INVENTORY VALUATION DECISIONS
AND STRATEGY ANALYSIS

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ABSTRACT

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In this study, a contingent claims model was developed to determine optimal inventory storing strategy. The model chooses the optimal inventory strategy that maximizes net present value (NPV) using the Black-Scholes option pricing model. The contingent claims approach maps the demand for an item carried in inventory onto an underlying state variable. In this case, price for wheat storage was mapped onto the underlying commodity that is being milled.

The primary objective of this research is to develop a real options inventory management tool to determine the optimal inventory level along with determining the option value while addressing risk in an economically feasible way. A secondary objective was to compare the contingent claims model results to the economic order quantity (EOQ) model results.

Separate models were conducted for three different grain commodities with high volatility. High-protein spring wheat, amber durum, and milling quality oats are the three commodities used in this study, each of which have their own unique market characteristics.

Results revealed that in the model most closely related to actual industry characteristics, the contingent claims model resulted in greater inventories. However, those results were highly sensitive to market price, volatility, flour price, salvage value, and marginal increases in sales related to an increase in the underlying asset price.
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CHAPTER 1
STATEMENT OF PROBLEM

Introduction

Inventory management is very important to agribusiness firms, with inventory typically comprising between ten to forty-five percent of the total assets. Historically inventory has been looked at as a major cost and source of uncertainty due to the volatility within the commodity market and demand for the value-added product.

The economic order quantity model (EOQ) is one of the fundamental models used in logistics management to determine the optimal order pattern. It seeks to maximize expected profits based on a given distribution of the item under varying types of demand. This conventional approach does not place any asset or option value on inventory carried within the firm.

Standard tools used for strategic investments are limited and might not have managers asking the right questions for investments to succeed and allow future growth. A firm must have a strategic vision that is integrated into a framework of market analysis so they know when their strategic value is increasing.

Also, uncertainty within the marketplace has forced many agribusinesses to find other means of managing inventory. One of these tools is the contingent claims approach using real options. A contingent claims approach investment decision is one that depends on an uncertain outcome. Many managers of strategic investment decisions think of uncertainty as costly, but using the real options approach, this uncertainty creates opportunities for profit.

Agribusiness firms carrying inventory have options. They can either process the inventory or sell the inventory at the market price; therefore, this option creates a
value of the inventory. Using the contingent claims approach, an option is the opportunity to make a decision after the payoffs are known. The real options approach uses financial market values in order to place an accurate value on the real asset. There are three steps to designing and managing strategic investments using real options. First, identify value options in a strategic investment. Second, redesign the investment to better use options. Third, manage the investment proactively through the options created (Amram and Kulatilaka, 1999).

According to Amram and Kulatilaka (1999), the real options approach creates a valuable connection between project level analyses of strategic investments and the corporate strategic vision. First, from the corporate level down to the project level, the real options approach consistently asks the following questions: What value-creating opportunities are unique to this firm? What amount and type of risk must be borne to create this value? What risk can be cast aside? Second, from the project level up, the real options approach creates a framework in which to aggregate project value, risk, and the structure to manage a company’s net exposure to risk. This bottom-up approach is where uncertainty plays a key role in creating a vision for corporate managers.

This contingent claims approach maps the demand and payoffs from inventory policy into the price of the underlying asset. It builds a portfolio of options that replicate the payoffs and then derives the value of an inventory policy using option pricing models in which the value of the policy is contingent on the financial characteristics of the underlying asset.
The contingent claim analysis along with a prototypical agribusiness company with a supply function that contains uncertainty will be simulated in this study. The commodities researched in this study all have high uncertainty in price, quality, and quantity in the marketplace to exploit the value of carrying an inventory. Commodities will not have direct hedge alternatives but they could be cross-hedged. 14+ percent protein spring wheat, durum, and oats will be researched. These commodities have variability in quantity and quality and thus are very problematic.

**Research Problem and Justification**

Historically the EOQ model has been very valuable to agribusinesses in determining the minimum cost of inventory. The EOQ model uses economies of scale to exploit fixed costs. This model seeks to minimize total cost subject to the fixed costs within the supply chain, with the objective of the EOQ model being to optimize lot sizing in order to reduce the cost of satisfying demand. There are many trade-offs between fixed costs that need to be looked at when implementing this model.

Minimizing costs is very beneficial to agribusinesses in order to maximize profits, but these businesses also have to realize the potential for profit from uncertainty. This is where the contingent claim approach of managing inventory using real options will have a strategic advantage over the EOQ model.

This contingent claims approach with options gives the agribusiness that is holding a readily marketable commodity the option to process or sell the commodity on the marketplace after the value is realized. This option creates a value to the
agribusiness in which they can maximize their profit with respect to the uncertainty in the market and their demand.

Currently most agribusiness firms use the economic order quantity model to determine the optimal level of inventories held by the firm under varying types of demand uncertainty. Implementing a recent alternative model using the contingent claims approach of real options is going to help solve the problem of placing an option or strategic value on the inventory rather than treating inventories strictly as a cost. Once the inventories have an option value, the optimal inventory size of a prototypical agribusiness firm will be obtained using the real options model.

Contingent claims strategy is important for several reasons. Commodity inventories have a value. They are valuable to an agribusiness whether they are storing with intent to process or to sell on the open market. The strategic value of this option will depend strongly on the uncertainty of the commodity market. This uncertainty will create value in part because the agribusiness will see a greater investment return. Real options provide the agribusinesses with a strategic opportunity that can affect how inventory is managed.

