, the effects of
hedge-horizon and input-output price correlations are incorporated into the decision-making process.

First, the hedging model is developed; then, results from a numerical analysis are presented to explain effects of key parameters on the hedging decision. At the end of the chapter, a short summary outlines differences between previous models and the model developed here. Details of mathematical derivations and some extensions to the model are explained in Appendix A.

The Model

The model is developed by extending the framework used by Nayak and Turvey (2000), Blank et al. (1991), Vukina et al. (1996), and Lapan et al (1991). Some important differences relate to including both the revenues from selling output, and the costs of inputs and production. Inclusion of both input and output prices allows the explicit modeling of the relationship between those markets. Input cash and futures prices, as well as output prices, are stochastic variables which the firm cannot forecast accurately. The firm’s profit function is expressed as

\[
\Pi = \left( Q_{t+n} \times \tilde{P}_{O,t+n} \right) - \left( Q_{t+n} \times \tilde{P}_{I,C,t} \right) + F_{1} \left( \tilde{P}_{1,F,t} - P_{1,F,t-m} \right) - \left( Q_{t+n} \times C_{S/O} \right)
\]

(1)

\[
E(\Pi) = Q_{t+n} \times \tilde{P}_{O,t+n} - Q_{t+n} \times \tilde{P}_{I,C,t} + F_{1} \left[ E(\tilde{P}_{1,F,t}) - P_{1,F,t-m} \right] - \left( Q_{t+n} \times C_{S/O} \right)
\]

(2)

where \( Q_{t+n} \) is the number of units of output planned to be produced from day \( t \) to day \( t+n \) and \( Q_{t+n} \) is the quantity of inputs needed to produce the desired output from \( t \) to \( t+n \). The relationship between the quantity of inputs and outputs is assumed to be fixed (constant conversion ratio). \( F_{1} \) is the quantity of inputs purchased in the futures market on
day $t-m$; $\tilde{P}_{O_{t+n}}$ is the price of processed products (output) sold at time $t+n$; $\tilde{P}_{I,F,t}$ is the price of inputs in the spot market at time $t$; $\tilde{P}_{I,F,t}$ is the futures price at time $t$; and $P_{I,F,t-m}$ is the futures price at time $t-m$. The non-ingredient cost of production is assumed to be a non-stochastic constant and is represented by $C_{S/O}$ (costs in dollars per unit of output).

Stochastic variables are represented by tildas. While many of the variables included in this model are similar to those in traditional models, there are also fundamental differences. The cost of hedging in terms of fees and margin calls is not explicitly included in the model, but it can be reflected in the futures price as explained by Anderson and Danthine (1981).

One of the differences relates to the firm’s operations. In this model, input and output decisions are not viewed separately, but as separate components of a single process. Therefore, both input and output prices are represented by random variables. The degree to which these variables are correlated has an effect on the risk-management strategies chosen by the firm. This model is built with the goal of explicitly including these interactions and effects.

The first term in the profit equation represents the revenue from selling output at time $t+n$. In other words, a production decision is made at $t-m$ which will result in sale of output at time $t+n$. The second term is the price paid for cash inputs at time $t$ in the cash markets when the firm acquires the physical inputs. The third term represents the payoffs from hedging activities in the futures markets. Here, payoffs from the positions in futures markets offset the price fluctuations in cash markets. Consequently, the effective purchase cost of cash inputs can remain relatively stable. The higher the correlation between input
cash and futures markets, the more effective hedging will be at reducing the fluctuation in profits.

The last term is the cost of production other than ingredient costs. There are several approaches to treating the category of production and other costs. Frequently, for the sake of simplicity, these costs are assumed to be a constant amount per unit, up to the point where total capacity is exhausted. A more realistic assumption would be that the marginal cost of producing additional units begins to rise beyond some point as total capacity is approached. In that case, the link between quantity produced and production costs cannot be represented by a linear relationship. In this model, per unit production costs are assumed to be constant.

Frequently, payoff functions do not include both costs and revenues. Models of producers tend to focus on hedging output prices while models of processors are concerned primarily with input costs. An important distinction between previous models and the model developed in this chapter is that both input and output prices are considered at the same time, regardless of whether the firm is a producer or a processor.

All decisions are made at time $t-m$ for the production period from $t$ to $t+n$, where the sequence of days is given by $t-m < t < t+n$. Inputs are purchased at time $t$ in the spot market, and if a hedge was placed in the futures market at time $t-m$, it is also lifted at $t$. Risk can be described as the change in input prices from time $t-m$ until time $t$, when actual spot inputs are purchased, as well as changes in output prices from day $t-m$ until day $t+n$.

There are several possible combinations of $m$ and $n$ time periods, and as a result, both inputs and outputs may be “priced” at different times. The length of periods $m$ and $n$ may depend on the particular industry, or it may be the firm’s decision. However, in the
model, it is assumed that output prices are random until the sale at \( t+n \). The timeline of the decision-making process is illustrated by Figure 3.1.

![Figure 3.1. Timeline of Hedging and Processing Periods.](image)

Firms could also sell their output forward at \( t \), and in this case, \( \tilde{P}_{O_{t+n}} \) would no longer be a random variable. Output may be contracted at a specific price; therefore, part of the price risk could be eliminated. Alternatively, a processor can contract inputs, which makes \( \tilde{P}_{I,C,t} \) a non-random variable. These issues are considered in the empirical analysis and simulations.

The mathematical derivations are very similar to previous studies for mean-variance utility maximizing hedge ratios, except that the variance of output prices, \( Var(P_{O_{t+n}}) \), and the covariance between input and output prices, \( Cov(P_{O_{t+n}}, P_{I,F,t}) \), are non-zero.

The variance of profits is given by

\[
Var(\Pi) = \left( Q_{O,t+n} \right)^2 Var(P_{O_{t+n}}) + \left( Q_{I,F,t} \right)^2 Var(P_{I,F,t}) + F_i^2 Var(P_{I,F,t}) - 2Q_{O,t+n} Q_{I,F,t} Cov(P_{O_{t+n}}, P_{I,F,t}) + 2Q_{O,t+n} F_i Cov(P_{O_{t+n}}, P_{I,F,t}) - 2Q_{I,F,t} F_i Cov(P_{I,C,t}, P_{I,F,t})
\]  

(3)
The covariances among input cash, futures, and output prices are included in the variance equation. The formulation allows explicit modeling of input and output price interaction and incorporates the manner in which price changes in input markets translate into price changes in output markets. Measuring this relationship in a particular industry is an empirical issue; and may depend on the competitive nature of the industry, standard business practices, or the length of the time lag. Appendix A includes the full development and derivations of the model as well as some extensions.

The optimal futures position is found through maximizing the mean-variance utility function of the firm, by choosing the appropriate level of futures position (Garcia et al., 1994). In this case, the utility of the hedger increases with expected profit but decreases with risk, depending on the risk-aversion factor.

The risk-aversion of the firm is characterized by the parameter $\lambda$, where $\lambda > 0$. The larger the value of $\lambda$, the more risk-averse the firm is. When this parameter equals zero, the firm is risk-neutral, and as it approaches infinity, the firm becomes extremely risk-averse. A realistic value of $\lambda$ for a particular firm may be difficult to estimate, but conducting the analysis with a variety of values offers insight about why hedging strategies may be different for firms with different tolerance for risk. Parameter $\lambda$ can also be thought of as the rate at which the firm is willing to take on risk in order to earn additional profits.

The problem is represented as

$$\text{Max } J = E(\Pi) - \frac{\lambda}{2} \text{Var}(\Pi)$$  \hspace{1cm} (4)$$

The hedger selects the optimal size of the futures position in order to maximize the above function. The first-order condition is given by
\[
\frac{\partial J}{\partial F_I} = E[P_{I,F,t} - P_{I,F,t-m}] - \frac{\lambda}{2} \left[ 2F_I Var(P_{I,F,t}) + 2Q_{I,t+i+n} Cov(P_{O,t+i+n}, P_{I,F,t}) - 2Q_{I,t+i+n} Cov(P_{I,C,t}, P_{I,F,t}) \right] = 0
\]

Rearranging the terms and solving for \( F_I \), the optimal futures position is given by

\[
F_I = \frac{Q_{I,t+i+n} Cov(P_{I,C,t}, P_{I,F,t})}{Var(P_{I,F,t})} + \frac{E[P_{I,F,t}] - P_{I,F,t-m}}{\lambda Var(P_{I,F,t})} - \frac{Q_{O,t+i+n} Cov(P_{O,t+i+n}, P_{I,F,t})}{Var(P_{I,F,t})}
\]

In the above form, \( F_I \) is interpreted as the number of units purchased in futures markets. To arrive at the hedge ratio, both sides of the equations are divided by \( Q_I \):

\[
\frac{F_I}{Q_{I,t+i+n}} = \frac{Cov(P_{I,C,t}, P_{I,F,t})}{Var(P_{I,F,t})} + \frac{E[P_{I,F,t}] - P_{I,F,t-m}}{Q_{I,t+i+n} \lambda Var(P_{I,F,t})} - \frac{Q_{O,t+i+n} Cov(P_{O,t+i+n}, P_{I,F,t})}{Q_{I,t+i+n} Var(P_{I,F,t})}
\]

These expressions are similar to other utility-maximizing models. The important differences are the inclusion of the relationship between input and output prices, the time lag between purchasing, processing, and selling the commodity, and explicitly incorporating the quantity of inputs and outputs. In the next section, the effects of key variables are examined through a numerical analysis.

The first term of equation (5) is often labeled as the hedging demand for futures (Blank et al., 1991). The ratio of the covariance of cash and futures input prices to the variance of futures prices is also the solution to the risk-minimizing approach to finding hedge ratios. Since equation (5) defines the optimal futures position in terms of units instead of a ratio, equation (6) is derived to show the hedge ratio. Since \( F_I \) is the number
of units purchased in the futures markets, the hedge ratio as a percentage of the cash input position is given by  \( HR = F_i / Q_{t,t+1} \), as shown by equation (6). In order for the hedging demand to be 100% of the underlying position (when the hedge ratio is 1.0), the following equation must hold:  

\[
Cov(P_{t,F}, P_{t,F}) = Var(P_{t,F}).
\]

Ignoring the second and third terms, an increase (decrease) in \( Cov(P_{t,F}, P_{t,F}) \) leads to a proportionate increase (decrease) in the minimum-risk hedge ratio. When the covariance term is zero the hedging demand is also zero since there is no benefit gained from hedging with zero correlation. An increase (decrease) in \( Var(P_{t,F}) \) results in a decline (rise) in the optimal hedge ratio.

The second term of equation (5) is conventionally referred to as the speculative demand for futures. The numerator is the bias in the futures markets, and the denominator is the product of the risk-aversion coefficient and the variance of futures prices. If futures are unbiased, the numerator becomes zero; the second term cancels out; and the futures position is the same as the risk-minimizing hedge ratio (ignoring the third term for the moment). In a similar way, as the agent’s level of risk-aversion increases, the denominator becomes very large, and the second term approaches zero.

The last term of equation (5), which we refer to as the \textit{strategic demand} for futures, includes the covariance (which in concept is related to the correlation) of input futures and output product prices. These correlations are not necessarily contemporaneous, as the time lag, \( n \), does not have to be zero. It is important to analyze how this term affects the overall hedging strategy and futures positions. Output quantity is multiplied by the ratio of the covariance between input futures and output prices, where the correlation is determined
over \( n \), the length of the hedge horizon, and the variance of futures prices. In durations over which the covariance (or correlation) between input and output markets is very low, this term converges to zero. However, greater correlations result in a reduction in the optimal hedge ratio. In this case, profit margins are protected by similar fluctuations in input and output prices.

The optimal hedge ratio maximizes the expected utility of the firm when both future input and output prices are uncertain. For a firm with a short spot input position, the hedge ratio at \( t-m \) would presumably be positive as inputs are purchased in the futures markets. However, companies may also be exposed to price risk after purchasing inputs at time \( t \) in the spot market. In that case, they have a long spot ingredient position at \( t \left( Q_{t,t} > 0 \right) \), which may pose additional risks. At that point, the firm is already protected against rising input prices. However, a decline in input costs may give an advantage to its competitors who will be relatively lower cost producers.

The firm could also use call options instead of futures contracts and pay a premium at \( t-m \). Another possible way to address the issue of the open “long output” position is by using the same hedge ratio model developed in this chapter and recognizing that the initial long hedge now becomes a short-hedge problem at \( t \). Output could be sold under a forward agreement to reduce the risk of declining prices as \( \tilde{P}_{0,t+n} \) becomes non-random. In that case, the strategic demand component of the hedge position is zero.

**Numerical Analysis**

A numerical analysis is conducted for a processor’s long-hedge strategy (short-cash and long-futures position) in order to explore the response of the model to key variables and parameters, and to highlight the relationship between these variables using hypothetical
parameter values. In this section, the values are not as important as the relationship they reveal through sensitivity analysis. Chapters IV, V, and VI apply the model to particular situations with statistical data from real-world markets. All of the analysis below is *ceteris paribus*, holding all other variables constant.

To carry out the numerical analysis, base values were assumed for each of the parameters. As a further restriction, initially, it is assumed that a unit of input will be processed into a single unit of output (conversion ratio = 1). The initial values are

\[
\begin{align*}
\text{Cov}\left(P_{I,C,t}, P_{I,F,t}\right) &= 230 \\
\text{Cov}\left(P_{O,j+n}, P_{I,F,t}\right) &= 50 \\
\text{Var}\left(P_{I,F,t}\right) &= 250 \\
E[P_{I,F,t}] &= 5.30 \\
P_{I,F,t-m} &= 5.25 \\
\lambda &= 0.5
\end{align*}
\]

Given the initial values of variables, the hedging demand is 0.92; the speculative demand is 0.0004; and the strategic demand is 0.2. The sum of these three components results in the optimal hedge ratio, which is 0.72. This ratio means that 72% of the spot input position should be hedged in the futures markets. Even this simple case demonstrates the effects of correlation between input and output markets on the optimal hedge ratio, which is presented in Figure 3.2.

There is a substantial adjustment in the optimal hedge ratio as a result of including the input-output price relationship. When \( \text{Cov}\left(P_{O,j+n}, P_{I,F,t}\right) \) is zero, the strategic hedge component is also zero, and the optimal hedge is 0.92, as in the traditional approach. However, as the relationship between input and output prices strengthens (higher correlation), the optimal futures position is reduced. Measuring the correlation of input and output prices is an empirical question. The correlation should also be industry and product specific, and related to the level of competition.
The time lag between input and output decisions is also an important factor when measuring the correlation of input and output prices. In any case, it is important to include the relationship between input and output markets in the formulation of hedging strategies in the procurement decisions of the firm. At the extreme, it is conceivable that a very well-defined positive relationship between input and output prices may lead to an optimal risk-management strategy which is simply not to hedge in the futures markets.

Changes in the value of the risk-aversion coefficient, $\lambda$, also illustrate some interesting results. As the firm becomes more risk-averse, the speculative demand for futures approaches zero, and the hedge ratio is 0.72. As $\lambda$ decreases, the firm is less risk-averse, and both the speculative demand and the hedge ratio increase. At the extreme, when $\lambda$ is very small but not yet zero (i.e., 0.0000001), the firm would assume a large position in the futures market. One implication of this analysis is that the value of the risk-
Aversion coefficient can change by a large percentage before having a material effect on the hedge-ratio calculation. This result may make it somewhat easier to assume a certain value of $\lambda$ for a representative firm. The relationship is illustrated in Figure 3.3.

Figure 3.3. The Impact of the Level of Risk-Aversion on Hedge Ratios.

The variance of futures prices, $Var(P_{t,F,t})$, also has an asymptotic effect on the hedge ratio. When the variance of futures prices increases, the hedge ratio approaches zero. The opposite is true when the variance term declines since, as $Var(P_{t,F,t})$ gets closer to zero, the hedge ratio approaches infinity. The results suggest that, all else held constant, if futures markets become extremely volatile, their use as a risk management instrument may decline. For relatively small percentage changes in the variance, the change in the
hedge ratio is more moderate, however, the negative relationship between \( \text{Var}(P_{t,F,t}) \) and the hedge ratio is well defined. This relationship is shown in Figure 3.4.

![Figure 3.4. The Impact of the Variance of Futures Prices on Hedge Ratios.](image)

The relationship between spot and future input markets is reflected by the value of \( \text{Cov}(P_{t,F,t}, P_{t,t}) \). The numerical analysis showed that increases in \( \text{Cov}(P_{t,C,t}, P_{t,F,t}) \) result in increased hedge ratios, as shown by Figure 3.5. One interpretation of this result is that a closer relationship between spot and futures markets leads to a more effective hedge mechanism.