**Problem Elements**

A potential problem element of the theory is that most of the agribusiness firms potentially will not want to change their inventory management ways in part because they cannot see the value of the contingent claims approach using real options. Using real options requires managers to strategically think in a different way than traditional theory. Real options are a way of thinking in that the commodity has two or more options within the firm. These two options in this case could be to
process or sell the product. This decision depends on the strategic management of the firm. The firms might be in a commodity market that has a low level of uncertainty. Thus, implementing this model will have a smaller effect on the optimal inventory level.

The higher uncertainty within the commodity market creates a higher value using the real options.

**Objective**

The objective of this research is to determine the optimal level of inventory held by a prototypical agribusiness firm under varying types of demand uncertainty and procurement using the contingent claims approach. The contingent claims approach uses the options on future sales. The objective would state the level of inventory which is most valuable to the firm and place an option value on the inventory. To accomplish this objective, the specific sub-objectives of this study are as follows:

1. Determine the relationships between price and quality attributes of the given commodities.

2. Using the information gathered, find the optimal level of inventory using both contingent claims and the EOQ model.

3. Specifically consider which model is most effective in predicting the profitability of inventory decisions, thus providing information for which the inventory model should be used.

This study provides a mechanism for developing a better understanding of the information requirements necessary for agribusinesses to evaluate inventory
management strategies. These information requirements form the basis for implementing the contingent claims approach using real options into the agribusinesses corporate management idea. This information would also help the firm to make more precise decisions when working with commodities that have great volatility in the marketplace.

**Research Method**

Several methods will be used to develop a better understanding of the information requirements for a continent claims approach to inventory management. These methods include a literature review, development of a theoretical model, development of a spreadsheet model, and the sensitivity analysis of the model results for selected scenarios.

The inventory management literature has evolved from several subject areas. The purpose of the literature review is to provide an overview of stockholding agricultural commodities, logistical models of storage, and real options. In addition, insight into firm decision-making will be gained by a review of industrial literature. These literatures, as well as those specifically related to inventory industries in the commodity market, will be integrated into this study.

Based on the literature review, a theoretical model will be developed using real options to find the optimal level of inventory within a firm. The model will reflect logistical costs, procurement costs, and operations within the firm. Once the theoretical model is developed, a spreadsheet-based empirical model will be developed. The model will have areas of uncertainty that will be addressed using a simulation program.
A prototypical agribusiness firm is going to be used in this study. The firm will have an uncertain supply function with a normal distribution. A salvage value will be given to any excess inventory the firm carries. A shortage cost will also be an added cost if the firm does not carry enough products in inventory to meet demand. The prototypical firm will have all the same logistical costs as the majority of agribusiness firms.

Commodity price data will have to be gathered in order to address the problems of volatility in the market. Market information gathered from the exchange is then entered into the model to show how different commodities affect the outcomes. A higher volatility in the marketplace should create a higher option value, thus creating more opportunity for profit within the firm.

A journal article titled “A Contingent Claims Approach to the Inventory Stocking Decisions” by John D. Stowe and Tie Su (1997) is going to be the base article for this model. Research into optimizing this model given a prototypical firm will be added into their current model.

**Thesis Organization**

The remainder of this study is divided into five parts. Chapter 2, titled Review of Literature, examines the previous studies in stockholding agricultural commodities, logistics models of storage, and real options. Chapter 3, titled Theoretical Model, discusses how the model is set up and the reasons behind the model. Chapter 4, titled Empirical Model, places the model into a prototypical agribusiness firm. Chapter 5 explains the results of the empirical model. Finally, Chapter 6 will sum up the study and offer a conclusion.
CHAPTER 2
REVIEW OF LITERATURE

Introduction

Decreased quality and quantity of specific commodities can have a large impact on the overall cost of the grain inventories. An increased amount of research has been conducted on how to properly hedge an inventory. Hedging inventories using the futures markets plays a vital roll in the operating costs of agribusinesses.

The basic inventory decisions show how much inventory to purchase and or carry when facing uncertain demand. Conventional inventory management techniques suggest stocking an inventory level that maximizes expected profits. This approach does not handle risk or the time value of money in a theoretically acceptable manor.

An appropriate way to incorporate risk and the time value of money is the real options approach. Real options are seen as a way to properly manage an inventory that cannot be hedged directly. Real options place an option value on the inventory instead of the traditional inventory management techniques which link inventory directly to cost. The financial approach using real options relies on the relationships between demand and the price of underlying marketable securities.

This chapter focuses on the problematic risks of hedging, inventory, and real options. Hedging strategies are presented along with inventory strategies used to discover the optimal inventory policy. Real options are then introduced with some basic knowledge.
Stockholding of Agricultural Commodities

Theory of Price of Storage

Inter-temporal price relations are defined by Working as a relationship at a given time between prices at different times (1949). An example is the relation at a given time between a spot price and a forward price for the same commodity or the relation between two forward prices. The relationship between the prices for delivery at two different dates is commonly regarded as the cost of carrying the stocks according to Working (1949).

The basic economic theory of commodity markets and price relationships are explained in the law of one price (LOP). The law of one price states that there is only one price for commodities, that being the cash price of the commodity (Blank et al., 1991). All of the other prices are related to the cash price through storage, transportation, and processing costs. This theory holds in domestic market analysis, but it does have some flaws when trying to empirically support it in the international markets (Blank et al., 1991).

Prices of commodities are also partially dependent on whether the commodity is storable or non-storable. Blank (1985) classifies a commodity as being storable when flexible production and a marketing alternative exist. The differences between storable and non-storable commodities are very significant, and they should be understood in order to fully comprehend the price relationships.