The bias, or expected change in futures between \( t-m \) and \( t \), is the numerator of the speculative demand component of the hedge ratio. When the bias is zero, the speculative demand is also zero. On the other hand, when the bias is large and the risk-aversion parameter is close to zero, the speculative demand increases.
Figure 3.5. The Impact of Cash and Futures Price Covariances on Hedge Ratios.

To conclude, the initial restriction on input-output conversion was lifted. When a single unit of input can be converted into multiple units of outputs, the direction of changes in the number of futures sold still holds for each of the parameters. The only difference is that, as more units of outputs can be produced from a single unit of input, fewer inputs will be needed to produce the same amount of output. The analysis in this area is more relevant when it is applied to actual market data from a particular industry. This analysis will take place in Chapter V.

**Hedging with Multiple Output Products**

Instead of considering only a single output, by-products (such as millfeed) can also be included in the model. The analytical model developed earlier in this chapter can be extended for multiple outputs. Using the same notation as in the previous derivations, with subscripts $I$ and 2 designating outputs 1 and 2, the profit function of the firm is given by
\[
\Pi = \left[ (Q_{O_1,t \rightarrow t+n} + \tilde{P}_{O_{1,t+n}}) + (Q_{O_2,t \rightarrow t+n} + \tilde{P}_{O_{2,t+n}}) \right] - (Q_{I,t \rightarrow t+n} + \tilde{P}_{I,C,t}) + F_t (\tilde{P}_{I,F,t} - P_{1,F,t-m}) - (Q_{I,t \rightarrow t+n} (C_{P/I}))
\] 

(8)

The variance of the profit function is shown by

\[
Var(\Pi) = (Q_{O_1,t \rightarrow t+n})^2 Var(P_{O_{1,t+n}}) + (Q_{O_2,t \rightarrow t+n})^2 Var(P_{O_{2,t+n}}) + (Q_{I,t \rightarrow t+n})^2 Var(P_{I,C,t})
\]

\[
+ F_t^2 Var(P_{I,F,t}) + 2Q_{O_{1,t \rightarrow t+n}}Q_{O_{2,t \rightarrow t+n}} Cov(P_{O_{1,t+n}}, P_{O_{2,t+n}}) - \left[ 2Q_{O_{1,t \rightarrow t+n}}Q_{I,t \rightarrow t+n} Cov(P_{O_{1,t+n}}, P_{I,C,t}) \right]
\]

\[
+ 2Q_{O_{2,t \rightarrow t+n}} F_t Cov(P_{O_{2,t+n}}, P_{I,F,t}) - 2Q_{I,t \rightarrow t+n} F_t Cov(P_{I,C,t}, P_{I,F,t}) \]

(9)

The first order conditions are

\[
\frac{\partial J}{\partial F_t} = E [P_{I,F,t} - P_{1,F,t-m}] - \frac{\lambda}{2} \left[ 2F_t Var(P_{I,F,t}) + 2Q_{O_{1,t \rightarrow t+n}} Cov(P_{O_{1,t+n}}, P_{1,F,t}) \right]
\]

\[
+ 2Q_{O_{2,t \rightarrow t+n}} Cov(P_{O_{2,t+n}}, P_{I,F,t}) - 2Q_{I,t \rightarrow t+n} Cov(P_{I,C,t}, P_{I,F,t}) \right] = 0
\]

(10)

Solving for the futures position \( F_t \) gives

\[
F_t = \frac{Q_{I,t \rightarrow t+n} Cov(P_{I,C,t}, P_{I,F,t}) + E [P_{I,F,t}] - P_{I,F,t-m}}{Var(P_{I,F,t})} - \frac{Q_{O_{1,t \rightarrow t+n}} Cov(P_{O_{1,t+n}}, P_{I,F,t})}{Var(P_{I,F,t})}
\]

\[
- \frac{Q_{O_{2,t \rightarrow t+n}} Cov(P_{O_{2,t+n}}, P_{I,F,t})}{Var(P_{I,F,t})}
\]

(11)

There is a further adjustment in the quantity of inputs purchased in the futures markets as a result of correlations between both output product prices and input futures prices. Extending the model to \( n \) outputs would result in \( n \) strategic demand terms.
Hedging Multiple Risks

For importers involved in international transactions, an additional source of uncertainty stems from fluctuation in the exchange rate between their home currency and the currency in which their accounts have to be settled. The profit function is altered to accommodate the additional parameters:

\[
\Pi = \left( Q_{1,t-m+n} \tilde{P}_{1,t+n} \right) + \left( Q_{2,t-m+n} \tilde{P}_{2,t+n} \right) - \left( Q_{1,t-m+n} \tilde{P}_{1,C,t} \tilde{P}_{FX,S,t} \right) + F \left[ \left( \tilde{P}_{FX,S,t} \right) \left( P_{FX,S,t-m} \right) \right] + F \left[ \left( \tilde{P}_{FX,F,t} \right) \left( P_{FX,F,t-m} \right) \right] - \left( Q_{1,t-m+n} \right) \left( C_{1,t} \right)
\]

where the notation is the same as earlier, with the addition of subscripts 1 and 2 representing quantity and price of multiple outputs, \( P_{FX,S,t-m} \) and \( \tilde{P}_{FX,S,t} \) are spot USD/peso exchange rates at times \( t-m \) and \( t \), and \( P_{FX,F,t-m} \) and \( \tilde{P}_{FX,F,t} \) are futures rates for USD/peso at times \( t-m \) and \( t \). Equation 12 allows for two separate output products and for the hedging of the exchange rate risk with a USD/peso futures contract. Exchange rate risk also enters the formulation as part of the wheat futures transactions since the wheat futures are priced and marked-to-market in U.S. dollars.

Summary

The analysis in this chapter illustrates that, in the case when the speculative demand is zero, the minimum variance hedge is not simply the first term (hedging demand), but a combination of the hedging demand and the strategic demand for futures. With a high level of correlation between input and output prices for a given time lag, the minimum variance hedge ratio will be lower than suggested by simply looking at the hedging demand. When the correlation of inputs and outputs is very low, the strategic hedge
component approaches zero, and the risk-minimizing hedge is the same as the hedging
demand.

The model developed in this chapter is similar to previous models in some respects.
However, by including input and output price correlations, it may explain observed
behavior of hedgers better than current models. These questions are discussed in the
context of empirical cases in Chapters IV, V, and VI.

Other relevant theoretical questions relate to finding the optimal amount of output
the company should plan on producing over a given time period. This area is further
explored mathematically in Appendix A.
IV. EMPIRICAL PROCEDURES

The empirical model applies the theoretical framework to real-world situations in the form of business cases. The theoretical model developed in Chapter III is applied to successive stages of the grain handling and processing industry in order to illustrate the effects of input-output price correlations and the hedge horizon on the minimum-risk and utility-maximizing hedge ratios. Four sectors were selected to represent successive stages of the value chain in agribusiness. The specific sectors covered are U.S. domestic grain trading, flour milling, and bread baking as well as Mexican flour milling. A detailed description of the data and procedures used for each case are presented in this chapter.

The purpose of the empirical analysis is to examine the effectiveness of the theoretical model. The effectiveness of the model may be evaluated in terms of its ability to explain the observed risk management behavior of firms. In addition, through empirical analysis, specific predictions or recommendations may be formulated on risk management strategies to enhance the performance of a firm. This empirical analysis does not focus on price-forecasting techniques or complex econometric estimation of model parameters. Instead, the goal of the analysis is to evaluate the effect of the strategic demand component and the hedge horizon on the risk management strategies of firms in selected industries.

This chapter describes the salient features of four selected case studies, the data, and the process used for parameter estimation. Tables with statistical parameters are included in Chapters V and VI, along with figures representing each price series. The results of the analysis are presented and explained in Chapters V and VI.
Grain Trading

Grain trading was selected as a case study to represent the extreme case where inputs and outputs are essentially the same. The firm performs minimal (if any) processing or value-added activities. Grain traders purchase wheat from farmers or other firms, and sell wheat at other locations, either domestic or export destination at a later time.

Hedging may be motivated by several different reasons. Working (1962) described different categories of hedging, including carrying-charge or arbitrage hedging, operational hedging, selective hedging, anticipatory hedging, and pure risk-avoidance hedging. In the case of grain-traders, the motivation for hedging is most often operational or carrying-charge (Working, 1962). The hedging behavior of flour mills and bakery firms may be better characterized by operational hedging. These firms are typically not trying to gain through arbitrage activities between cash and futures markets; rather, their primary objective is the transference of risk and flexibility in procurement (Blank et al., 1991).

Wheat prices in cash and futures markets typically have a strong positive relationship over time, but the correlation of “input” and “output” prices generally declines as the time lag increases. The correlation between a time series and its lagged values is also referred to as the autocorrelation coefficient (Hanke and Reitsch, 1995).

The empirical models developed in this chapter do not explicitly include transaction costs incurred when buying and selling futures contracts or potential margin calls that may have to be met by the hedger. The model also omits the possibility of savings associated with not having to buy cash inputs and store them during the hedging period.

Daily data were collected for Minneapolis Hard Red Spring (HRS) 14% protein wheat cash prices and hard red spring wheat futures prices from the Minneapolis Grain
Exchange (MGE). The data set contained few missing observations; less than 1% of the total closing prices were missing and were omitted from the analysis altogether. The data cover the period beginning January 2, 1990, and ending August 29, 2000, for a total of 2678 observations for each variable. The grain trader is assumed to have a short-cash wheat position and seeks protection from price fluctuations by taking a long position in wheat futures to reduce his/her price risk. This relatively long time-period was selected to contain a wide variety of market environments, including stable as well as volatile market cycles.

The data set used for this analysis includes the relatively more volatile period of 1996 when both futures and cash prices increased significantly. While including this section may suggest that the market is more volatile than what has been the experience during the last three years, it is important to recognize that highly volatile periods could occur again in the future. While market volatility was relatively stable for some years, periods of high price volatility also occurred on a fairly regular basis.

**U.S. Flour Milling**

Flour milling is an important point in the value chain where wheat, the raw commodity, is processed into a value-added product. In this case, weekly data were collected for bulk spring standard patent flour from an industry publication, *Milling & Baking News*, for Fridays between January 5, 1996, and August 25, 2000, for a total of 237 observations for each variable. The prices are in dollars per hundredweight for Minneapolis spring standard patent flour and dollars per bushel for wheat. The miller uses 14% protein Minneapolis HRS cash wheat and MGE wheat futures to hedge the resulting risk exposure.
Statistics for average mill size and processing capacity were taken from the 2000 Grain & Milling Annual published by Milling & Baking News. The average production capacity per mill was approximately 7,500 hundredweight of flour per day. It was assumed that the processing ratio is 1.38 units of wheat per one unit of flour and that a bushel of wheat weighs approximately 60 lbs.

Given these relationships, the number of bushels of wheat required to produce the daily flour output is found by multiplying each hundredweight of flour by 2.3, which gives the daily input requirement to be approximately 17,250 bushels of wheat for the mill. The volumes of output and input of the mill are given by Table 4.1.

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Days</th>
<th>Flour cwt.</th>
<th>Wheat bu.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>37,500</td>
<td>86,250</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>75,000</td>
<td>172,500</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>150,000</td>
<td>345,000</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>375,000</td>
<td>862,500</td>
</tr>
</tbody>
</table>

In this example, if management of the mill decides to purchase one month’s total input needs at once at a future date, it would have a short-cash wheat position of 345,000 bushels of wheat, which will be processed into 150,000 hundredweight of flour. With an equal and opposite position in the futures markets, this hedge would consist of purchasing 69 futures contracts (5,000 bushels of wheat per contract).

**U.S. Bread Baking**

Bread baking was selected to represent a grain-based processing sector in the value chain closer to the consumers. The availability of reliable and consistent data dictated the direction of this case study in the analysis.
For the output price series, U.S. city monthly average white pan bread prices were collected for the same period from the Bureau of Labor Statistics. Each series includes 128 observations. Since white pan bread prices were available from the Bureau of Labor Statistics Consumer Price Index only as monthly averages, the flour and wheat futures price series were also converted to a monthly average. Bread prices represent average bread prices from across the United States.

Daily data were collected for Kansas City Board of Trade (KCBT, 2000) nearby Hard Red Winter wheat futures, which were converted into a monthly average for the period January 1990 to August 2000. Weekly data were collected and also converted to monthly averages for Kansas City bakers standard patent bulk flour for same time period. For the input price series (flour), Kansas City flour was used since that variety is often purchased by bakery operations (BakingBusiness.com, 2000). The time series of the data are shown in Figure 5.9 in Chapter V.

The following assumptions were made in order to calculate production capacity of the bakery; approximately 9 ounces of flour were used to produce a 1 pound loaf of white pan bread; a large bakery used approximately 8,000 hundredweight of flour per week; a medium-size bakery used approximately 3,000 hundredweight of flour per week. Conversions between hundredweight of flour and bushels of wheat were calculated using the same ratios as in the case of the flourmill. Tables 4.2 and 4.3 show the bread production, flour needs, and the corresponding bushels of wheat for a representative large and mid-size bakery.
Table 4.2. Production Volume of a Large Bakery.

<table>
<thead>
<tr>
<th>Months</th>
<th>Loafs (millions)</th>
<th>Flour cwt.</th>
<th>Wheat bu. (futures)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.422</td>
<td>8,000</td>
<td>18,400</td>
</tr>
<tr>
<td>2</td>
<td>2.844</td>
<td>16,000</td>
<td>36,800</td>
</tr>
<tr>
<td>3</td>
<td>4.266</td>
<td>24,000</td>
<td>55,200</td>
</tr>
<tr>
<td>6</td>
<td>8.533</td>
<td>48,000</td>
<td>110,400</td>
</tr>
</tbody>
</table>

Table 4.3. Production Volume of a Small Bakery.

<table>
<thead>
<tr>
<th>Months</th>
<th>Loafs (millions)</th>
<th>Flour cwt.</th>
<th>Wheat bu. (futures)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.533</td>
<td>3,000</td>
<td>6,900</td>
</tr>
<tr>
<td>2</td>
<td>1.066</td>
<td>6,000</td>
<td>13,800</td>
</tr>
<tr>
<td>3</td>
<td>1.600</td>
<td>9,000</td>
<td>20,700</td>
</tr>
<tr>
<td>6</td>
<td>3.200</td>
<td>18,000</td>
<td>41,400</td>
</tr>
</tbody>
</table>

**Mexican Flour Milling**

In the case of international transactions, foreign currency exchange rates represent an additional source of risk. This scenario models a Mexican flour-milling operation as an international version of the U.S. flour-milling case. Modeling the hedging decision from the perspective of a Mexican flour mill is similar to modeling a domestic bread baking operation with the addition of exchange rate risk. The domestic case is extended along two dimensions, including the exposure to fluctuations of the US$/peso exchange rate and the presence of multiple output products.

The firm is in the business of milling wheat into flour in Mexico and selling its output for Mexican pesos. Milling by-products (such as mill-feeds) are sold for pesos in Mexico. The processing ratio from wheat to flour is assumed to be approximately 73.5% with the remaining 26.5% accounted for as mill-feed. Inputs are purchased in the United States cash wheat markets and are paid for in U.S. dollars.
The exchange rate risk relates to the price paid for the cash input at time \( t \) and also to the use of dollar denominated wheat futures contracts. It is assumed that the firm uses KCBT wheat futures and peso futures from the Chicago Mercantile Exchange (CME) in its risk management strategy. Transportation costs are implicitly included in the cash prices. The amount of flour produced and the corresponding input needs are calculated by using the numbers from Table 4.2 and the case description of the domestic flour-milling operation.

The sample statistics were measured from a data set provided by a Mexican flour-milling operation. The firm’s activities fit the international flour-milling scenario. The data include monthly average prices for flour and mill-feed, pesos/kilogram for the period, September 1996 to September 2000, for a total of 49 observations. Monthly observations for CME Mexican peso futures were collected for the same time period from TradingCharts.com. (TradingCharts.com, 2000) Monthly average $/bushel cash wheat prices were collected from U.S. Wheat Associates for HRW 11% protein, FOB US Gulf. Kansas City wheat futures prices were also collected for the same time period from the KCBT (KCBT, 2000).

Finding the variance of the products of random variables in an analytical solution is a complex exercise (in the presence of multiple risks). An alternative approach to solve for hedge ratios and gain additional insight into the effects of input-output correlations is utilizing a Monte Carlo stochastic simulation model. This model was developed using the @Risk™ simulation package. As an alternative, it is possible to take an analytical approach to finding optimal hedge ratios in wheat and peso futures. However, because the profit equation involves products of random variables, an analytical solution is only an
approximation at best, rather than an exact solution. The procedure for finding the variance of the profit equation with multiple outputs and multiple hedgeable risk involves calculations with covariance matrices of the random variables, as described in Greene (Greene, 2000). The results from stochastic simulation are also approximations. In this case, the accuracy can be improved by increasing the number of iterations in the simulation.

In order to simulate the outcome of the profit function, probability distributions had to be estimated for the six uncertain variables, including wheat cash and futures, flour and mill-feed prices, and exchange rates for pesos. Instead of estimating distributions of price levels, in this case, the expected change in the price of a variable was used as the basis of the distribution. The software application, @Risk™, allows for the specification of correlations between random variables by using a function which includes a correlation matrix. The model for generating random prices follows the form

$$\tilde{P}_{i,t} = P_{i,t-1}(\tilde{\varphi}_i),$$

where the random variable, $P_i$ (price), at time $t$ depends on the outcome at $t-1$ and a multiplier, $\varphi_i$. The multiplier, $\varphi_i$, is a normally distributed variable, with mean 1 (for the case of zero bias) and a standard deviation measured from price changes from historical data. The outcomes of each of the $P_i$ variables are correlated through the multiplier, $\varphi_i$. This formulation also includes expectations of price movements by adjusting the mean of the multiplier’s distribution.

In order to find minimum-variance and utility-maximizing hedge positions, a Monte Carlo simulation was run with combinations of hedge ratios. Each simulation
involved 2,000 iterations to approximate the profit distribution. As an alternative approach, another simulation software from Palisade Corporation, Risk Optimizer™, was used. This approach may be more accurate, however, it requires several hours of computation for each of the cases, and specific time lags and price expectations. Regardless, it was used as a check against the simulation results.

**Summary**

Four empirical cases were examined in Chapter IV for application of the theoretical model from Chapter III and for a comparison of hedging strategies in successive states of the grain handling and processing industry. Price data were collected and analyzed for statistical characteristics and to serve as inputs for the analytical and simulation models. Chapter V presents the results of the empirical analysis for grain trading, domestic flour milling, and bread baking. The results of the Mexican flour-milling case are presented in Chapter VI.
V. EMPIRICAL RESULTS FOR GRAIN TRADING, DOMESTIC FLOUR MILLING, AND BREAD BAKING

The theoretical model developed in Chapter III was applied to the four case studies which were covered in detail in the preceding chapter. The results of the empirical analysis for the first three cases are reported in this chapter. The results for Mexican flour milling are reported in the following chapter.

A base scenario is constructed, and the results are presented for each case. Departures from the base-scenario are then applied in terms of changes in the bias, price expectations, the hedger’s level of risk-aversion, and some other parameters. Figures and tables for selected results are also included.

The results are reported in the following order. First, figures of price series are shown; then, the statistical characteristics of the markets are summarized. Hedging strategies are also reported and compared in tabular, as well as graphical, format for each case. A sensitivity analysis of crucial parameters is conducted, and the effects on the hedge strategies are reported.

Grain Trading

Sample statistics

Cash and futures prices for Minneapolis wheat are shown in Figure 5.1. There has been a close relationship between Minneapolis 14% protein HRS cash and futures prices over the past 5 years. Due to the high degree of correlation, hedging the price risk of a cash wheat position by using MGE futures appears to be favorable.
Sample statistics were calculated for various combinations of \( m \) (hedging) and \( n \) (processing) periods using price levels. The sample statistics for a four-week hedge are reported in Table 5.1. The variances of cash and futures input prices, \( \text{Var}(P_{t,C,t}) \) and \( \text{Var}(P_{t,F,t}) \), respectively, and the covariance, \( \text{Cov}(P_{t,C,t}, P_{t,F,t}) \), are constant since the time lag, \( n \), does not enter these parameters. These estimates reflect the relationship between cash and futures input prices at a point in time without reference to the output market. The sample statistics of the output price series are then measured relative to cash and futures input prices for various \( n \) period time lags. As \( n \) increases, the correlation between input and output prices typically declines.
Table 5.1. Four Week Hedge Sample Statistics for Grain Trading.

<table>
<thead>
<tr>
<th>n (weeks)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variances:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Var}(P_{I,C,t}) )</td>
<td>0.7540</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Var}(P_{I,F,t}) )</td>
<td>0.5068</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Var}(P_{O,t+n}) )</td>
<td>0.7540</td>
<td>0.7444</td>
<td>0.7357</td>
<td>0.7216</td>
<td>0.6654</td>
</tr>
<tr>
<td><strong>Covariances:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Cov}(P_{I,C,t}, P_{I,F,t}) )</td>
<td>0.6044</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Cov}(P_{I,C,t}, P_{O,t+n}) )</td>
<td>0.7505</td>
<td>0.7317</td>
<td>0.7168</td>
<td>0.6864</td>
<td>0.5689</td>
</tr>
<tr>
<td>( \text{Cov}(P_{I,F,t}, P_{O,t+n}) )</td>
<td>0.6044</td>
<td>0.5947</td>
<td>0.5857</td>
<td>0.5645</td>
<td>0.4691</td>
</tr>
<tr>
<td><strong>Correlations:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Correl}(P_{I,C,t}, P_{I,F,t}) )</td>
<td>0.9822</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Correl}(P_{I,C,t}, P_{O,t+n}) )</td>
<td>1.0000</td>
<td>0.9811</td>
<td>0.9668</td>
<td>0.9348</td>
<td>0.8069</td>
</tr>
<tr>
<td>( \text{Correl}(P_{I,F,t}, P_{O,t+n}) )</td>
<td>0.9822</td>
<td>0.9726</td>
<td>0.9635</td>
<td>0.9378</td>
<td>0.8116</td>
</tr>
</tbody>
</table>

When the sample statistics were measured on price levels for different time-lags, some difficulties were encountered. As the time lags \( m \) and \( n \) are changed, the subset of the data on which the analysis was run also changed. Small shifts in data sets led to some of the time-lags including more volatile periods than others, and in some cases, given the peculiarities of the sample data, volatility declined as the time-lag increased. The variance of output prices, \( \text{Var}(P_{O,t+n}) \), at time \( t+n \) changed with the time-lag since the observations varied slightly with each scenario.

**Base case values**

The base case scenario for grain trading was set up as a benchmark for the analysis. Initial parameters were chosen to allow for zero bias, where \( P_{I,F,t-m} \) equals \( E[P_{I,F,t}] \). In the case of zero bias, the speculative demand is nil; therefore, the level of risk-aversion \( (\lambda) \)
does not have an impact on the hedge ratio. Results were found for combinations of \( m = 1, 2, 4, \) and 10 week hedging periods and \( n = 0, 1, 2, 4, \) and 10 week processing (storage) periods. Since the trader does not process the grain, the input is the same as the output; therefore, \( Q_{I,t \rightarrow t+n} \) equals \( Q_{O,t \rightarrow t+n} \). In the case of the grain trader, positions in futures markets were calculated per bushel of input; therefore, the hedge ratio in percentage terms is the same as the futures position in terms of bushels.

Parameter values for the base case are shown below. The expected values of input and output (buy and sale) prices are not particularly important in terms of the hedge ratio results since these parameters do not enter the equation derived for finding the hedge ratio. They are, however, important in terms of the expected outcome, or profitability, of the transaction. Possible reasons why \( E[P_{I,F,t}] \neq E[P_{I,C,t}] \) may be if the underlying of the futures contract is not exactly the same as the commodity being hedged (cross hedging) or if the futures contract matures later than time \( t \).

\[
\begin{align*}
Q_{I,t \rightarrow t+n} &= 1 & Q_{O,t \rightarrow t+n} &= 1 & E[P_{I,F,t}] &= P_{I,F,t-m} = 2.97 \\
E[P_{I,C,t}] &= 3.37 & E[P_{O,t+n}] &= 3.56 & \lambda &= 0.5
\end{align*}
\]

The two covariance terms are equal in the special case when processing period \( n \) equals zero. In this case, the firm buys and sells the same commodity at the same instant at the same price. This type of transaction would not be likely since, considering the costs associated with trading, at least some spread would be normally required. Since the correlation between input and output prices with a zero lag for the same commodity is 1.00, the covariance terms, \( \text{Cov}(P_{I,C,t}, P_{I,F,t}) \) and \( \text{Cov}(P_{O,t+n}, P_{I,F,t}) \), are equal. As time lag \( n \) increases, the covariance term, \( \text{Cov}(P_{O,t+n}, P_{I,F,t}) \), typically declines.
Hedging results

Table 5.2 shows the calculated values for $F_i$, the futures position for a combination of $m$ and $n$ hedging and processing periods. Equation 5 from Chapter III can be decomposed into hedging, speculative, and strategic demands, and Table 5.2 shows the values of each component for the base scenario. First, the components of the hedge position are shown separately then summed to obtain the values for $F_i$. The sign of the hedging demand is positive, but the sign of the strategic demand is negative. In the case of zero bias, the risk-minimizing and utility-maximizing (optimal) hedge ratios are equal.

Table 5.2. Base Case Results for Grain Trading.

<table>
<thead>
<tr>
<th>m (weeks)</th>
<th>n=0</th>
<th>n=1</th>
<th>n=2</th>
<th>n=4</th>
<th>n=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedging Demand (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
</tr>
<tr>
<td>2</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
</tr>
<tr>
<td>4</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
</tr>
<tr>
<td>10</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
</tr>
<tr>
<td>Speculative Demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Strategic Demand (-)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.1945</td>
<td>1.1792</td>
<td>1.1653</td>
<td>1.1222</td>
<td>0.9400</td>
</tr>
<tr>
<td>2</td>
<td>1.1937</td>
<td>1.1793</td>
<td>1.1627</td>
<td>1.1182</td>
<td>0.9365</td>
</tr>
<tr>
<td>4</td>
<td>1.1926</td>
<td>1.1734</td>
<td>1.1557</td>
<td>1.1139</td>
<td>0.9258</td>
</tr>
<tr>
<td>10</td>
<td>1.1865</td>
<td>1.1676</td>
<td>1.1508</td>
<td>1.1008</td>
<td>0.8218</td>
</tr>
</tbody>
</table>

$F_i$

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<td>0.0153</td>
<td>0.0291</td>
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<td>0.2545</td>
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<td>0.0310</td>
<td>0.0755</td>
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<td>4</td>
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<td>0.0369</td>
<td>0.0787</td>
<td>0.2668</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>0.0189</td>
<td>0.0358</td>
<td>0.0857</td>
<td>0.3648</td>
</tr>
</tbody>
</table>
As shown by Table 5.2, when \( n = 0 \), the optimal position in futures, \( (F) \), is zero since buying and selling the same commodity simultaneously at the same price involves no risk. As the holding period \( n \) increases, \( F \) also increases. The increase in hedging is due to the declining correlation between “input” and “output” prices as time-lag \( n \) increases. Since speculative demand is driven by expectations, when the bias term equals zero, the speculative demand also equals zero.

In this case, the optimal (minimum risk) hedge ratio is the combination of the hedging and strategic demand components, which partially offset one another. For any hedging period, \( m \), the hedging demand remains the same for each possible holding period, \( n \). The strategic demand causes a significant downward adjustment of the overall hedge position. The size of this adjustment (the strategic demand) depends on the correlation between “input” and “output” markets and tends to decline as \( n \) increases. Figure 5.2 shows the results from Table 5.2.

In most trading operations, the combination of highly uncertain markets and low operating margins leads to firms taking “equal and opposite” positions in cash and futures markets. Equal and opposite positions mean that the trader is only considering hedging demand and that hedging demand is equal to 100% of the underlying cash position. The fully hedged strategy is especially relevant when a firm makes a sale for forward delivery and purchases cash “inputs” close to the time of making delivery on the contract. In this scenario, output prices are fixed at the contracted level, and input prices remain uncertain. In other words, input and output prices have a zero correlation when outputs are sold forward at a fixed price, leading to a reduction in the size of the strategic demand factor and increasing the importance of hedging demand.
Since the variance of output prices and, thus, input-output price correlations are zero in the case of fixed output prices (Strategic demand is also nil.), the firm takes a position in futures markets equal to the hedging demand. However, in the formulation of the theoretical model, the firm is allowed to take a position in anticipation of making a sale in cash output markets at a future time. Outputs are priced at $t+n$. The resulting adjustment in the hedge position is due to changes in the strategic demand factor.

In the case where input and output positions are zero, both the hedging demand and the speculative demand must be zero. In this case, the only reason for futures trading would be for speculative purposes, as long as the speculative demand is non-zero.

![Figure 5.2. Comparison of Optimal Hedge Ratios for Grain Trading Base Case.](image)

Figure 5.2 shows a graphical representation of the results from Table 5.2. As the time-lag increases, the strategic demand declines since there is a weaker relationship between input and output prices. Also, when the bias term is zero, the E-V curve becomes
a single point, which represents the lowest risk portfolio for that particular level of expected profit. These results illustrate the fact that the optimal hedge ratio for each value of $\lambda$ must be the same as the risk-minimizing hedge ratio when the bias is zero. Hedge ratios tend to increase with longer time horizons as the correlation between “input” and “output” prices declines with longer time lags. Lower input-output correlations result in a larger futures position as the size of strategic demand declines.

When $\lambda$, the risk-aversion parameter, approaches infinity, the agent wants to minimize risk. In that case, the hedger would take a zero speculative position. Therefore, the combination of hedging and strategic demand factors would be referred to as the minimum-risk hedge ratio.

**Sensitivity analysis**

Holding everything else constant, if hedgers have bullish price expectations about futures markets ($P_{t,F,t-m} < E(P_{t,F,t})$), their hedging strategies would change. Given the base-case parameters and assuming that the hedger expects the value of futures at $t$ to be $P_{t,F,t} = 3.10$ (futures prices expected to increase), the optimal hedge ratio reflects the speculative expectations and may no longer be risk-minimizing. The hedge positions for different time horizons are shown by Table 5.3.

For a given level of risk-aversion, the hedge ratio changes as the hedger takes speculative positions based on his/her price expectations. The size of the hedging demand and strategic demand are not affected by the bias, only the speculative demand term, as shown by Table 5.3. As a result of positive bias, the speculative demand increases with $m$ but does not change by changing $n$. This adjustment reflects a long position taken in futures due to expectations of increasing futures prices. The speculative demand does not
vary with the time horizon since there is no direct relationship between the bias and output prices (no direct link to the length of \( n \)).

Table 5.3. Results with Positive Bias for Grain Trading for \( \lambda = 0.5 \).

<table>
<thead>
<tr>
<th>M (weeks)</th>
<th>n=0</th>
<th>n=1</th>
<th>n=2</th>
<th>n=4</th>
<th>n=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedging Demand (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
</tr>
<tr>
<td>2</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
</tr>
<tr>
<td>4</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
</tr>
<tr>
<td>10</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
</tr>
<tr>
<td>Speculative Demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.5032</td>
<td>0.5032</td>
<td>0.5032</td>
<td>0.5032</td>
<td>0.5032</td>
</tr>
<tr>
<td>2</td>
<td>0.5062</td>
<td>0.5062</td>
<td>0.5062</td>
<td>0.5062</td>
<td>0.5062</td>
</tr>
<tr>
<td>4</td>
<td>0.5130</td>
<td>0.5130</td>
<td>0.5130</td>
<td>0.5130</td>
<td>0.5130</td>
</tr>
<tr>
<td>10</td>
<td>0.5490</td>
<td>0.5490</td>
<td>0.5490</td>
<td>0.5490</td>
<td>0.5490</td>
</tr>
<tr>
<td>Strategic Demand (-)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.1945</td>
<td>1.1792</td>
<td>1.1653</td>
<td>1.1222</td>
<td>0.9400</td>
</tr>
<tr>
<td>2</td>
<td>1.1937</td>
<td>1.1793</td>
<td>1.1627</td>
<td>1.1182</td>
<td>0.9365</td>
</tr>
<tr>
<td>4</td>
<td>1.1926</td>
<td>1.1734</td>
<td>1.1557</td>
<td>1.1139</td>
<td>0.9258</td>
</tr>
<tr>
<td>10</td>
<td>1.1865</td>
<td>1.1676</td>
<td>1.1508</td>
<td>1.1008</td>
<td>0.8218</td>
</tr>
<tr>
<td>( F_1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.5032</td>
<td>0.5185</td>
<td>0.5323</td>
<td>0.5754</td>
<td>0.7577</td>
</tr>
<tr>
<td>2</td>
<td>0.5062</td>
<td>0.5205</td>
<td>0.5371</td>
<td>0.5817</td>
<td>0.7633</td>
</tr>
<tr>
<td>4</td>
<td>0.5130</td>
<td>0.5322</td>
<td>0.5500</td>
<td>0.5917</td>
<td>0.7798</td>
</tr>
<tr>
<td>10</td>
<td>0.5490</td>
<td>0.5679</td>
<td>0.5848</td>
<td>0.6347</td>
<td>0.9138</td>
</tr>
</tbody>
</table>

As the risk-aversion coefficient decreases (the hedger is less risk-averse), the speculative demand increases, and the hedger takes a larger long position in futures, based on bullish price expectations. The size of hedging demand and strategic demand are not affected by the level of risk-aversion. Figure 5.3 shows the data from Table 5.3. The optimal (utility-maximizing) hedge position \( (F_1) \) is still increasing with the hedge horizon.
as a result of declining correlations. The effects of a positive speculative demand are shown by higher hedge ratios.