If a commodity is storable, there might be storage costs that have to be implemented. If the storage cost is positive, the price when received would have to be higher than the current price. Positive storage costs exist because of handling,
interest, and other expenses (Blank et al., 1991). This positive storage cost must be paid to the producers that are willing to hold inventory from one time period to another. Price changes in one period may not be directly reflected in another time period.

The domestic grain market is a very good example of temporal prices within a single production period (Blank et al., 1991). Domestic grain markets have a very seasonal pattern. This seasonal pattern is called systematic fluctuation within the marketing year (Blank et al., 1991). The quantity supplied is available in periods after the grain harvest only through storage of the commodity. Commodities are stored in inventories until the time in which they are used. Because of this, the lowest price for the commodity appears during harvest. Usually increasing cash prices can be observed until the next year’s crop is nearing harvest.

A positive carrying charge market can be seen when the futures contracts with later maturity dates are steadily increasing. The price of storage is the difference between the prices of alternative contracts indicating what the futures market is willing to pay inventory holders for storing the commodity (Blank et al., 1991).

An inverted market can be seen when the price for contracts with the later delivery dates have a lower price than the contracts that are expiring earlier (Blank et al., 1991). This market is said to have a discount for storing commodities. If a market experiences inverted prices, there is an economic incentive to deliver the commodity immediately. “A normally inverted market arises when there is a relatively low carryover of stocks (and a normal production year coming up) or,
alternatively, when there is a temporary storage of stocks in a position to be delivered against a futures contract” (Blank et al., 1991, p70).

Normal backwardation occurs when the positive carrying charge is insufficient to cover all the storage costs (Blank et al., 1991). The storage cost has to be greater than the interest required to cover the cost of carrying the inventory, along with insurance and warehousing costs (Blank et al., 1991). This is where arbitrage opportunities exist for grain merchants if the carrying charge between futures contracts exceeds the storage costs. Hence, storage costs represent the maximum difference which will be observed in an efficient market. The Keynes’s theory that hedgers must compensate speculators for assuming the price risk associated with futures contracts can be one explanation for backwardation (Blank et al., 1991). Holbrook Working gave an alternative explanation which was the concept of the supply of storage (Blank et al., 1991).

**Keynes’s Theory of Normal Backwardation**

Keynes’s theory of backwardation emphasized the financial burden posed by the necessity for carrying inventories and suggested that futures markets exist to facilitate hedging (Blank et al., 1991). Working promoted the idea that the function of futures markets was the provision of returns for storage services (Blank et al., 1991).

The futures price was an unreliable estimate of the cash price at the expiration date of the futures contract according to Keynes. Keynes believed the futures price to be a downward biased of the upcoming cash price. “This theory argues that fact that speculators sell insurance to hedgers and that the market is normally inefficient
because the futures price is a biased estimate of the subsequent cash price” (Blank et al., 1991, p72).

“Three critical assumptions of the theory of normal backwardation are: that speculators are net long, they are risk averse (i.e. they require positive profits) and they are unable to forecast prices (i.e. all their profits can be viewed as a reward for risk bearing). Given these assumptions, two major implications are associated with the theory. The first is that over time speculators will earn profits by merely holding long positions in futures markets. The second implication is that there is an upward trend in futures prices, relative to spot prices, as the contract approached maturity” (Blank et al., 1991, p72).

Hedgers generally sell to speculators, and then speculators will generally later sell the contracts to offset their futures position. Futures contract prices must increase in order for speculators to profit, and in order for speculators to achieve this compensation constantly, there must be a risk premium. The risk premium results in a reduction in price of what the speculators pay to purchase the futures from the hedgers (Blank et al., 1991)

Several researchers have gathered empirical data to try and prove the theory of backwardation but found little support for the theory. The empirical data which were gathered from profit and loss data show that large commercial agriculture hedgers earn substantial profits from futures trading.

**Working’s Theory of the Price of Storage**

Holbrook Working’s theoretical extension to backwardation has also proved to be very important. “Workings theory was critical to the view that futures markets
existed solely for the purposes of transferring risk from the hedger to the speculator and critical of the view that cash and futures markets are autonomous” (Blank et al., 1991, p73). The price of storage developed by Working said that intertemporal price relationships are determined by the net costs of carrying stocks.

Commodities are frequently demanded by consumers throughout the year even though production is seasonal, so this product is placed in storage until demanded. The term storage refers to the level of inventories; thus, the theory of storage refers to the demand and supply of commodities as inventories (Blank et al., 1991). The term does not relate to how much storage is available or the price charged for the storage. As inventories increase, this lowers the price for storage, which implies that the price of storage is a downward-sloping curve (Blank et al., 1991).

In Working’s (1949) paper “The Theory of Price of Storage,” inter-temporal price relations are clearly defined. Working defines inter-temporal price relations as “relations at a given time between prices applicable to different times, for example the relation at a given time between a spot price and a forward price for the same commodity or the relation between two forward prices” (Working, 1949, p1).

Inter-temporal prices examine the present compared to the future changes between the present and the past or two time periods in the past that are classified as price changes (Working, 1949). The relation between two future periods usually remains a constant percentage. An example of this is that April futures are lower than the May futures by a fairly constant percentage when comparing various years over time (Working, 1949).
Using inter-temporal price relations, the difference between the December futures and the May futures is commonly called the cost of carrying stocks (Working, 1949). If the supplies of stocks are fairly large, the return for carrying stocks may exceed the cost of storage. With that theory in mind, when supplies of stocks are smaller, the return for carrying stocks is usually higher than the cost of storage (Working, 1949).