Figure 5.3. Utility-Maximizing Hedge Ratios for Grain Trading with Positive Bias.

The E-V frontier was traced out for each scenario to show optimal combinations of expected return and variance with alternative levels of risk-aversion. Figure 5.4 shows an example of the E-V frontiers for a two-week hedge and various $n$ “processing” periods. Typically, the lower segment of the E-V curve is not shown, but it can be traced out and would be optimal for a risk-seeking agent ($\lambda < 0$). The results of sensitivity analysis show, among other things, that the hedge ratio (for $F_1 > 0$) rises consistently as one moves higher along the upper segment of the E-V frontier.

Each point on the E-V frontier represents the expected profit and variance for a particular possible “optimal” portfolio (or combination) of cash and futures positions. In
other words, each point corresponds to a particular optimal hedge ratio. The curves can be traced out by minimizing the variance of profit for each possible level of expected profit.

The E-V frontiers differ for each holding period, $n$, since the correlation between input and output prices changes with the time-lag. In other words, the expected profit and the corresponding variance changes for the same optimal hedge ratio over different time periods.

Figure 5.4. Two-Week Hedge E-V Frontiers for Grain Trading with Positive Bias.

Alternatively, given a specific value of risk-averison, $\lambda$, $F_{r}$ is calculated and the expected value and variance derived. Since each value of $\lambda$ corresponds to a particular optimal hedge position (or each level of $F_{r}$ has a corresponding $\lambda$), the upper segments of the curves (for which $\lambda > 0$) can be called efficient frontiers. Table 5.3 shows that, as the
hedge horizon increases, the minimum-variance hedge ratios increase as a result of reduced strategic demands.

Another transaction routinely carried out in the grain trading industry is the forward sale of wheat at a fixed price. Outputs are priced at \( t-m \) as explained earlier. This case can be interpreted as \( t = t+n \), or, in other words, \( n = 0 \). Since the sale price is fixed, the covariance (or correlation) between “input” and “output” prices, \( \text{Cov}(P_{t,F,t}, P_{O,t+n}) \), is zero, and, as a result, the strategic demand is also zero. The grain trader is only concerned with the risk of fluctuating input prices and wants to seek protection (coverage) from the risk of rising prices. Again, protection is achieved by a hedge ratio that equals the sum of hedging and speculative demands.

**U.S. Flour Milling**

**Sample statistics**

Minneapolis cash and futures wheat prices, and Minneapolis spring standard patent flour prices are shown in Figure 5.5. The relationship between wheat and flour prices appears to be strong, suggesting that wheat futures may be a good risk-management alternative for flour milling operations.

Visual examination of the three price series suggests a strong relationship between wheat and flour prices, and that changes in the price of wheat translate into changes in flour prices. Sample statistics for a four-week hedge \( (m = 4) \) are given in Table 5.4. In this case, mills plan four weeks ahead in their cash input purchase and hedge this short-cash input position in futures markets. The hedging strategy is carried out by purchasing Minneapolis cash wheat at time \( t \) (in the future) and taking a long position in MGE wheat futures at the
present time, \( t-m \). As in the case of the grain trader, the behavior of the mill may be seen as hedging \textit{in anticipation} of selling (pricing) output at a future time, \( t+n \).

![Figure 5.5. Minneapolis Wheat and Flour Prices.](image)

As with the case of a grain trader, volatility was measured on price levels. The time lag of \( n = 0 \) represents the instantaneous relationship between input cash, and futures and output prices. The measure of volatility is an approximation of the average volatility over the time period. A possible improvement could be achieved by calculating an exponentially weighted-average volatility estimate, giving greater weight to more recent observations.

While correlations were not explicitly used in the hedge-ratio calculations, the statistics are included in the table to illustrate the weakening relationship between flour and wheat markets as the time-lag increases. With a time-lag of 10 weeks, Minneapolis flour
prices and wheat futures prices still have a strong correlation of 0.76 versus 0.97 for concurrent prices.

Table 5.4. Four Week Hedge Sample Statistics for Flour Milling

<table>
<thead>
<tr>
<th>n (weeks)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var(P_{I,C,t})</td>
<td>0.7541</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var(P_{I,F,t})</td>
<td>0.5068</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var(P_{O,t+n})</td>
<td>2.7408</td>
<td>2.7058</td>
<td>2.6704</td>
<td>2.6084</td>
<td>2.4137</td>
</tr>
</tbody>
</table>

Covariances:

| Cov(P_{I,C,t}, P_{I,F,t}) | 0.6044 |       |       |       |       |
| Cov(P_{I,C,t}, P_{O,t+n}) | 1.4123 | 1.3719 | 1.3352 | 1.2628 | 1.0102 |
| Cov(P_{I,F,t}, P_{O,t+n}) | 1.1413 | 1.1174 | 1.0958 | 1.0450 | 0.8374 |

Correlations:

| Correl(P_{I,C,t}, P_{I,F,t}) | 0.9823 |       |       |       |       |
| Correl(P_{I,C,t}, P_{O,t+n}) | 0.9869 | 0.9649 | 0.9453 | 0.9046 | 0.7522 |
| Correl(P_{I,F,t}, P_{O,t+n}) | 0.9729 | 0.9586 | 0.9463 | 0.9131 | 0.7606 |

As in the case of the grain trader, the variances of cash and futures markets, \( \text{Var}(P_{I,C,t}) \) and \( \text{Var}(P_{I,F,t}) \), respectively, and the covariance, \( \text{Cov}(P_{I,C,t}, P_{I,F,t}) \), are constant since \( n \), the time lag, does not affect these parameters.

**Base case values**

In the base case, the parameter values reflect a zero bias (\( E[P_{I,F,t}]=P_{I,F,t-m} \)) that leads to the mill optimally taking a risk-minimizing hedge position in the futures markets. Risk management strategies were examined for combinations of \( m = 1, 2, 4, \) and 10 week hedging periods and \( n = 0, 1, 2, 4, \) and 10 week processing periods before output is priced.
Since the mill performs considerable processing of the base commodity, it must purchase 2.3 bushels of wheat to produce 1 hundredweight of flour. Inputs are no longer the same as outputs, and some of the wheat is “lost” in the form of millfeeds. By-products are taken into consideration explicitly in the Mexican flour milling case but are not included in this analysis. Since futures positions are calculated to cover production needs, a hedge ratio in percentage terms can be found by dividing the recommended futures position by the cash input needs that give a hedge ratio of 100% \( \left( \frac{F_t}{Q_{t,t+n}} \right) \).

Parameter values for the base case were selected as:
\[
Q_{t,t+n} = 2.3 \quad Q_{0,t,t+n} = 1 \quad E[P_{t,F,t}] = P_{t,F,t-m} = 2.97
\]

\[
E[P_{t,C,t}] = 3.30 \quad E[P_{O,n}] = 9.10 \quad \lambda = 0.5
\]

Statistics for wheat cash and futures markets are the same as in the grain trading case since price observations were taken for the same time period. The value of the covariance between wheat futures and flour prices, \( Cov(P_{O,t+n},P_{t,F,t}) \), changes with the length of both periods, \( m \) and \( n \). Since the base commodity is processed into flour, the covariance terms, \( Cov(P_{t,C,t},P_{t,F,t}) \) and \( Cov(P_{O,t+n},P_{t,F,t}) \), are no longer equal for \( n = 0 \). The mill is not buying and selling the same good. Again, increasing time-lag \( n \) will typically result in a declining correlation between wheat and flour prices.

Hedging results

Base case results are shown in Table 5.5. As with the grain trading case, when the bias equals zero, the speculative demand is also zero; therefore, the level of risk-aversion does not have an impact on the hedge ratio. The optimal hedge position \( (F_t) \) is the combination of the three components, where the signs of the hedging and speculative
demands are positive and the sign of the strategic demand is negative. Since the three components of the hedge ratio are given in terms of units of inputs needed to produce 1 hundredweight of flour, to get the hedge ratio in percentage form, it must be divided by 2.3. See Chapter IV for a discussion of conversion factors.

It is important to note that, in the scenarios with \( n = 0 \), the input needs are also equal to zero. An instantaneous transaction does not allow the mill to produce any amount of flour. In other words, for the case of \( n = 0 \), the underlying short-cash position is zero; therefore, the futures position is also zero.

Table 5.5. Base Case Results for Flour Milling.

<table>
<thead>
<tr>
<th>m (weeks)</th>
<th>N=0</th>
<th>n=1</th>
<th>n=2</th>
<th>n=4</th>
<th>n=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedging Demand (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
</tr>
<tr>
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<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
</tr>
<tr>
<td>4</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
</tr>
<tr>
<td>10</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
</tr>
<tr>
<td>Speculative Demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0000</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
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<tr>
<td>2</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>0.0000</td>
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<td>Strategic Demand (-)</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0.9486</td>
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<td>0.7302</td>
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<td>0.9781</td>
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<td>0.9466</td>
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<td>0.7272</td>
</tr>
<tr>
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<td>0.9401</td>
<td>0.8965</td>
<td>0.7184</td>
</tr>
<tr>
<td>10</td>
<td>0.9710</td>
<td>0.9504</td>
<td>0.9334</td>
<td>0.8853</td>
<td>0.6240</td>
</tr>
<tr>
<td>( F_1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2177</td>
<td>0.2329</td>
<td>0.2459</td>
<td>0.2879</td>
<td>0.4643</td>
</tr>
<tr>
<td>2</td>
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<td>0.2304</td>
<td>0.2470</td>
<td>0.2912</td>
<td>0.4664</td>
</tr>
<tr>
<td>4</td>
<td>0.2135</td>
<td>0.2340</td>
<td>0.2525</td>
<td>0.2961</td>
<td>0.4743</td>
</tr>
<tr>
<td>10</td>
<td>0.2155</td>
<td>0.2361</td>
<td>0.2531</td>
<td>0.3012</td>
<td>0.5625</td>
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</table>
The strategic demand component represents a significant adjustment to the hedge ratio. The risk-minimizing (and optimal) hedge position varies with the time horizon between 21% and 56% of the underlying short-cash wheat position. It is important to note the effect of the hedge horizon on the minimum-risk hedge ratio. As the time-lag, \( n \), increases, the correlation between input and output prices declines, and the strategic demand component decreases.

As the time-lag from purchasing cash inputs to selling the processed output increases, the strategic demand becomes weaker. The change in strategic demand is due to a weaker relationship between wheat and flour markets over an extended period of time. The hedging demand does not change with the time-lag, \( n \), since it is not included in the covariance parameter.

Similar to the grain trading case, the E-V frontier was traced out for each scenario to show combinations of expected profit and variance from alternative hedge positions. When the bias term is zero, the E-V curve is again reduced to a single point presenting the minimum-variance hedge portfolio. As Figure 5.6 shows, the results from Table 5.5 indicate that hedge ratios tend to increase with longer time horizons. The increase in hedge ratios is due to the declining correlation between input and output prices.

Hedge positions were converted to bushels of wheat using the production figures from Table 4.2. Given the production capacity of a typical mill, the corresponding wheat requirement is first calculated. The wheat futures position is found by multiplying the minimum-risk hedge ratio by the cash wheat requirement. Table 5.6 shows the number of futures contracts the mill would buy, given a four-week hedge and alternative processing
periods. Futures positions are rounded down to the nearest 5,000 bushel since the mill cannot purchase a fraction of a contract.

Figure 5.6. Comparison of Hedge Ratios for Flour Milling Base Case.

In Table 5.6, the ratio between inputs (wheat) and outputs (flour) is fixed, and as the length of the processing period increases, both input and output quantities also increase. It is interesting to note the importance of the hedge horizon in the outcome of the mill’s risk management and procurement strategies. While the hedge ratio changes slightly from 0.2340 for a 1-week processing period to 0.4743 for a 10-week processing period, the amount of futures purchased increases much more significantly from 4 to 81 contracts. In essence, the question of how far forward the mill should purchase its inputs in futures markets not only depends on its attitude toward risk, but also on its ability to finance and maintain futures positions through the period of the hedge.
Table 5.6. Hedging Results for Flour Milling Base Case.

<table>
<thead>
<tr>
<th>m=4 (weeks)</th>
<th>n=0</th>
<th>N=1</th>
<th>N=2</th>
<th>n=4</th>
<th>N=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bu wheat purchased at t:</td>
<td>0</td>
<td>86,250</td>
<td>172,500</td>
<td>345,000</td>
<td>862,500</td>
</tr>
<tr>
<td>Cwt flour priced at t+n:</td>
<td>0</td>
<td>37,500</td>
<td>75,000</td>
<td>150,000</td>
<td>375,000</td>
</tr>
<tr>
<td>Min. Risk Hedge Ratio:</td>
<td>0</td>
<td>0.2340</td>
<td>0.2525</td>
<td>0.2961</td>
<td>0.4743</td>
</tr>
<tr>
<td>Bu wheat futures bought at t-m:</td>
<td>0</td>
<td>20,185</td>
<td>43,563</td>
<td>102,162</td>
<td>409,055</td>
</tr>
<tr>
<td>Number of futures contracts:</td>
<td>0</td>
<td>4</td>
<td>8</td>
<td>20</td>
<td>81</td>
</tr>
</tbody>
</table>

Sensitivity analysis

Holding all other variables constant, expectations of rising futures prices (positive bias) will lead to positive speculative demand. Depending on the level of risk-aversion, the hedger will try to take advantage of his/her expectations by increasing the position in futures contracts. Using the base case parameters, it is assumed that the hedger formulates his/her expectation about the futures price at \( t \) as \( E[P_{t,F,P}]=3.10 \). The utility-maximizing optimal hedge ratios are shown in Table 5.7.

Expectations of rising futures prices (positive bias) lead to a positive speculative demand for processing periods other than \( n = 0 \). The hedging demand and strategic demand did not change from the base case scenario. Since the sign of the speculative demand is positive, it will increase the hedge ratio and, therefore, increase the size of the futures position.
Table 5.7. Results with Positive Bias for Flour Milling for $\lambda=0.5$.

<table>
<thead>
<tr>
<th>m (weeks)</th>
<th>n=0</th>
<th>n=1</th>
<th>n=2</th>
<th>n=4</th>
<th>n=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedging Demand (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
<td>1.1945</td>
</tr>
<tr>
<td>2</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
<td>1.1937</td>
</tr>
<tr>
<td>4</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
<td>1.1926</td>
</tr>
<tr>
<td>10</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
<td>1.1865</td>
</tr>
<tr>
<td>Speculative Demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2188</td>
<td>0.2188</td>
<td>0.2188</td>
<td>0.2188</td>
<td>0.2188</td>
</tr>
<tr>
<td>2</td>
<td>0.2201</td>
<td>0.2201</td>
<td>0.2201</td>
<td>0.2201</td>
<td>0.2201</td>
</tr>
<tr>
<td>4</td>
<td>0.2231</td>
<td>0.2231</td>
<td>0.2231</td>
<td>0.2231</td>
<td>0.2231</td>
</tr>
<tr>
<td>10</td>
<td>0.2387</td>
<td>0.2387</td>
<td>0.2387</td>
<td>0.2387</td>
<td>0.2387</td>
</tr>
<tr>
<td>Strategic Demand (-)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.9768</td>
<td>0.9615</td>
<td>0.9486</td>
<td>0.9066</td>
<td>0.7302</td>
</tr>
<tr>
<td>2</td>
<td>0.9781</td>
<td>0.9632</td>
<td>0.9466</td>
<td>0.9025</td>
<td>0.7272</td>
</tr>
<tr>
<td>4</td>
<td>0.9791</td>
<td>0.9586</td>
<td>0.9401</td>
<td>0.8965</td>
<td>0.7184</td>
</tr>
<tr>
<td>10</td>
<td>0.9710</td>
<td>0.9504</td>
<td>0.9334</td>
<td>0.8853</td>
<td>0.6240</td>
</tr>
<tr>
<td>$F_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.4364</td>
<td>0.4517</td>
<td>0.4647</td>
<td>0.5066</td>
<td>0.6830</td>
</tr>
<tr>
<td>2</td>
<td>0.4356</td>
<td>0.4505</td>
<td>0.4671</td>
<td>0.5112</td>
<td>0.6865</td>
</tr>
<tr>
<td>4</td>
<td>0.4365</td>
<td>0.4571</td>
<td>0.4756</td>
<td>0.5192</td>
<td>0.6973</td>
</tr>
<tr>
<td>10</td>
<td>0.4542</td>
<td>0.4748</td>
<td>0.4918</td>
<td>0.5399</td>
<td>0.8012</td>
</tr>
</tbody>
</table>

Figure 5.7 gives a visual representation of the data from Table 5.7, and Figure 5.8 shows an example of E-V frontiers for a two week hedge ($m = 2; n = 0, 1, 2, 4, 10$). The results show that increasing the time-lag, or the duration of the hedge, results in a larger proportion of input needs being purchased in futures markets at time $t-m$. 

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Figure 5.7. Utility-Maximizing Hedge Ratios for Flour Mill with Positive Bias.

Figure 5.8. Two-Week Hedge E-V Frontiers for Flour Mill with Positive Bias.
Table 5.8 shows the number of futures contracts the hedger would purchase, given the expectation of increasing futures prices. For each processing period (other than \( n = 0 \)), due to the speculative demand, the firm will take a larger optimal futures position than under a purely risk-minimizing scenario (or under zero bias).

**Table 5.8. Results with Bias for Flour Milling.**

<table>
<thead>
<tr>
<th>m=4 (weeks)</th>
<th>n=0</th>
<th>n=1</th>
<th>n=2</th>
<th>n=4</th>
<th>n=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bu wheat purchased at ( t ):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>86,250</td>
<td>172,500</td>
<td>345,000</td>
<td>862,500</td>
</tr>
<tr>
<td>Cwt flour produced by ( t+n ):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>37,500</td>
<td>75,000</td>
<td>150,000</td>
<td>375,000</td>
</tr>
<tr>
<td>Utility Max. Hedge Ratio:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.4571</td>
<td>0.4756</td>
<td>0.5192</td>
<td>0.6973</td>
</tr>
<tr>
<td>Bu wheat futures bought at ( t-m ):</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>39,423</td>
<td>82,039</td>
<td>179,115</td>
<td>601,436</td>
</tr>
<tr>
<td>Number of futures contracts:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>7</td>
<td>16</td>
<td>35</td>
<td>120</td>
</tr>
</tbody>
</table>

Similar to the grain trader, the flour mill normally has the option to forward contract its output (for delivery at \( t+n \)) by pricing it at \( t-m \). By fixing flour prices, the variance of output prices, \( \text{Var}(P_{O,t+n}) \), and the covariance between input futures and outputs, \( \text{Cov}(P_{O,t+n}, P_{I,F,t}) \), are both zero. In this case, the strategic demand diminishes, and the mill takes a futures position equal to the sum of its hedging and speculative demands.

**U.S. Bread Baking Statistics**

Figures 5.9 and 5.10 show monthly average prices for Kansas City bakers standard patent flour, KCBT HRW wheat futures, and white pan bread (U.S. average, dollars per one pound loaf). Flour (dollars per 0.005625 cwt) and wheat (dollars per 0.0129375 bushel) prices are shown for the approximate equivalent of inputs for baking a one pound loaf.
white pan bread. In Figure 5.9, wheat and flour prices are measured against the left vertical axis, and bread prices are measured against the right vertical axis.

The relationship between wheat and flour prices is apparent, however, neither of the two series appear to be strongly related to bread prices. The low correlation between flour and bread prices is likely caused by fluctuations in some of the other cost factors in bread production, such as energy, labor, distribution, or capital expenditures.

Figure 5.10 shows the relative weight of flour prices in the cost structure of a typical bakery operation. All of the price series are graphed relative to the left axis. While wheat and bread prices fluctuate over time (with a high correlation), flour prices represent only a minor and relatively stable portion of total costs. Depending on the profit margins of the firm, it may still be advantageous to protect itself from increasing flour prices by hedging in wheat futures markets.
Sample statistics for a three-month hedge \( (m = 3) \) are given in Table 5.9. The parameters from Table 5.9 show that, by the time a base commodity (wheat or flour) is processed into a consumer good (bread), the correlation between input and output prices is weaker than higher up in the value chain. In this case, flour input needs are cross-hedged in KCBT wheat futures contracts. While the relationship between bread prices (the output) and wheat futures (the hedge instrument) is weaker, wheat prices are highly correlated with flour prices.

As with the previous two cases, the variance of cash and futures markets, 
\[
Var(P_{t,C,t}) \quad \text{and} \quad Var(P_{t,F,t}),
\]
and the covariance, 
\[
Cov(P_{t,C,t}, P_{t,F,t}),
\]
remain constant since \( n \), the time-lag, does not affect these parameters.

Table 5.9. Three-Month Hedge Sample Statistics for Bread Baking.
Base case values

The base case scenario for the bread baking case assumes zero bias by the hedger. In that case, the expected futures price at the time the hedge is lifted, \( E[P_{t,m}] \), equals the futures price when the hedge is entered, \( P_{t,m} \), and the speculative demand component of the hedge strategy becomes zero. The base case is set up to allow for the examination of a minimum risk hedge ratio without reference to the hedger’s level of risk-aversion.

As in the case of the grain trader and flour miller, the baker makes a hedging decision at time \( t-m \). At this time, he or she anticipates selling flour at a future date, \( t+n \). Inputs (flour and other inputs) will be purchased at \( t \). The hedge is placed in wheat futures from \( t-m \) to \( t \).

Results were found for \( m = 1, 2, 3, \) and 6 month hedging periods and \( n = 0, 1, 2, 3, \) and 6 month processing periods. In this case, the firm purchases flour at time \( t \) and processes it into bread during \( m \) periods. The output (bread) is priced at \( t+n \). Inputs,
represented by \( Q_{I,t \rightarrow t+n} \), are not the same commodity as outputs, \( Q_{O,t \rightarrow t+n} \), and there is significant additional value among wheat, flour, and bread in the vertical value chain.

The parameter values for the base case were selected as

\[
Q_{I,t \rightarrow t+n} = 0.005625 \quad Q_{O,t \rightarrow t+n} = 1 \quad E[P_{I,F,t}] = P_{I,F,t-m} = $2.80/bu
\]

\[
E[P_{I,C,t}] = $8.5/cwt \quad E[P_{O,t+n}] = 0.60/\text{lb bread} \quad \lambda = 0.5
\]

The input price of 8.5 dollars per hundredweight of flour translates into approximately 4.7 cents of flour-related expenses per loaf of bread. In addition, 45 cents are assumed to represent all other processing-related costs, including those for other ingredients. The output price, 60 cents, represents typical wholesale prices per loaf of bread.

Table 5.10 shows the results for \( F_i \), the optimal futures position for a combination of \( m \) and \( n \) hedging and processing periods. The hedge ratios can be interpreted as the percentage of wheat equivalents of inputs (flour) to produce a one-pound loaf bread that is purchased in futures markets. In other words, for 1 loaf of bread, 0.005625 hundredweight of flour is used, which is equivalent to around 0.0129375 bushels of wheat. Therefore, the value of \( F_i / 0.0129375 \) gives the optimal hedge ratio in percentage form.

Results are also shown in Figure 5.11 and indicate that the bakery would minimize overall risk by taking a short position in wheat futures. While these findings seem counter-intuitive, further investigation of the hedging and strategic demand factors illuminates the situation. Since the correlation between KCBT wheat futures and bread prices has been positive over the past decade, the strategic demand term has a negative sign, or in other words, it represents a downward adjustment to the overall hedge position.
Table 5.10. Base Case Results for Bread Baking.

<table>
<thead>
<tr>
<th>m (months)</th>
<th>n=0</th>
<th>n=1</th>
<th>n=2</th>
<th>n=3</th>
<th>n=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedging Demand (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.9003</td>
<td>0.9003</td>
<td>0.9003</td>
<td>0.9003</td>
<td>0.9003</td>
</tr>
<tr>
<td>2</td>
<td>0.8995</td>
<td>0.8995</td>
<td>0.8995</td>
<td>0.8995</td>
<td>0.8995</td>
</tr>
<tr>
<td>3</td>
<td>0.8954</td>
<td>0.8954</td>
<td>0.8954</td>
<td>0.8954</td>
<td>0.8954</td>
</tr>
<tr>
<td>6</td>
<td>0.8922</td>
<td>0.8922</td>
<td>0.8922</td>
<td>0.8922</td>
<td>0.8922</td>
</tr>
<tr>
<td>Speculative Demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Strategic Demand (-)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.2777</td>
<td>2.4587</td>
<td>2.6251</td>
<td>2.7184</td>
<td>2.7159</td>
</tr>
<tr>
<td>2</td>
<td>2.1881</td>
<td>2.3652</td>
<td>2.5308</td>
<td>2.6197</td>
<td>2.5966</td>
</tr>
<tr>
<td>3</td>
<td>2.0536</td>
<td>2.2306</td>
<td>2.3876</td>
<td>2.4604</td>
<td>2.4529</td>
</tr>
<tr>
<td>6</td>
<td>1.6488</td>
<td>1.8037</td>
<td>1.9489</td>
<td>2.0312</td>
<td>2.0318</td>
</tr>
<tr>
<td>$F_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-1.3774</td>
<td>-1.5584</td>
<td>-1.7249</td>
<td>-1.8181</td>
<td>-1.8156</td>
</tr>
<tr>
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<td>-1.2887</td>
<td>-1.4657</td>
<td>-1.6313</td>
<td>-1.7203</td>
<td>-1.6971</td>
</tr>
<tr>
<td>3</td>
<td>-1.1581</td>
<td>-1.3351</td>
<td>-1.4922</td>
<td>-1.5649</td>
<td>-1.5574</td>
</tr>
<tr>
<td>6</td>
<td>-0.7566</td>
<td>-0.9116</td>
<td>-1.0567</td>
<td>-1.1390</td>
<td>-1.1397</td>
</tr>
</tbody>
</table>

Since the strategic demand exceeds hedging demand, the firm takes a net short-wheat futures position. It is important to note that the sign of the strategic demand depends on the correlation between bread and wheat futures prices. When the correlation is positive, the strategic demand causes a negative adjustment of the futures position, but it causes a positive adjustment when correlations are negative.
Sensitivity analysis

The outcome of the hedging strategy is highly dependent on the correlation between inputs and outputs. To contrast with the base case, the same analysis was repeated on a section of the data from January 1996 to August 2000. During this period, bread prices exhibited a general upward trend while wheat and flour prices were gradually declining. These relationships resulted in a negative correlation between input and output prices, and had the net effect of suggesting a larger long position (hedge ratio between 2.00 and 3.00). These results were mainly due to the negative sign of strategic demand, which represented an upward adjustment of the futures position.

To examine the effects of changing correlation on the hedge position, the covariance between input and output prices was first converted into a correlation coefficient using the expression $\rho_{xy} = \sigma_{xy} / \sigma_x \sigma_y$. For illustration, the results for a three-week hedge and two-week processing period ($m = 3$ and $n = 2$) are presented, assuming
that the base case variances do not change. The correlations and covariances as well as the results for the optimal hedge position are presented in Table 5.11.

Table 5.11  Sensitivity Analysis of Correlations for Bread Baking.

<table>
<thead>
<tr>
<th></th>
<th>$\rho = -0.4$</th>
<th>$\rho = 0$</th>
<th>$\rho = 0.4$</th>
<th>$\rho = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Var(P_{t,F,t})$</td>
<td>0.5832</td>
<td>0.5832</td>
<td>0.5832</td>
<td>0.5832</td>
</tr>
<tr>
<td>$Var(P_{O,t+n})$</td>
<td>0.0047</td>
<td>0.0047</td>
<td>0.0047</td>
<td>0.0047</td>
</tr>
<tr>
<td>$Cov(P_{t,F,t}, P_{O,t+n})$</td>
<td>-0.0209</td>
<td>0.0000</td>
<td>0.0209</td>
<td>0.0418</td>
</tr>
<tr>
<td>Hedging Demand</td>
<td>0.8954</td>
<td>0.8954</td>
<td>0.8954</td>
<td>0.8954</td>
</tr>
<tr>
<td>Speculative Demand</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Strategic Demand</td>
<td>-2.7754</td>
<td>0.0000</td>
<td>2.7754</td>
<td>2.5507</td>
</tr>
<tr>
<td>Min. Var. Hedge Ratio</td>
<td>3.6707</td>
<td>0.8954</td>
<td>-1.8799</td>
<td>-4.6553</td>
</tr>
</tbody>
</table>

A logical explanation of the results is that the bakery does not simply hedge input costs but rather takes an operation-wide view of its risk management strategy. Given the upward trend in bread prices, if correlations between input and output markets are expected to be positive, the firm would take a short position in wheat futures. In other words, not only the input side, but the input and the output side are considered together in the formulation of the overall risk management strategy.

The firm is buying wheat futures to hedge its input needs and selling wheat futures effectively to hedge its output price risk. The net position depends on the size of the strategic demand which, in turn, depends on the degree of correlation between input and output markets. Table 5.12 shows the number of futures contracts bought (sold) at time $t-m$ for the base case scenario.
Table 5.12  Three-Month Hedge Summary of Results for Bread Baking.

<table>
<thead>
<tr>
<th></th>
<th>m=3 (months)</th>
<th>n=0</th>
<th>n=1</th>
<th>n=2</th>
<th>n=3</th>
<th>N=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cwt flour purchased at ( t ):</td>
<td></td>
<td>0</td>
<td>8,000</td>
<td>16,000</td>
<td>24,000</td>
<td>48,000</td>
</tr>
<tr>
<td>Millions of loafs of bread produced by ( t+n ):</td>
<td></td>
<td>0</td>
<td>1.422</td>
<td>2.844</td>
<td>4.286</td>
<td>8.533</td>
</tr>
<tr>
<td>Hedge Ratio:</td>
<td></td>
<td>-1.1581</td>
<td>-1.3351</td>
<td>-1.4922</td>
<td>-1.5649</td>
<td>-1.5374</td>
</tr>
<tr>
<td>Bu wheat futures bought at ( t-m ):</td>
<td></td>
<td>0</td>
<td>-24,566</td>
<td>-54,913</td>
<td>-86,382</td>
<td>-169,729</td>
</tr>
<tr>
<td>Number of futures contracts:</td>
<td></td>
<td>0</td>
<td>-4</td>
<td>-11</td>
<td>-17</td>
<td>-33</td>
</tr>
</tbody>
</table>

In the special case that output price (bread) is set by a forward contract, the price of output would be known with certainty at the time the hedging decision is made (at \( t-m \)). In this case, there would be no output price risk. Similar to the results from the grain-trading and flour-milling industries, the variance of output prices and the covariance converge to zero, which leads to a zero strategic demand. Therefore, when output is priced, the hedge ratio is the sum of hedging and speculative demands.

The bakery firm can also form an opinion about the direction of prices in wheat futures markets and take its expectations into account through a non-zero bias. Holding all else constant (\textit{ceteris paribus}), non-zero bias leads to non-zero speculative demand as the firm acts on its opinion through taking a speculative position in wheat futures. The size of this position is determined by the firm’s attitude towards risk as characterized by the risk aversion parameter, \( \lambda \) (and by its corporate risk committee that limits speculative activities). Assuming expected futures prices are \( E[P_{f,F}]=2.82/bu \), (a two cent increase), the results are presented in Table 5.13.
Table 5.13. Results with Positive Bias for Bread Baking for $\lambda=0.5$

<table>
<thead>
<tr>
<th>m (months)</th>
<th>n=0</th>
<th>n=1</th>
<th>N=2</th>
<th>n=3</th>
<th>n=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedging Demand (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.9003</td>
<td>0.9003</td>
<td>0.9003</td>
<td>0.9003</td>
<td>0.9003</td>
</tr>
<tr>
<td>2</td>
<td>0.8995</td>
<td>0.8995</td>
<td>0.8995</td>
<td>0.8995</td>
<td>0.8995</td>
</tr>
<tr>
<td>3</td>
<td>0.8954</td>
<td>0.8954</td>
<td>0.8954</td>
<td>0.8954</td>
<td>0.8954</td>
</tr>
<tr>
<td>6</td>
<td>0.8922</td>
<td>0.8922</td>
<td>0.8922</td>
<td>0.8922</td>
<td>0.8922</td>
</tr>
<tr>
<td>Speculative Demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5.4183</td>
<td>5.4183</td>
<td>5.4183</td>
<td>5.4183</td>
<td>5.4183</td>
</tr>
<tr>
<td>2</td>
<td>5.3657</td>
<td>5.3657</td>
<td>5.3657</td>
<td>5.3657</td>
<td>5.3657</td>
</tr>
<tr>
<td>3</td>
<td>5.3016</td>
<td>5.3016</td>
<td>5.3016</td>
<td>5.3016</td>
<td>5.3016</td>
</tr>
<tr>
<td>6</td>
<td>5.1549</td>
<td>5.1549</td>
<td>5.1549</td>
<td>5.1549</td>
<td>5.1549</td>
</tr>
<tr>
<td>Strategic Demand (-)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.2777</td>
<td>2.4587</td>
<td>2.6251</td>
<td>2.7184</td>
<td>2.7159</td>
</tr>
<tr>
<td>2</td>
<td>2.1881</td>
<td>2.3652</td>
<td>2.5308</td>
<td>2.6197</td>
<td>2.5966</td>
</tr>
<tr>
<td>3</td>
<td>2.0536</td>
<td>2.2306</td>
<td>2.3876</td>
<td>2.4604</td>
<td>2.4529</td>
</tr>
<tr>
<td>6</td>
<td>1.6488</td>
<td>1.8037</td>
<td>1.9489</td>
<td>2.0312</td>
<td>2.0318</td>
</tr>
<tr>
<td>$F_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.0410</td>
<td>3.8599</td>
<td>3.6935</td>
<td>3.6002</td>
<td>3.6027</td>
</tr>
<tr>
<td>2</td>
<td>4.0771</td>
<td>3.9000</td>
<td>3.7344</td>
<td>3.6455</td>
<td>3.6686</td>
</tr>
<tr>
<td>3</td>
<td>4.1434</td>
<td>3.9665</td>
<td>3.8094</td>
<td>3.7366</td>
<td>3.7441</td>
</tr>
<tr>
<td>6</td>
<td>4.3983</td>
<td>4.2434</td>
<td>4.0982</td>
<td>4.0159</td>
<td>4.0153</td>
</tr>
</tbody>
</table>

The expectation of a relatively small increase (two cents) in wheat futures prices leads to a rather large hedge position, resulting in an overall long position which is several times as large as the size of underlying input needs. As in the previous cases, the bias has no effect on either hedging or strategic demands. Since the sign of the speculative demand is positive, it will increase the hedge ratio and, therefore, increase the size of the futures position.
These results seem to indicate that the firm is taking rather large positions based on their expectation of rising futures prices. On the other hand, as shown in Figure 5.10, the portion of input costs represented by flour (and implicitly by wheat) are only a small portion of total costs, as opposed to flour milling where a similar position would represent a large portion of overall activities.