A competitive industry will determine the necessary return for storage using the inter-temporal price relations in the futures market. The competitors competing for the storage of the commodity have different costs, so they will hedge at different levels, therefore exposing the necessary return for storage. Each competitor will have different hedging strategies and will adjust the strategies if they do not like their current strategy. For example, the returns for storage would equal the amount by which the futures price exceeds the current price. The expected return for storage is equal to the expected futures price in May minus the expected futures price in April, for example (Working, 1949).

There are two limitations to Working’s (1949) theory of price of storage. The first one is that a lot of the storage is supplied by people who do not hedge and who decide to store or not regardless of the price of storage. The second is that people who hedge earn returns which are not exactly equal to the market price of storage. People are going to store regardless of the quoted price because they are expecting a higher return.
Convenience Yield

The futures market along with hedging shows the approximate return that can be expected from storing wheat. The price of storage is not a direct number because it is based on two quoted prices given in the futures market. This price of storage when positive represents a price that is determined in a free market though competition of those who seek to supply the service of storage (Working, 1949). Figure 2.1 shows the relationship between price of wheat stored and amount of wheat storage supplied.

![Figure 2.1. Price of storage (Working, 1949, p5).](image)

The figure above, quoted directly from Working (1949), is a general representation of the price of storage and the amount of storage supplied. In this figure, all of the values are scale values except at the origins. The origins have a value of zero. The price scale is dependant on the time interval, and the quantity scale depends on the time of year and storage facilities directly and indirectly used for the particular commodity.

The above figure is true for a normal market when the current price is lower than the expected futures price, but what happens when there is an inverted market?
The inverted market is when the current price is above the expected futures price. Therefore, a negative price of storage is experienced. Note the difference between Figure 2.1 and Figure 2.2. The graph below shows the complete storage supply curve with the negative price of storage included.

![Figure 2.2. Price of storage with convenience yield (Working, 1949, p6).](image)

The second graph shows that there is a very large amount of storage that is supplied even when the price of storage is at or below zero. One factor that might explain the negative price of storage is the fact that most costs are fixed in the short run (Working, 1949). Another factor is that most potential suppliers have joint costs in that the owners of large storage facilities are mostly engaged either in merchandising or in processing and maintain storage facilities necessary for their merchandising or processing (Working, 1949). If this is explanatory of the business, then the direct costs of storing can be charged against the associated business which is profitable, therefore appearing as a direct loss on the storage operation. Carrying stocks at this lower price level is called the convenience yield. The convenience yield
may off set what appears to be a loss from the storage function itself (Working, 1949).

Storage is an adjunct business for most suppliers as they have production, processing, and/or merchandising facilities. Thus, the suppliers of storage gain a convenience yield from holding stocks because of the uncertainties in receiving raw materials and uncertainties in consumer demand for products (Cordier et al., 1989, p47). Loss to businesses can be substantial if fixed costs are high and the production process gets interrupted from the lack of raw commodities. Holding inventories can also give wholesalers a convenience, as it increases the chance for additional business.

Expectations of the futures availability of a commodity is reflected in the convenience yield. The greater the possibility that shortages will occur during the life of the futures contract, the higher the convenience yield (Hull, 2000). Thus, once again, Hull (2000) states the point that when inventories are high with little chance of shortages, the convenience yield is low, but on the other hand, when inventories are low and shortages exist, convenience yields tend to be high. A more substantial convenience yield per unit can be seen when stocks are lower. Thus, Cordier et al. (1989, p47) state that the total convenience yield of stocks increases with quantities held until marginal convenience becomes zero at some large level of stocks. This means there is a much higher convenience yield at lower stock levels that is negatively sloped as it approaches zero at some large level of stocks.

Producers may be encouraged to hold inventories because of government pricing and storage programs, causing current grain users to bid up cash prices above
the futures price; thus, an inverted market results. Quality and transportation also have the same effects of government pricing and storage programs according to Cordier et al. (1989).

Stulz (2003, p125) states that the convenience yield is the benefit one derives from holding the commodity physically. The convenience yield reduces the forward price relative to the spot price, but the storage costs increase the forward price relative to the spot price. Stulz (2003) uses the example of having some gold which you could melt down to create a gold chain to wear. If the cash buyer has no use for the gold, they could lend it, and whoever borrows it would pay the convenience yield to the lender.

Firms also hold inventories of commodities much the same way they hold money, according to Williams (1986). The difficulty of moving inventory quickly to where it is needed explains some of the reasoning behind why firms hold inventory when cash price is above futures price.

The convenience yield may fluctuate over time in the agricultural futures market. With fluctuating convenience yields, the positions nearby are subject to basis risk, which is why investors will generally hedge the nearby positions using the distant futures (Hong, 2001). With that in mind, the distant contract is less sensitive to underlying shocks than nearby contracts, so the nearby contracts have higher volatility. Markets with more volatile convenience yield shocks are more likely to have a larger futures risk premium, and the futures risk premium increases with the persistence of convenience yield shocks (Hong, 2001). Hong (2001) states that because convenience yield fluctuations generate basis risk to trading futures,
investors continuously facing spot risk over time have to simultaneously trade futures of different maturities to optimally hedge their spot positions.

**Logistics Models of Storage**

**Appraisals of Inventory**

Ballou (1999) states that there are two main reasons for carrying inventory, and they are to improve customer service and reduce costs. Inventories provide a level of product or service availability which, when located in proximity of the customer, can meet a high customer service requirement.