Summary

This chapter presented the results from the grain-trading, domestic flour-milling and bread-baking cases. The hedging behavior of these segments in the agribusiness industry was examined and explained from the perspective of hedging, speculative, and strategic demand factors. The correlation between input and output markets, as well as the time-horizon of the decision-making process, was shown to be crucial parameters.

The results indicate that hedging behavior varies across industry segments and that the correlation between “input” and “output” prices changes not only over time, but throughout the value chain. Depending on the industry, outlooks or opinions (the bias) about the direction of movements in futures prices lead to varying degrees of speculative trading. The strategic demand component typically represents a negative adjustment to the conventional hedge position, as the “natural protection” of margins through the parallel fluctuation of input and output prices is taken into account.

As firms determine their hedge horizon, they implicitly determine how much of the future input needs they wish to purchase at a locked in price. Prices are locked in by hedging in futures markets, where the effectiveness of risk management strategies is improved by a close correlation between the cash (or physical) market and the futures contract price.
VI. EMPIRICAL RESULTS FOR MEXICAN FLOUR MILLING

The empirical analysis on Mexican flour milling was conducted to examine the effects of multiple risks on a importer firm’s risk management strategies. In addition to the risk of changing input prices, exchange rate fluctuation and multiple outputs were added to the model. Results were obtained by stochastic simulation of the firm’s payoff function using @Risk™ and Risk Optimizer™ to find optimal positions in both peso and wheat futures contracts. The simulation procedure and the generation of random variables were explained in Chapter IV.

First, the data and the statistical characteristics are reviewed; then, the results of a base case scenario simulation are presented. Various figures show the effects of hedge ratios on the variance of profits, and the cumulative probability distribution functions from alternative hedging strategies. Finally, sensitivity analysis is carried out on several variables.

Statistical Behavior of Key Variables

Figure 6.1 shows Mexican flour and millfeed prices, exchange rates, and U.S. Gulf of Mexico Freight On Board import-port cash wheat prices. All variables except U.S. Gulf cash wheat prices are measured in Mexican pesos on the left vertical axis. Wheat prices are measured on the right vertical axis in U.S. dollars. Sample statistics were calculated for various combinations of \( m = 1, 2, \) and 4 month and \( n = 0, 1, 2, \) and 4 month periods. In order to generate random variables (random prices), the distribution of percentage price changes and correlations were measured from each price series. This estimation process was essentially done on a time series representing percentage returns in all of the variables over each particular combination of \( m \) and \( n \) periods. The standard deviation of returns and
A correlation between the returns in each variable was then calculated to estimate probability distributions.

Figure 6.1. Mexican Flour, Millfeed, U.S. Gulf HRW, and CME Peso Futures.

Sample statistics for a two-month hedge \((m = 2)\) are reported in Table 6.1. The notation is the same as that used in Chapters III and IV. There was a strong positive correlation \((0.9092)\) between Kansas City wheat futures \(\left(P_{1,F,t}\right)\) and U.S. Gulf cash wheat prices \(\left(P_{1,C,t}\right)\). As time-lag \(n\) increases, there is a weakening relationship between input (cash wheat) and output (Mexican flour) prices. The correlation between \(P_{1,C,t}\) and \(P_{0,1+n}\) declined from 0.4539 for \(n = 0\) to 0.2172 for \(n = 4\). The contrary is true for the by-product (Mexican mill-feed), where the correlation between \(P_{1,C,t}\) and \(P_{0,2+n}\) is first negative then slightly positive with a longer time-lag. There was a very weak relationship between spot or futures exchange rates and wheat prices. There is a weak relationship between Mexican
flour and by-product prices which may be explained by the effect of alternative feeds (such as corn) on mill-feed prices.

Table 6.1. Mexican Flour Milling, Two-Month Hedge Sample Statistics.

<table>
<thead>
<tr>
<th>Variables</th>
<th>n=0</th>
<th></th>
<th>n=1</th>
<th></th>
<th>n=2</th>
<th></th>
<th>n=4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash wheat input*</td>
<td>-0.01920</td>
<td></td>
<td>0.0749</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KCBT wheat futures*</td>
<td>-0.01799</td>
<td></td>
<td>0.0701</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexican flour</td>
<td>-0.03021</td>
<td>0.0485</td>
<td>0.0594</td>
<td>0.0677</td>
<td>0.0824</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexican by-product</td>
<td>0.02542</td>
<td>0.1026</td>
<td>0.1352</td>
<td>0.1611</td>
<td>0.1811</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spot peso/$ exch. Rate</td>
<td>0.01008</td>
<td>0.0367</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CME peso futures</td>
<td>0.00864</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0475</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Corr</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr(P_{I,C,t}, P_{I,F,t})</td>
<td>0.9092</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{I,C,t}, P_{O1,t+n})</td>
<td>0.4539</td>
<td>0.3338</td>
<td>0.2784</td>
<td>0.2172</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{I,C,t}, P_{O2,t+n})</td>
<td>-0.1369</td>
<td>-0.0619</td>
<td>0.0864</td>
<td>0.2518</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{C,3}, P_{FX,S,t})</td>
<td>-0.0009</td>
<td>-0.1724</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{I,F,t}, P_{O1,t+n})</td>
<td>-0.4082</td>
<td>0.3189</td>
<td>0.2524</td>
<td>0.1912</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{I,F,t}, P_{O2,t+n})</td>
<td>-0.0348</td>
<td>-0.0297</td>
<td>0.0760</td>
<td>0.2485</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{I,F,t}, P_{FX,S,t})</td>
<td>0.0996</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0896</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{O1,t+n}, P_{O2,t+n})</td>
<td>-0.1024</td>
<td>-0.0603</td>
<td>-0.0340</td>
<td>0.0173</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{O1,t+n}, P_{FX,S,t})</td>
<td>0.3640</td>
<td>0.4623</td>
<td>0.3782</td>
<td>0.3739</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{O1,t+n}, P_{FX,F,t})</td>
<td>0.0724</td>
<td>0.3192</td>
<td>0.3283</td>
<td>0.2654</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{O2,t+n}, P_{FX,S,t})</td>
<td>-0.1601</td>
<td>-0.1363</td>
<td>-0.0464</td>
<td>0.1817</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{O2,t+n}, P_{FX,F,t})</td>
<td>-0.1781</td>
<td>-0.1776</td>
<td>-0.1808</td>
<td>0.0653</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr(P_{FX,S,t}, P_{FX,F,t})</td>
<td>0.7111</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Values are in U.S. dollars. (All other variables are measured in terms of Mexican pesos.)

**Base Case Simulation Results**

The base case scenario was used to determine minimum-variance hedge ratios over various hedge horizons and production periods. Base case values at \( t-m \), the time hedging decisions are made, are shown in Table 6.2.
The simulation procedure was described in detail in Chapter IV. The probability
distribution and correlations of the random variables were estimated from historical data.
The simulation model generated the random variables from a normal distribution, with the
specific correlation, and was used to find the optimal combination of positions in KCBT
wheat and CME peso futures.

Table 6.2. Mexican Flour Milling, Base Case Values.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Gulf FOB import port</td>
<td></td>
</tr>
<tr>
<td>Cash wheat</td>
<td>3.39 $/bushel</td>
</tr>
<tr>
<td>KC wheat futures</td>
<td>3.90 $/bushel</td>
</tr>
<tr>
<td>Mexican flour</td>
<td>2.16 MP/kg</td>
</tr>
<tr>
<td>Mexican by-product</td>
<td>1.11 MP/kg</td>
</tr>
<tr>
<td>Spot exchange rate</td>
<td>9.45 MP/$</td>
</tr>
<tr>
<td>CME MP futures</td>
<td>9.48 MP/$</td>
</tr>
</tbody>
</table>

@Risk™ was used to simulate the firm’s payoff function and find the combination
of hedge ratios in KCBT wheat futures and CME Mexican peso futures contracts that
results in a minimum variance hedge portfolio (risk-minimizing positions). Risk
Optimizer™ was then used to verify the results from the @Risk™ simulations. Table 6.3
shows the simulation results for combinations of \( m \) and \( n \) time periods.

Results from Table 6.3 indicate that the firm should optimally take a long position
in KCBT wheat futures, and either a long or a short position in CME peso futures,
depending on the time horizon of the decision and the correlation of the random variables.
The futures positions for wheat are given as a percentage of the wheat-equivalent cash flour
inputs. Wheat futures appear to be effective in reducing the firm’s risk (variance of profits). The size of the long wheat futures position is different for each combination of \( m \) and \( n \) periods, but generally is approximately 40 to 50% of the underlying cash position.

Table 6.3. Mexican Flour Milling, Base Case Results.

<table>
<thead>
<tr>
<th>m (months)</th>
<th>n=0</th>
<th>n=1</th>
<th>n=2</th>
<th>n=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>m=1</td>
<td>0.3645</td>
<td>0.3739</td>
<td>0.3178</td>
<td>0.1892</td>
</tr>
<tr>
<td>m=2</td>
<td>0.4816</td>
<td>0.5046</td>
<td>0.4771</td>
<td>0.3560</td>
</tr>
<tr>
<td>m=4</td>
<td>0.4533</td>
<td>0.4458</td>
<td>0.4594</td>
<td>0.4895</td>
</tr>
</tbody>
</table>

Min Risk hedge ratio in KC wheat futures (\( F_I \)):

Min Risk hedge ratio in CME peso futures (\( F_{FX} \)):

The estimated hedge ratios in Table 6.3 show that, generally, as the time horizon is increased, less of the input needs (wheat) are hedged. These findings are consistent with the results from Chapter V and the notion that, over a longer time period, output prices would generally adjust to changes in input prices. However, it is important to keep in mind that these results refer to the particular time period over which the data were collected and that the sample statistics, including volatilities and correlations, could change over time, resulting in different minimum-risk hedge ratio estimates.

The size of the position in peso futures is less stable, ranging from small long positions to relatively large short positions. The positions are given as a percentage of the...
underlying spot currency position that is purchased in *CME* peso futures. These results depend on the levels of correlation and probability distributions of each random variable, and do not reflect any expectations about future price changes.

In addition to finding minimum-risk hedge ratios for each time horizon, [@Risk™](#) was also used in select scenarios to find a large number of expected return and variance combinations. The large movements in the size of peso futures positions may better be explained by Figure 6.2 which shows the variance for various (possible optimal) combinations of peso and wheat hedge ratios for \( m = n = 2 \).

![Figure 6.2. Variance of Profits as a Function of Peso and Wheat Hedge Ratios with Zero Bias.](image)
The edges of the graph in Figure 6.2 were highlighted to show the top left and the front side. The variance is interpreted in terms of pesos squared. The arrow indicates the relative position of the surface to the floor of the graph. It is apparent that the choice of hedge ratios in KCBT wheat futures has a large impact on the variance of payoffs. The minimum variance of payoff (in terms of pesos squared) is 8.0185, which is achieved by taking a long position in wheat futures \( F_I = 0.4771 \) and a short position in peso futures \( F_{FX} = -1.6089 \). Deviating from either of these hedge ratios results in an increased variance.

The impact of the peso hedge ratio is much smaller. The results suggest that the risk faced by the Mexican flour milling operation (in terms of pesos squared) can be reduced from 10.3877 without hedging to 8.0185 with the minimum variance hedge ratio. It is not possible to decompose the relative size of hedging, speculative, and strategic demand when using stochastic simulation.

As shown by Figure 6.3, the relationship between expected profits and the hedge ratios is somewhat peculiar compared to a single-hedge ratio model. See grain trading or bread baking. In the case of zero bias, the single-hedge ratio model with no exchange rate risk results in expected profits that are the same for all hedge ratios. In that case, the E-V frontier is reduced to a single point (and the only one optimal hedge portfolio is also the risk-minimizing portfolio).

In the Mexican flour-milling case, where the expectation of futures prices, \( E[P_{I,F,I}] \), is multiplied by the expectation of spot exchange rates, \( E[P_{FX,S,I}] \), the covariance (or correlation) between the random variables results in a slightly different expected profit for
each combination of optimal hedge ratios. This result may be explained by taking the expectation of the firm’s profit function. Given

\[
\Pi = \left[ Q_{O_1,t-n+n} \tilde{P}_{O_1,t+n} \right] + \left[ Q_{O_2,t-n+n} \tilde{P}_{O_2,t+n} \right] - \left( Q_{I_1,t-m} \tilde{P}_{I_1,C,t} \tilde{P}_{FX,S,t} \right) \\
+ F_I \left[ \tilde{P}_{I_1,F,t} \tilde{P}_{FX,S,t} \right] - \left( P_{I_1,F,t-m} P_{FX,S,t-m} \right) + F_{FX} \left[ \tilde{P}_{FX,F,t} \right] - \left( P_{FX,F,t-m} \right) \\
- \left( Q_{I_2\rightarrow n+n} \right) C_{P/1}.
\]

Applying the expectations operator, \( E(xy) = E(x)E(y) + Cov(x,y) \), results in

\[
E(\Pi) = Q_{O_1,t-n+n} E(P_{O_1,t+n}) + Q_{O_2,t-n+n} E(P_{O_2,t+n}) \\
- Q_{I_1,t-m} E(P_{I_1,C,t}) E(P_{FX,S,t}) - Q_{I_2\rightarrow n+n} Cov(P_{I_1,C,t}, P_{FX,S,t}) \\
+ F_I E(P_{I_1,F,t}) E(P_{FX,S,t}) + F_I Cov(P_{I_1,F,t}, P_{FX,S,t}) - F_I \left( P_{I_1,F,t-m} \right) P_{FX,S,t-m} \\
+ F_{FX} E(P_{FX,F,t}) - F_{FX} \left( P_{FX,F,t-m} \right) - \left( Q_{I_2\rightarrow n+n} \right) C_{P/1}.
\]

When bias is zero, \( E[P_{I_1,F,t}] = P_{I_1,F,t-m} \) and \( E[P_{FX,S,t}] = P_{FX,S,t-m} \), we are still left with the covariance term. Since the wheat hedge ratio \( (F_i) \) is multiplied by the covariance term, the expected value differs as the hedge ratio changes. Figure 6.3 shows the values of expected profits for combinations of wheat and peso hedge ratios, again for the case where \( m = n = 2 \). If there is no exchange rate risk, the plane would be reduced to a single point since the hedge ratios would have no effect on the level of expected profits.

While the expected profit changes with different levels of hedge ratios, these changes are comparatively small. The corresponding changes in variance, however, are comparatively large as shown by Figure 6.2 for the same scenario. These relationships and the effects of hedge ratios on risk reduction are better illustrated by Figure 6.4 where expected profit and variance of profit are plotted for each combination of wheat and peso hedge ratios. Each point is the result of an \( @Risk^\text{TM} \) stochastic simulation with a
corresponding portfolio of cash and futures positions. Figure 6.4 illustrates that changes in wheat hedge ratios \( F_i \) have a larger effect on the variance than changes in peso hedge ratios \( F_{FX} \). The graph could be “filled-in” completely within the E-V frontier running simulations with very small incremental changes in both hedge ratios. The “jumps” between clusters of observations are due to the intervals selected for the hedge ratios from simulation to simulation.