There are five main reasons that inventories can reduce overall cost (Ballou, 1999). The first is economies of size in the production runs because carrying an inventory will allow longer, larger, and more level production runs. Second, firms may purchase in larger quantities seeing an economy in purchasing and transportation. Third, firms may purchase greater quantities at the current price if the product price is anticipated to rise. Fourth, inventories can be used to buffer the variability and shocks in the marketplace. The fifth reason is that inventories can help hold a buffer stock to help against natural disasters.

Ballou (1999) lists three reasons inventories draw skepticism. The first is that inventories are considered wasteful because they absorb capital that might otherwise be put towards a different use. The second is inventories can mask quality problems. The last reason is that using inventories promotes insular attitudes about the
management of the logistics channel as a whole. This may encourage individual firms in the supply chain to act inefficiently.

**Types of Inventories**

Ballou (1999, p. 311) lists inventories in five different forms. The first is pipeline. These are inventories that are in transit between stocking or production points because movement is not instantaneous. Speculation is the second type. These are inventories purchased for price speculation. Regular or cyclical is the third type. These are inventories necessary to meet the average demand during the time between successive replenishments. Safety stock is the fourth type, and these inventories are created as a hedge against variability in demand for the inventory. Obsolete, dead, or shrinkage stock is the fifth type, and this inventory is out of date, stolen, or deteriorated.

**Push and Pull Approaches**

There are two approaches in logistical strategies, labeled as the push and pull approaches. The pull approach according to Ballou (1999) is a philosophy which views each stocking point as independent of all others in the channel. Forecasting demand and determining replenishment quantities are done on an individual level, thus not taking into consideration the economies of the supply.

Ballou (1999) describes the push approach as an inventory management strategy when decisions about each inventory are made independently. Inventory levels are set collectively across the whole system. The push system works well when there are purchasing or production economies that outweigh the benefits of minimum collective inventory levels as achieved by the pull method. Using this
method, inventories can be managed centrally, taking full advantage of economies that can be used to lower costs.

Materials Requirements Planning

Materials requirements planning (MRP) methods try to avoid as much as possible carrying items in inventory through precise timing of material flows to meet requirements (Ballou, 1999). MRP manages inventory in supply chains with the help of time-phased inventory levels. It is a preferred method when demand is reasonably known due to uncertainty of the forecasting component. If demand is forecasted to change, MRP planning is able to adapt to this new level.

MRP is defined as a mechanical accounting method of supply scheduling whereby the timing of purchases or of production output is synchronized to meet period by period operations requirements by offsetting the request for supply from the requirements by the length of the lead time (Ballou, 1999). The trade off in costs associated with MRP concepts is between having the materials arrive before they are needed, in which case they are subject to a holding charge, and the expected cost of the materials arriving after they are needed so the materials are subject to a late charge. The challenge of scheduling models (MRP) is to determine the optimal time to request materials ahead of requirements (Ballou, 1999).

A major problem with the MRP modeling is that not all uncertainties are taken into account. Ballou (1999) mentions that the challenge of MRP is to find the optimal release time of materials to meet requirements. There is uncertainty associated with that release time and the required time for the transportation of that product between points in the supply chain.
Economic Order Quantity Model

The economic order quantity model (EOQ) was developed to find the optimal order quantity. Alstrom (2001) found that 84 percent of the firms surveyed used the EOQ model for inventory control. Annual holding cost will increase with an increase in lot size, but in contrast, the annual order cost decreases with an increase in lot size. Material cost is independent of lot size because it is assumed to have a fixed price. The concept of the EOQ is that there is a trade off between the fixed order cost and the holding cost.

The EOQ assumes the four following inputs: annual demand of the product, fixed cost incurred per order, cost per unit, and holding cost per year as a fraction of product cost. Using those inputs, there are three costs that must be considered when deciding on lot size. The first is annual material cost which is annual cost of material purchased. Annual order cost is the annual order cost for the lot ordered. The last is the annual holding cost which is the annual cost of holding inventory.

Lot or batch size is the quantity that a stage of the supply chain either produces or purchases at a given time. Ballou (1999) defines cycle inventory as the average inventory that builds up in the supply chain because a stage of the supply chain either produces or purchases in lots greater that those demanded by consumers. The key to reducing cycle inventory according to Ballou (1999) is the reduction of lot size. The key to reducing lot size without increasing costs is to reduce the fixed cost associated with each lot as stated by Chopra and Meindl (2001). Aggregation across multiple products, customers, or suppliers works best in trying to help reduce overall fixed cost.
There are several problems associated with the EOQ model. Ballou (1999) mentions that between the time that the replenishment order is placed at the reorder point and when it arrives in stock, there is a risk that demand will exceed the remaining amount of inventory. The probability of this occurring can be controlled by raising or lowering the reorder point and by adjusting your optimal order quantity. Ballou (1999) also states that it is not unusual for the reorder point quantity to exceed the order quantity. This frequently happens when lead times are long and/or demand rates are high.

**Demand Uncertainty**

Safety inventory is inventory carried for the purpose of satisfying demand that exceeds the amount forecasted for a given period. Chopra and Meindl (2001) state that the appropriate level of safety inventory is determined by two factors; these are uncertainty of demand or supply and the desired level of product availability. As the uncertainty of demand or supply increases, the level of safety inventory also increases. If the desired level of product availability increases, the safety inventory must also increase. Thus, these two factors must be understood and minimized to lower the overall amount of safety inventory carried.

Minimizing the amount of safety inventory is a matter of minimizing the total of two different costs, the first one being the cost of holding safety stocks, and the second is the cost of being out of stock. The two costs have an inverse relationship: as one decreases, it increases the other. The risk of being out of stock depends on the chance of consumption in a delivery period being unusually large.
Maia and Qassim (1999) came to the conclusion that the optimal strategy for safety inventories is to eliminate stock-outs or not to hold safety stocks at all. In this article, opportunity costs are seen as stock-out costs. They stated that intermediate inventory levels are uneconomical. Also noted is that the products that should be stocked are those in which the opportunity costs are higher than inventory costs.