Figure 6.3. Expected Profits as a Function of Peso and Wheat Hedge Ratios with Zero Bias (for \( \lambda = 0.5 \) and m=n=2)
When interpreting this graph, it is important to note that the variance is measured in pesos squared. To convert the unit of measurement to pesos, the square root can be taken to arrive at standard deviations. The minimum-variance hedge position is indicated by the arrow on the left side of Figure 6.4. It corresponds to a hedge ratio of 0.4771 for KC wheat and 1.6089 for CME peso futures. Values of the peso hedge ratio are indicated on the right side of the graph and show that most of the effect on expected profit and the variance is due to the size of the wheat futures position.

Figure 6.4. Expected Profits and Variance for Mexican Flour Base Case (Zero Bias)

The E-V frontier is found by tracing out portfolios where variance is minimized for each possible level of expected profit. The combinations of wheat and peso hedge futures positions in Figure 6.4 only show an interval close to the “optimum” risk-minimizing portfolio under the base case assumptions. In this case, the E-V frontier is not well visible.
since relatively large peso futures positions would have to be taken to be on the frontier. This issue is revisited later in this chapter in the sensitivity analysis.

An additional question with hedging involves the amount of risk reduction when compared to a “no-hedge” scenario \( F_i = 0 \) and \( F_{FX} = 0 \). \@Risk™ was used again to simulate the outcome for the no-hedge and minimum-variance \( F_i = 0.4771 \) and \( F_{FX} = 1.6089 \) hedge positions. The resulting cumulative probability density functions are reported by Figure 6.5.

Figure 6.5 shows an approximation of the probability density function of payoffs without hedging and with minimum-variance hedge ratios. The graph illustrates that minimum-variance hedging reduces the dispersion (the variance) in the distribution of profits. The degree of risk reduction is related to the level of correlation between the cash commodity (inputs) and the hedging instrument. The graph also points out that hedging not only reduces the “downside” potential, but also the “upside” potential of profits.
Figure 6.5. Base Case Cumulative Probability Density Functions.

Table 6.4 compares the outcomes of alternative hedging strategies with the minimum-variance hedge ratios. If no hedges were placed, the expected profit would be slightly lower and the variance higher than the minimum risk hedge ratios. These results reflect the correlations between not only the input spot and futures prices, but also the correlation between wheat and peso futures.

Table 6.4. Mexican Flour Milling, Comparison of Hedging Strategies.

<table>
<thead>
<tr>
<th>FI</th>
<th>FFX</th>
<th>E(P)</th>
<th>VaR(P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>9.21</td>
<td>10.39</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>12.91</td>
<td>11.83</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>9.22</td>
<td>10.24</td>
</tr>
<tr>
<td>0.48*</td>
<td>-1.61*</td>
<td>11.01</td>
<td>8.02</td>
</tr>
</tbody>
</table>

*Minimum variance hedge ratios.
Sensitivity Analysis

The analysis was expanded to include the case where the firm expects wheat futures prices to increase during $m$, the hedging period $\left(P_{I,F,t-m} < E[P_{I,F,b}]\right)$. The model was adjusted to include expectations of a 10% increase in Kansas City (KC) wheat futures prices over the hedging period. In this case, the firm may take these expectations into account in its hedge ratio formulation.

For the base case, $\lambda$, the value of the risk-aversion coefficient was set to 0.5. @Risk™ was used to generate expected profits and variances for combinations of hedge ratios in KC wheat and CME peso futures. These numbers were then inserted into the mean-variance utility function for a given level of risk-aversion to find utility-maximizing hedge positions. Figure 6.6 shows the level of utility from each hedge-ratio combination for the firm with increasing wheat futures price expectations.
Figure 6.6. Utility-Maximizing Hedge Ratios for $\lambda = 0.5$.

Figure 6.6 shows how utility is maximized for a particular value of risk-aversion by choosing hedge ratios in KCBT wheat and CME peso futures. The firm with $\lambda = 0.5$ would maximize its utility by taking a long position in wheat futures ($F_t = 1.25$) and a short position in peso futures ($F_{FX} = -2$). By incorporating the expectation of increasing prices, the firm takes a larger long position than when its goal was risk minimizing ($F_t = 0.4771$). The larger long position in wheat futures benefits the firm if wheat futures increase since, if prices increase, a long position would be profitable. Figure 6.6 also shows that the firm’s
utility is not as sensitive to the size of the peso futures position. Even relatively large changes in the position in peso futures (the hedge ratio) lead to small changes in utility.

The risk-aversion coefficient, $\lambda$, could be represented by a different value, depending on the attitude of the firm towards risk. Sensitivity analysis was conducted to examine the adjustment in the size of hedge ratios as a result of changing attitudes toward risk. Beginning with the initial value of $\lambda = 0.5$, higher and lower values of risk-aversion were also tested.

Table 6.5 shows the results for the sensitivity analysis. As the hedger becomes more risk-averse ($\lambda$ increases), both the hedge ratios in KC wheat and CME peso futures approach the risk-minimizing hedge positions. As $\lambda$ nears positive infinity, the utility-maximizing hedge ratios match the risk-minimizing hedge ratios. However, when the firm is less risk-averse ($\lambda$ decreases), it takes a larger long position in wheat futures and also a larger short position in peso futures. These actions reflect the expectation (bias) that wheat futures prices would increase, and given the past correlations between the markets, it also means that a short peso position would benefit the hedger.

Table 6.5. Mexican Flour Milling, Sensitivities for Risk-Aversion.

<table>
<thead>
<tr>
<th>$\lambda$ (risk aversion)</th>
<th>$F_t$ (KC wheat futures)</th>
<th>$F_{FX}$ (CME peso futures)</th>
<th>$E(\Pi)$</th>
<th>VaR(\Pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07</td>
<td>6</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>2</td>
<td>-4.5</td>
<td>16.61</td>
<td>29.41</td>
</tr>
<tr>
<td>0.5</td>
<td>1.25</td>
<td>-3</td>
<td>13.74</td>
<td>13.32</td>
</tr>
<tr>
<td>1</td>
<td>0.75</td>
<td>-2</td>
<td>12.01</td>
<td>8.68</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>-1.75</td>
<td>10.17</td>
<td>8.05</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.47</td>
<td>-1.6</td>
<td>9.24</td>
<td>8.02</td>
</tr>
</tbody>
</table>

* The futures position becomes very large with respect to the underlying exposure.
The expectation of rising wheat futures prices \((ceteris paribus)\) leads the hedger to take positions in both wheat and peso futures in anticipation of an expected return which would be higher than under the “no-bias” scenario. The degree to which firms take their expectations (market outlook) into consideration in the hedging decision again depends on the risk-aversion parameter. The extent to which the positions would be adjusted from the minimum-variance hedge ratio is demonstrated by the size of futures positions shown in Table 6.4. The results were obtained from \(\text{@Risk}^{TM}\) simulations and were approximated to the nearest 10%.

**Strategic Decisions on Prices and Contracting Output**

The relationship between input and output prices is an important factor in the formulation of hedging strategies. To extend the sensitivity analysis to input-output price correlations, two additional cases were considered: output contracting and output price regulation. These two cases involve setting the standard deviation of output prices to zero, and also reducing the correlation between output prices and the random price variables to zero. The miller could contract his output (flour or by-products) before he produces it. Alternatively, in certain countries, government price regulations attempt to fix the price of goods, which has a similar effect on price correlations.

Sensitivity analysis was conducted assuming that the standard deviations of output prices are zero and that all of the correlation terms involving the outputs, \(P_{O1,t+n}\) and \(P_{O2,t+n}\), are also zero. \(\text{Risk Optimizer}^{TM}\) was used to find the minimum-variance hedge ratios under the above restrictions. While changing the hedging period, \(m\), resulted in a different hedge ratio for both wheat and peso futures, changing the processing period, \(n\), did not have an effect on the outcome. Table 6.6 reports the results.
Table 6.6. Mexican Flour Milling, Price Regulation and Output Contracting.

<table>
<thead>
<tr>
<th>$m$ (months)</th>
<th>$F_I$ (KC wheat futures)</th>
<th>$F_{FX}$ (CME peso futures)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.709</td>
<td>-0.084</td>
</tr>
<tr>
<td>2</td>
<td>0.837</td>
<td>-0.368</td>
</tr>
<tr>
<td>4</td>
<td>0.923</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Comparing the results from Table 6.6 with those for the base case in Table 6.3 shows the increase in the long wheat futures position for each hedging period, $m$. This increase is due to the reduced risk in output prices, as firms now take futures positions strictly to manage their input costs. (i.e., Strategic demand is zero.)

Cumulative probability functions are compared in Figure 6.7 for “no-hedge” scenarios for the base case and fixed output prices, as well as minimum-variance hedges for the fixed output price case. When the firm chooses not to hedge, overall risk is as if output prices are fixed (or contracted) without much reduction in the level of expected profits. However, when input prices are hedged in wheat and peso futures, overall risk is substantially reduced.

Again, the potential negative effect of hedging is the reduction of not only the “downside” but also the “upside” potential for profits. These results illustrate that, if the firm is able to lock in favorable output prices through forward contracting, then input cost hedging would be a beneficial and a very important risk-management strategy.
Summary

This chapter extended the empirical analysis to include foreign currency exchange rates and multiple output products. A base case scenario was set up to examine risk-minimizing hedge strategies with sample statistics calculated from historical observations. Additional analysis was conducted on price expectations and the level of risk-aversion. The effects of contracting output prices and government price regulation were also illustrated through an examination of hedging, speculative, and strategic demand factors. It was shown that hedging behavior changes with the length of the time period as well as with the correlation and volatility of markets.

Figure 6.7. Cumulative Probability Density Functions for Fixed Output Prices.
The impact of the hedge horizon on hedging strategies is measured by the size of the strategic demand for futures. This factor also incorporates input-output price correlations, which tend to reduce the need for hedging. The sample statistics measured from market data suggest that the need to cover input positions declines as the time horizon increases (e.g., typically more important to cover the next few week’s exposure than 12 months out).

The need for hedging is significantly influenced by the ability to forward contract outputs. In this case, one can speak of locking in a margin, whereby input costs can be fixed relative to a fixed output price. In this case, both legs of the transaction are priced, and a profit margin will be realized when the hedge is lifted. The only relevant risk faced by the firm is reduced to basis risk, which is measured by the correlation between the underlying price and the relevant futures contract.
VII. CONCLUSION

Risk management is an important function in the agribusiness industry, including producers, traders, and processors. Firms typically hedge their exposure to price risk in futures and options markets to protect profit margins and, in some cases, to secure access to supplies or output markets. The traditional approach is offsetting an exposure in the underlying commodity by taking an *equal and opposite* position in a closely related derivative contract. The goal of the strategy is to match increases (decreases) in the value of the underlying position with decreases (increases) in the value of the derivative. With perfect correlations, this strategy would lead to a neutral outcome.

Since, in the real world, few commodities have an associated derivative contract with a perfect price correlation, hedge-ratio calculations have been the object of extensive research. The hedge ratio, or the size of the position in the derivative contract relative to the position in the underlying commodity, is selected to meet the objectives of the hedger. Two frequently studied objectives are risk minimization and profit maximization. The literature includes numerous derivations and formulations for finding optimal positions under both risk minimization and profit maximization models.

Past studies mostly focused on finding optimal hedge positions for producers and traders for managing the risk of price fluctuations in input or output markets. These models were frequently extended to include multiple inputs or outputs, and several sources of risk. However, it is important to model the decision-making process of a firm including its activities in both input and output markets at the same time.

The dynamic relationship between prices in input and output markets is influenced by several factors, including the nature of competition in a particular industry; the amount
of value added to the output product; and the period of coverage, or *hedge horizon*. All of these issues were examined by including input and output products in the theoretical model and by empirical case studies of successive stages of the agribusiness industry.

The amount of value added influences the degree of correlation between the price of the base commodity and the processed product. The more processing a product is subject to, typically, the lower the correlation between the source commodity and the good. This relationship can be observed when comparing the correlations between wheat prices and flour prices (high correlation) with the correlation between wheat prices and bread prices (lower correlation).

The *hedge horizon* has implications not only in terms of correlations, but also in terms of the amount of inputs (and the locked in price) the firm decides to purchase at a given time. In terms of correlations, the period of time between purchasing inputs and selling processed outputs is important. In some industries, changes in input prices translate into similar changes in output prices almost instantaneously (naturally protecting profit margins) while, in other industries, this adjustment may take longer. The hedge horizon also determines the length of time for which the firm seeks coverage by hedging. These two issues are closely related and are important aspects of this analysis.

The objective of this study was to develop a theoretical model for the risk management decisions of a firm considering *both* input and output markets at the same time within the framework of a single model. The crucial issues were to explore the way input-output price correlations and the hedge horizon affect risk-management strategies. While a number of potential derivative instruments are available for risk-management purposes, this study focused on the use of futures and forward contracts in some cases.
Summary of Results

First, the results of the theoretical model are presented, followed by the results of the analytical model for grain trading, U.S. domestic flour milling, and U.S. bread baking. Finally, the results from the Mexican flour-milling case are presented.

Theoretical model

The theoretical model was developed as an extension of previous models reviewed in Chapter II by explicitly including both input and output markets into the analysis. The firm was assumed to maximize its utility according to a mean-variance utility function by selecting an optimal position in futures contracts. This formulation allows for a risk-aversion parameter, \( \lambda \), which characterizes the degree to which a firm tolerates risk. In addition, the expectations of the hedger (the bias) are also taken into consideration.

The derivation for an optimal hedge ratio (or futures position) is similar to the results of previous models, with the addition of the strategic demand factor as a downward adjustment to the hedge position. The familiar terms from previous models, including the hedging demand and speculative demand, are part of the formulation. The hedging demand is the equivalent of the minimum-variance hedge ratio, and it only depends on the ratio of the covariance between input cash and futures markets to the variance of futures price movements. The speculative demand includes expectations of price movements in futures (the bias) as well as the risk aversion parameter, \( \lambda \).

The strategic demand includes the relationship between price changes in input and output markets. The sign of the strategic demand is negative, and a higher covariance leads to a reduced overall hedge ratio as shown by the numerical analysis in Chapter III. The relationship between input and output prices also depends on the time period, or lag
between procurement decisions and selling processed outputs. The empirical analysis considered these effects by measuring sample statistics on a variety of potential hedge horizons. While the theoretical model results include the relationship between input futures and output markets, the question of how far forward to purchase inputs is best explored within the context of empirical case studies.

The model was also extended to include the possibility of multiple output products. In this case, the results showed that the hedge ratio formulation includes the same hedging and speculative demand terms as well as a strategic demand term representing each of the output products. Additional outputs lead to additional strategic demand terms which, depending on the strength of the relationship between the particular output and input futures, would represent a further downward adjustment to the hedge position.

A model was also developed to include multiple risks, such as input price risk and the uncertainty stemming from foreign exchange rate fluctuations. Allowing for multiple uncertain variables is particularly relevant to importers who engage in international transactions and settle their accounts in a currency other than the currency in which profits are reported. In order to find optimal hedge positions under multiple risks, a Monte Carlo simulation model was developed and used for the Mexican flour-milling case.

**Analytical model results**

Analytical solutions were found for the grain-trading, U.S. flour-milling, and U.S. bread-baking cases. The hedge horizon in each case was defined to reflect the available data in terms of days, weeks, or months. Under the base case scenarios when the bias was assumed to be zero, speculative demand was always zero. The resulting hedge ratios were the combination of the hedging demand adjusted by a non-negative strategic demand.
Typically, the strategic demand had a negative sign, which resulted in a hedge position smaller than the traditional minimum variance hedge.

The hedging demand is not affected by the hedge horizon since parameters that determine the size of hedging demand do not include the time-lag. Similarly, the speculative demand is also unaffected by the hedge horizon. The strategic demand, however, changes with the time-lag since the relationship between input and output prices (as measured by the covariance or correlation) changes with the time-lag. In most instances, the longer the time-lag, the smaller the strategic demand and, therefore, the closer the optimal position is to a traditional risk-minimizing hedge ratio.

The sensitivity analysis showed that expectations of hedgers about movements in prices result in speculative demand. The size of speculative demand is determined not only by these expectations, but also by the degree of risk-aversion of the firm. There is a clear trade-off between expected profits and risk when the firm takes expectations into consideration in the formulation of hedging strategies.

There are no guarantees that expectations will turn out to be correct, and in the case when they turn out incorrect, the hedge can result in financial losses (something the firm wanted to avoid in the first place). The fear of financial loss is perhaps the reason why commodity-trading operations make a clear distinction between hedging and speculative trading. This distinction, or separation, manifests itself not only in the process of decision making, but also in accounting systems (i.e., The recent Financial Accounting Standards Board statement 133 defines specifically what activities can be accounted for as a hedge.) and lines of responsibility (FASB, 2000).
In the case when outputs are sold at a contracted price, the correlation of input and output prices is reduced to zero. Forward contracting outputs is a plausible scenario in all three of the case studies. When input prices are not correlated with output prices, the strategic demand term is zero, and the hedge ratio is determined as the sum of hedging and speculative demands.

As noted in Chapter V, the choice of hedge horizon is also limited by the ability of the firm to finance large futures positions for the duration of the hedge strategy. As the period of coverage of input needs increases, the number of futures used also increases, which could lead to a large capital reserve requirement to meet potential margin calls.

**Mexican flour milling results**

Monte Carlo simulation models were used for the Mexican flour-milling case to examine the effects of multiple risks and multiple output products on the firm’s hedging strategy. Random prices and exchange rates were generated from historical data, and incorporated volatility and correlation estimates among all of the stochastic variables.

The results show that the firm would benefit from hedging in KCBT wheat futures. The hedge ratio is affected by the strategic demand, which is determined by the relationship between input futures and output prices. The model is more complex since inputs and outputs are denominated in different currencies and the firm is faced with the additional risk of foreign exchange rate fluctuations. Mexican peso futures from the CME were considered as a potential risk management tool, but their effect on risk reduction was limited compared to wheat futures.
In addition to the goal of risk minimization, utility-maximizing strategies were also considered. The results indicated that the hedge ratio in KCBT wheat futures had a substantially larger impact on both risk and return than the hedge ratio in peso futures.

When comparing the cumulative probability function of expected profits from a hedged position with that of an unhedged position, it is apparent that risk can be reduced with hedging. While risk is reduced, it is at the “cost” of reduced expected profits since the probability of large profit/loss “swings” is smaller for a fully hedged position.

It was recognized that, in certain situations, output prices could be fixed either through contracting or government regulation. In that case, the correlation between inputs and outputs is reduced to zero, and the firm is exposed to the risk of rising input costs. The results showed that fixed output prices substantially reduced the variance of expected profits, but, at the same time, the probability of earning large profits also decreased.

**Limitations**

The empirical analysis was, in some cases, limited by the availability of reliable price data. In the case of U.S. bread baking and Mexican flour milling, the output price series (bread and flour) were available only as monthly averages. Therefore, futures prices and cash input prices were also converted into monthly averages for the same time period. A potential problem with this approach is that, by averaging prices, especially futures prices, the actual volatility of the price is reduced (averaged out). In other words, the sample statistics measured on monthly averages do not take into account the day-to-day fluctuations and may understate the actual volatility.

The models did not explicitly include transactions costs incurred in carrying out hedging strategies nor did they include costs such as the cost of capital and overhead. An
attempt was made to consider these factors by including a parameter for “all other costs,” but it does not fully reflect the effects of commissions and other trading related expenses on the firm’s profit margin. Throughout the study, the term “profits” was used to refer to the financial outcome of the particular transactions, not necessarily as the overall profitability of the operation over time.

When measuring sample statistics of volatilities and correlations, it is important to note that the results will be highly sensitive to the period selected for the sample. Since volatility and correlations do not hold constant over time, the resulting hedge ratios cannot be assumed to be correct in all situations. A hedge may be entered on the assumption that cash prices are highly correlated with futures prices, but there are no guarantees that, during the period of the hedge, prices will be correlated to exactly the same degree.

A similar issue arises when estimating the parameters of return distributions for the Mexican flour-milling case. Since changes in wheat futures are calculated on monthly averages, large daily movements are potentially not taken into account. Also, due to the relatively small number of observations, the distributions may underestimate the probability of larger market movements (fat tails), especially if the sample period did not include such events.

In the model formulation, it was assumed that all output is sold at time $t+n$ at the same time. A more reasonable assumption may be to recognize that, in many cases, output is sold more frequently, perhaps on a daily basis. In addition, assuming that all output is sold implicitly states that demand will be equal to or more than production.

Futures contracts (and in some cases forward contracts) were the only derivative instruments considered as risk-management tools in this analysis. In reality, a variety of
over the counter (OTC), contract embedded, or exchange traded options may be viable risk-management alternatives.

**Further Study**

An interesting extension of the analysis would be to derive an analytical solution to the multiple-risk hedging model (i.e., Mexican flour-milling case). In this analysis, a Monte Carlo stochastic simulation was used to estimate optimal futures positions under multiple risks. The model could also be extended to specifically address additional sources of risks, such as transportation costs, and to explicitly include transactions costs and commissions. Considering demand for output products as a known parameter may also be improved upon by treating it as a random variable.

The empirical analysis could extend to various stages of the agribusiness industry, which were not included in this study, perhaps considering products based on commodities other than wheat. In addition, other industries could be considered, including metals, energy, or other commodity-based sectors.

One of the most important areas for extension and further study would be the incorporation of options into the theoretical model. Options are widely used derivative instruments, and a comprehensive theoretical model should incorporate them as a viable risk management alternative. The difficulty associated with the nonlinearity found in options has to be overcome.

Creating a model which examines multiple decisions over time would also offer interesting insight. It is rare to find a business that makes one decision during its existence. If multiple decisions are modeled over time, it is conceivable that a firm may make a decision based, in part, by its expectation of the outcome of another decision.
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APPENDIX A
MODEL DERIVATIONS AND EXTENSIONS

Given the profit function
\[ \Pi = \left( Q_{O,J \rightarrow t+n} \tilde{P}_{O,J \rightarrow t+n} \right) - \left( Q_{I,J \rightarrow t+n} \tilde{P}_{I,C,J} \right) + F_t \left( \tilde{P}_{I,F,J} - P_{I,F,J - m} \right) - \left( Q_{O,J \rightarrow t+n} \right) C_{S/O} \]

the variance of profits is expressed as
\[ \text{Var}(\Pi) = \left( Q_{O,J \rightarrow t+n} \right)^2 \text{Var}(P_{O,J \rightarrow t+n}) + \left( Q_{I,J \rightarrow t+n} \right)^2 \text{Var}(P_{I,C,J}) + F_t^2 \text{Var}(P_{I,F,J}) - \\
- \left[ 2Q_{O,J \rightarrow t+n} Q_{I,J \rightarrow t+n} \text{Cov}(P_{O,J \rightarrow t+n}, P_{I,C,J}) \right] + \left[ 2Q_{O,J \rightarrow t+n} F_t \text{Cov}(P_{O,J \rightarrow t+n}, P_{I,F,J}) \right] - \\
- \left[ 2Q_{I,J \rightarrow t+n} F_t \text{Cov}(P_{I,C,J}, P_{I,F,J}) \right]. \]

The objective is to maximize a utility function of the form
\[ \text{Max } J = E(\Pi) - \frac{\lambda}{2} \text{Var}(\Pi). \]

In the case where the only decision variable is the hedge ratio, input and output quantities are considered given. The maximum value of $J$ is found from the first order condition
\[ \frac{\partial J}{\partial F_t} = E \left[ P_{I,F,J} - P_{I,F,J - m} \right] - \frac{\lambda}{2} \left[ 2F_t \text{Var}(P_{I,F,J}) + 2Q_{O,J \rightarrow t+n} \text{Cov}(P_{O,J \rightarrow t+n}, P_{I,F,J}) - \\
- 2Q_{I,J \rightarrow t+n} \text{Cov}(P_{I,C,J}, P_{I,F,J}) \right] = 0. \]

Rearranging the terms and solving for the futures position, the optimal hedge ratio is
\[ F_t = \frac{Q_{t,t+n} \text{Cov}(P_{I,C,t}, P_{I,F,t})}{\text{Var}(P_{I,F,t})} + \frac{E[P_{I,F,t}] - P_{I,F,t-m}}{\lambda \text{Var}(P_{I,F,t})} - \frac{Q_{t,t+n} \text{Cov}(P_{O,t+n}, P_{I,F,t})}{\text{Var}(P_{I,F,t})}. \]

Since the first-order condition is necessary but not sufficient in finding the maximum value of \( J \), the second-order condition is

\[
\frac{\partial^2 J}{\partial F_t^2} = -\lambda \text{Var}(P_{I,F,t}) < 0.
\]

Since \( \lambda \) is positive and the variance is also positive, the second derivative is negative, indicating that the function is concave and the solution represents a maximum. Since the optimal value of \( F_t \) maximizes the function, any other value of \( F_t \) would result in a less than optimal \( J \).

In the derivations above, \( F_t \) represents the number of units of inputs purchased in the futures markets. An alternative way to interpret \( F_t \) is by dividing both sides of the equation by \( Q_{t,t+n} \) to convert it to reflect a hedge ratio

\[
\frac{F_t}{Q_{t,t+n}} = \frac{\text{Cov}(P_{I,C,t}, P_{I,F,t})}{\text{Var}(P_{I,F,t})} + \frac{E[P_{I,F,t}] - P_{I,F,t-m}}{Q_{t,t+n} \lambda \text{Var}(P_{I,F,t})} - \frac{Q_{t,t+n} \text{Cov}(P_{O,t+n}, P_{I,F,t})}{Q_{t,t+n} \text{Var}(P_{I,F,t})}. \]

The results for both of the formulations above are the same. The only difference is that the first one is given in number of input units and that the second is given as the ratio of input futures to input cash commodity.
When the quantity of inputs purchased and output produced are considered decision variables, solving the maximization problem in Appendix A for not only $F_t$, but also $Q_{O,t\rightarrow t+n}$ and $Q_{I,t\rightarrow t+n}$, the first order conditions are

\[
\frac{\partial J}{\partial F_t} = E\left(\tilde{P}_{I,F,t} - P_{I,F,t-m}\right) - \lambda F_t Var\left(P_{I,F,t}\right) - \lambda Q_{O,t\rightarrow t+n} Cov\left(P_{O,t+n}, P_{I,F,t}\right) + \lambda Q_{I,t\rightarrow t+n} Cov\left(P_{I,C,t}, P_{I,F,t}\right) = 0
\]

\[
\frac{\partial J}{\partial Q_{O,t\rightarrow t+n}} = E\left(\tilde{P}_{O,t+n} - C_{S,O}\right) - \lambda Q_{O,t\rightarrow t+n} Var\left(P_{O,t+n}\right) + \lambda Q_{I,t\rightarrow t+n} Cov\left(P_{O,t+n}, P_{I,C,t}\right) - \lambda F_t Cov\left(P_{O,t+n}, P_{I,F,t}\right) = 0
\]

\[
\frac{\partial J}{\partial Q_{I,t\rightarrow t+n}} = E\left(-P_{I,C,t}\right) - \lambda Q_{I,t\rightarrow t+n} Var\left(P_{I,C,t}\right) + \lambda Q_{O,t\rightarrow t+n} Cov\left(P_{O,t+n}, P_{I,C,t}\right) + \lambda F_t Cov\left(P_{I,C,t}, P_{I,F,t}\right) = 0
\]

The system of three equations can be solved for $F_t$, $Q_{O,t\rightarrow t+n}$, and $Q_{I,t\rightarrow t+n}$. Since the resulting equations are considerably more complex and longer than the results in Appendix A, the variables were redefined in terms of $Q_{O,t\rightarrow t+n} = Q$, $Q_{I,t\rightarrow t+n} = R$, $F_t = F$,

\[
E\left[\tilde{P}_{O,t+n}\right] = A, \quad E\left[\tilde{P}_{I,C,t}\right] = S, \quad E\left[\tilde{P}_{I,F,t}\right] = G, \quad P_{I,F,t-m} = H, \quad C_{S,O} = C, \quad \lambda = g, \quad Var\left(P_{O,t+n}\right) = W, \quad Var\left(P_{I,C,t}\right) = T, \quad Var\left(P_{I,F,t}\right) = K, \quad Cov\left(P_{O,t+n}, P_{I,C,t}\right) = V, \quad Cov\left(P_{O,t+n}, P_{I,F,t}\right) = N, \quad Cov\left(P_{I,C,t}, P_{I,F,t}\right) = L.
\]
The closed form solutions to the system of three equations are given by

\[
Q = \frac{-L^2A + L^2C - LVH + LVG + LNS - NTG + NTH - KTC + KTA - VKS}{g(-L^2W + 2LNV + KTW - V^2K - N^2T)}
\]

\[
R = \frac{-(KSW - KVC + KVA + LNC + SN^2 - LHW - LNA - VNG + VNH + LGW)}{g(-L^2W + 2LNV + KTW - V^2K - N^2T)}
\]

\[
F = \frac{-(SLW - TNA + TGW + V^2H - THW - V^2G + VLA - VLC + TNC + SNV)}{g(-L^2W + 2LNV + KTW - V^2K - N^2T)}
\]

An interesting observation can be made about \( g \), the risk-aversion parameter. In contrast with previous models of hedging where an infinitely risk-averse agent’s hedge was the same as the minimum-risk hedge, in this formulation, an infinitely risk-averse agent would not even be involved with any production activities. A business with \( g \) approaching infinity would not be in business.

In order to prove that the solutions represent a maximum to the optimization problem, second order conditions were taken, and the Hessian matrix was constructed

\[
H = \begin{bmatrix}
-l \text{Var}(P_{O,t+n}) & l \text{Cov}(P_{O,t+n}, P_{I,c,j}) & -l \text{Cov}(P_{O,t+n}, P_{I,F,j}) \\
l \text{Cov}(P_{O,t+n}, P_{I,c,j}) & -l \text{Var}(P_{I,c,j}) & l \text{Cov}(P_{I,c,j}, P_{I,F,j}) \\
-l \text{Cov}(P_{O,t+n}, P_{I,F,j}) & l \text{Cov}(P_{I,c,j}, P_{I,F,j}) & -l \text{Var}(P_{I,F,j})
\end{bmatrix}.
\]

The conditions for maximum are

\[
|H| = -l \text{Var}(P_{O,t+n}) < 0
\]
\[ H_2 = \lambda^2 \left[ \text{Var}(P_{O,t+n}) \text{Var}(P_{I,C,t}) - \text{Cov}(P_{O,t+n}, P_{I,C,t}) \right]^2 > 0 \]

\[ H_3 = -\lambda \text{Var}(P_{O,t+n}) \left[ \lambda^2 \text{Var}(P_{I,C,t}) \text{Var}(P_{I,F,t}) - \lambda^2 \text{Cov}(P_{O,t+n}, P_{I,F,t}) \right] \]

\[ -\lambda \text{Cov}(P_{O,t+n}, P_{I,C,t}) \left[ \lambda^2 \text{Cov}(P_{O,t+n}, P_{I,C,t}) \text{Var}(P_{I,F,t}) - \lambda^2 \text{Cov}(P_{O,t+n}, P_{I,F,t}) \text{Cov}(P_{I,C,t}, P_{I,F,t}) \right] \]

\[ -\lambda \text{Cov}(P_{O,t+n}, P_{I,F,t}) \left[ \lambda^2 \text{Cov}(P_{O,t+n}, P_{I,C,t}) \text{Cov}(P_{I,C,t}, P_{I,F,t}) - \lambda^2 \text{Cov}(P_{O,t+n}, P_{I,F,t}) \text{Var}(P_{I,C,t}) \right] < 0. \]

The upper bound of covariance between two variables \( x \) and \( y \) is \( |\sigma_{xy}| = \sigma_x \sigma_y \).

Therefore, the conditions are met, with the exception of the special case when the covariance term equals the product of standard deviations between the two variables. This special case only happens when the correlation coefficient between \( X \) and \( Y \) equals 1.

Solving individually for quantity and futures position as in Appendix A gives

\[ Q_{O,t\rightarrow t+n} = \frac{\text{Cov}(P_{O,t+n}, P_{I,C,t})}{\text{Var}(P_{O,t+n})} + \frac{E[P_{O,t+n}] - C_{S/O}}{\lambda \text{Var}(P_{O,t+n})} - \frac{F_t \text{Cov}(P_{O,t+n}, P_{I,F,t})}{\text{Var}(P_{O,t+n})} \]

\[ Q_{I,t\rightarrow t+n} = \frac{\text{Cov}(P_{O,t+n}, P_{I,C,t})}{\text{Var}(P_{I,C,t})} + \frac{E[P_{I,C,t}] - F_t \text{Cov}(P_{I,C,t}, P_{I,F,t})}{\lambda \text{Var}(P_{I,C,t})} + \frac{F_t \text{Cov}(P_{I,C,t}, P_{I,F,t})}{\text{Var}(P_{I,C,t})} \]

\[ F_t = \frac{\text{Cov}(P_{I,C,t}, P_{I,F,t})}{\text{Var}(P_{I,F,t})} + \frac{E[P_{I,F,t} - P_{I,F,t-m}]}{\lambda \text{Var}(P_{I,F,t})} - \frac{Q_{O,t\rightarrow t+n} \text{Cov}(P_{O,t+n}, P_{I,F,t})}{\text{Var}(P_{I,F,t})}. \]