The firms’ operating risk associated with inventory policy can be decomposed into two components according to Kim and Chung (1989). The first is pure output market risk which is risk inherent in the demand uncertainty. The second is risk resulting from the need to determine the order quantity before the actual demand is revealed.

Kim and Chung (1989) concluded that the optimal inventory level of the risk-adjusted value-maximizing firm is lower than that of the expected profit-maximizing one, and the higher a firms output market uncertainty, the lower its optimal inventory level, where output market uncertainty is defined as the relative volatility of the demand for the firm’s output.

**Relevant Costs of Inventory**

Ballou (1999) suggests that there are three general classes of costs that are important to determining inventory policy. They are procurement costs, carrying costs, and stock-out costs. Procurement costs are those associated with the acquisition of goods for the replenishment of inventories. Carrying costs result from storing or holding goods for a period of time. Carrying costs may be space costs, capital costs, inventory service costs, and/or inventory risk costs. Stock-out costs are
incurred when an order is placed but cannot be filled from the inventory to which the order is normally assigned.

Heijden (2000) notes that customer service is usually included in conventional inventory models by imposing a penalty cost on shortages when minimizing inventories. This creates implementation problems in the application of the model because you now have penalty costs that have been distorted and are difficult to establish.

Unlike a fixed asset, stocks held for normal trading or manufacturing purposes can usually be realized in a matter of months without appreciable loss so that only a small premium over the marginal cost of borrowing would be required.

**Reliability of Delivery**

If a supplier is very unreliable, it will cause the purchaser to have higher costs, whether incurred though stock-outs, buying emergency lots from competitors, or expediting or carrying larger buffer stocks than would otherwise be necessary. Most of the reasons for unreliability, such as inefficient dispatch arrangements, insufficient buffer stocks of finished goods, and factories closing for holidays without warning being given to customers, are within the control of the supplier.

According to McClelland (1960), it is less usual for the cost of a stock-out to be estimated and used than for management to make a policy decision about the level of protection they want. Different levels of protection may be specified according to the type of commodity—high-profit goods, highly competitive goods, and goods with no close substitute or whose absence will disrupt production.
Real Options

Introduction

Options are the right, not the obligation, to acquire an asset by paying a certain amount of money on or before a certain time. Options are then opportunities to take some action in the future, so in that fashion, a capital investment is now an option. According to Dixit and Pindyck (1995), a company with an opportunity to invest is holding something much like a financial call option. If the option is not exercised, the lost value is the opportunity cost that must be included as part of the cost of the investment. The opportunity is highly sensitive to uncertainty over the future value of the project.

Real options or contingent claims occur when the underlying assets are real assets. Real options are held by companies to manage their business and allow for financial flexibility. Real options are not easy to deal with and often hard to identify. Managers need to have knowledge about real options as they may sometimes be tangled together.

Conventional net present value (NPV) is a tool that many managers rely on but might not always be the most accurate tool to use when dealing with uncertainty. An example of this is that the NPV of an investment might be negative. Going ahead with the investment may give rise to opportunities that will not otherwise occur, while foregoing the investment means giving up on the option to grow the business. In using NPV, a mistake has been made according to Turvey (2001) in that conventional NPV fails to take into consideration the value of growth options, even if the growth opportunities are uncertain. Real options are based on flexibility, irreversibility, and
uncertainty in investments. Irreversibility refers to the making of fixed investments in capital in which some of the investment costs are sunk, flexibility refers to the right that managers have to postpone making an irreversible decision until markets improve, and uncertainty causes irreversibility which creates value using real options.

**Types of Real Options**

Real options create flexibility for managers in their ability to respond to various investments and developments over time. Schwartz and Trigeorgis (2001) classify real options into seven general categories which are option to defer, time to build option (staged investment), option to alter operating scale, option to abandon, option to switch, growth options, and multiple interacting options. The options for reducing upside risk would be options to defer or expand, while options for reducing downside risk would be options to abandon for salvage value or switch use.

There is a large number of other combinations of these real options that managers can use in order to correctly assess opportunities. Many of the decisions managers make take into consideration various types of real options. The option used in this study is the option on future sales. This option on future sales is similar to a timing option to expand capacity. The option to expand inventory by assuming sales of inventory is related to the asset price of the inventory. The expansion option uses surplus and shortage variables to determine the option value. The surplus variable determines the value of the option through its net present value.

**Asset in Place vs. Options**

Assets in place are cash-producing assets that can be evaluated with discounted cash flow methodologies, while growth options or opportunities to make
future investments require option-pricing methodologies. Growth options and a few other decision opportunities are known as real options to distinguish them from financial options such as exchange traded puts and calls. According to Luehrman (1995), projects with high option content are likely to be misevaluated by discounted cash flow techniques: either the options will be ignored or they will be poorly approximated.

**Real Options and Uncertainty**

Real options exist because of uncertainty. Like financial options, if there is no risk, there is no option value or need for options. The real option is a contingent claim, which means that its values derived from positive value only. Dixit and Pindyck (1995) state that the opportunity cost is highly sensitive to uncertainty over the future value of that project; as a result, new economic conditions that may affect the perceived riskiness of future cash flows can have a large impact on investment spending, much larger than, say, a change in interest rates. Thus, viewing investments as options puts greater emphasis on the role of risk and less emphasis on interest rates and other financial variables.

The greater the uncertainty over the potential profitability of the investment, the greater the value of the option and the greater the incentive to wait and keep the opportunity alive rather than exercise it by investing at once.

**Real Options Payoff**

Real options have a nonlinear payoff structure. According to Turvey (2001), the reason for the nonlinear payoff structure is that waiting will only have a value if the expected value of waiting exceeds the expected value of not waiting. If this is
true, then postponing the investment means that the option to wait is being exercised. If it is false, then postponing the investment is not expected to increase value; the option to wait will be killed, and the project will be undertaken currently.

**Contingent Claims**

A contingent claim is an asset whose payoff depends upon the value of another underlying asset, the value of which is exogenously determined. “A valuation relationship is a formula relating the value of a contingent claim, or its derivates, to the value of the underlying assets and other exogenous parameters” (Brennan, 1979, p53).

Cox and Ross (1976) found that whenever a portfolio can be constructed which includes the contingent claim and the underlying assets in such proportions that the instantaneous return on the portfolio is non-stochastic, the resulting valuation relationship is risk neutral.

**Discrete Time Approach**

Brennan (1979, p54) claims one of the major advantages of using a discrete time approach to the pricing of contingent claims is that the restriction of investor preferences eliminates the requirement of the continuous time approach that an instantaneously riskless portfolio may be constructed which includes both the contingent claim and the underling asset. To construct such a portfolio, several key assumptions have to be made, the first one being that the underlying asset and contingent claim are traded assets which can be purchased and sold continuously in any proportions, and the second is restrictions on the stochastic process for the value
of the underlying asset (Brennan, 1979). The discrete time approach does not require these restrictions, thus extending the scope of the option model.

The following are a few examples of contingent claims explained by Brennan (1979). Different types of tax liability can be thought of as contingent claims in which the value depends on the accounting income of the firm. In this example of a contingent claim, neither the contingent claim nor the underlying asset can be traded. Costs that are only paid in the event of a bankruptcy are also thought of as non-traded contingent claims.

**Limits of Real Options**

There are several key limitations to real options as noted by Amram and Kulatilaka (1999). Amram and Kulatilaka (1999) state the limits to the powerful tool of real options to be model risk, imperfect proxies, lack of observable prices, lack of liquidity, and private risk. These limitations are not surprising due to the fact that the concept of real options is fairly new and the deriving of real options is not an exact science, but these limits need to be considered when modeling real options.

Once the real option is identified, a valuation model must be created, but the lack of information about the financial market may cause some bias in the models mathematics. The model risk is defined by Amram and Kulatilaka (1999) as the difference between the model answers and the theoretically correct answers. The short-term models usually represent less model risk, but the risk increases with time as there is more uncertainty in the future. Managers can counteract the model risk by being aware of it and taking it into account when looking at the model outputs.
Imperfect proxies also limit the manageability of real options. An example of an imperfect proxy is when a manager writes a contract based on the price of wheat in Fargo in December, but the only wheat future currently traded is based on delivery in Minneapolis. If wheat prices in December are different in Fargo than they are in Minneapolis, you have an imperfect proxy. This risk can be tailored to the business, thus lowering the risk of imperfect proxies.

A lack of observable prices is also a limit to real options. Price data may not be available as fast as needed in order to make a decision, or the relevant securities may not be traded frequently enough, so trades are delayed. If managers are forced to guess, this will increase the possibility that the options’ full value will not be realized. The fast access to information using electronics is helping to solve this problem, but it is still worth noting.

Lack of liquidity can be seen in real assets and thinly traded stocks. They are characterized by a low trade volume, and any sizable trade can move the market price. Traders have developed various techniques to minimize the distortions caused by the lack of liquidity, and some of them can be adopted by users of real options.

Private risk greatly affects the value of real options. Private risk is defined by Amram and Kulatilaka (1999) as risk that is peculiar to one company. Business managers are used to looking at private risk, so they tend to give it too much emphasis when making strategic decisions.

**Summary**

Factors discussed in this chapter are important in the inventory analysis of commodity marketing. Most relevant to this research is the defining of a convenience
yield. Suppliers of storage that also have adjoining businesses such as milling will 
carry inventory even when the price of storage is negative for the simple reason of 
convenience. Conventional inventory models are still used in many businesses today 
and are still very important in the agribusiness sector, but there is becoming an 
increasing awareness of the management tool called real options. The knowledge and 
concept of real options expressed in this chapter are very important to understand as 
the next chapter builds upon it.

Most of the conventional approaches to inventory in the past look at 
minimizing cost with no economical way of looking into the effects of convenience 
yields on the optimal inventory level. Past research in the business sector has shown 
that there is an increasing awareness of the downfalls of conventional inventory 
management tools. Real options are becoming a valuable tool as managers are 
seeking out a way to properly account for the optimal profit-maximizing level of 
inventory for their particular agribusiness.

The research reported in this thesis differs from past studies in several ways. 
Many of the past contingent claims studies have not included real data from 
aricultural commodities and have focused more on the theoretical and financial part. 
The following chapter will develop a theoretical model in which the optimal 
inventory level is defined.
CHAPTER 3
THEORETICAL MODEL

Introduction

Inventory strategies are increasing in importance in the commodity marketing system, and managing inventory uncertainties is becoming a topic of increasing significance. Two tools for inventory management are the EOQ and contingent claims. Inventory decisions and techniques affect the logistical flow of commodities and also the profit of agribusinesses. Inventory management tools are vital to the marketing system. The increasing variability of quality and demand of agribusiness adds to the problem of finding the optimal inventory level. Several studies have shown the relationship between contingent claims and inventory management. This variability has mangers looking for flexibility when managing inventories under uncertainty.

This chapter develops the theoretical background on how to define an inventory model that balances risk and profit. Logistical models are analyzed first. Second, the effects of the increased volatility in the marketplace are presented. Next, the contingent claims approach is discussed in the context of inventory decision.

Logistical Models

A variety of logistical techniques can be used to find inventory requirements. The overall goals of logistics are to get the right product in the right place; at the right time; and, in the process, at the lowest possible cost. The effect is to reduce cost and capital and improve customer service.

Conventional approaches use the economic order quantity (EOQ) model. The EOQ model minimizes the total annual costs for purchase order and holding costs.
The EOQ is used to find an optimal order quantity by evaluating the trade off between order costs and holding costs. The optimal order quantity under the EOQ model is represented by the following equation:

$$Q = \left(\frac{2DS}{IC}\right)^{1/2},$$  \hspace{1cm} 3.1

where

- $Q$ = Optimal order quantity (or lot size),
- $D$ = Annual demand for the item,
- $S$ = Procurement cost or order cost,
- $I$ = Carrying cost as a percentage of the items value, and
- $C$ = Value of the item carried in inventory.

Results from EOQ models provide the order quantity that minimizes cost. The equation is the first derivative for a specific cost function, which includes procurement cost and inventory carrying costs. The optimal order quantity exists when the summation of the two costs is minimized (Ballou, 1999). The number of times per year a replenishment order is placed is represented by the term $D/Q$. The average amount of inventory on hand is represented by the term $Q/2$.

The optimal order time between orders is therefore

$$T = \frac{Q}{D},$$  \hspace{1cm} 3.2

and the number of times per year to place an order is

$$N = \frac{D}{Q}.$$  \hspace{1cm} 3.3

Costs and demand cannot always be known exactly; however, the economic order quantity model is not very sensitive to over- or underestimating data. An example of this from Ballou (1999) is if demand is in fact 10 percent higher than anticipated, $Q$ should only be increased by $(1.10)^{1/2} = 4.88$ percent. If the carrying cost is 20 percent lower than assumed, $Q$ should be increased by only $(1/(1-0.20))^{1/2} = 5.39$ percent. The equation is:

$$Q = \left(\frac{2DS}{IC}\right)^{1/2},$$

where

- $Q$ = Optimal order quantity (or lot size),
- $D$ = Annual demand for the item,
- $S$ = Procurement cost or order cost,
- $I$ = Carrying cost as a percentage of the items value, and
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The optimal order time between orders is therefore

$$T = \frac{Q}{D},$$

and the number of times per year to place an order is

$$N = \frac{D}{Q}.$$
11.8 percent. This percentage change is inserted into the EOQ formula without changing the remaining cost and/or demand factors since they remain constant.

Reorder Point

The EOQ formula for basic inventory control results in a saw-tooth pattern of inventory depletion and replenishment. The quantity to which the inventory is allowed to fall before reordering more is called the reorder point. Because there is generally a time lapse between when the order is placed and when the items are available in inventory, the demand that occurs over this lead time must be anticipated. The reorder point (ROP) is

\[
\text{ROP} = d \times LT, \tag{3.4}
\]

where

- \( \text{ROP} \) = Reorder point quantity units;
- \( d \) = Demand rate, in time units; and
- \( LT \) = Average lead time, in time units.

The average lead time (LT) and the demand rate (d) must be expressed in the same time dimension. In Figure 3.1, the ROP is 1 and the quantity ordered is 4 for all four periods on the horizontal axis. This is representative of the saw-tooth pattern created by the depletion and stocking of inventories.

The effective inventory level at a particular point in time is the quantity on hand plus the stock on order less any commitments against the inventory such as customer back orders or allocations to production. The time between when the order is placed and received is defined as lead time. There is a risk that during this lead time, demand will exceed the remaining amount of inventory. This problem can be controlled by raising or lowering the reorder point and by adjusting Q (Ballou, 1999).
Ballou (1999) defines the operation of the reorder point system for a single item where demand during the lead time is known only to have a normal probability distribution. The demand during lead time (DDLT) distribution has a mean of $X'$ and a standard deviation of $s'_d$ (Ballou, 1999). The values for $X'$ and $s'_d$ are usually not directly known, but they can be easily estimated by summing a single period demand distribution over the length of the lead time. The mean of the DDLT distribution is simply the demand rate $d$ times the lead time $LT$, or $X' = d \times LT$ (Ballou, 1999). The variance of DDLT distributions is found by adding the variances of the weekly demand distributions. That is, $s'^2_d = LT(s^2_d)$. The standard deviation is the square root of $s'^2_d$ which is $s'_d = s_d(LT)^{1/2}$ (Ballou, 1999, p329).

Finding $Q$ and the ROP is mathematically complex, but a satisfactory approximation can be found if we first determine $Q$ according to the basic EOQ formula (Ballou, 1999, p329). Then the ROP is found by

$$\text{ROP} = d \times LT + z(s'_d).$$

Figure 3.1. Inventory supply and usage over time.
The number of standard deviations from the mean of DDLT distribution is defined as the variable $z$ to give us the desired probability of being in stock during the lead time period ($P$). The value of $z$ is found in a normal distribution table for the fraction of the area $P$ under the DDLT distribution.

Extending the ROP model to account for lead time uncertainties adds to the realism of the model. To do this, the standard deviation ($s'_{d}$) of the DDLT distribution based on uncertainty in both demand and lead time has to be found (Ballou, 1998). This can be found by adding the variance of demand to the variance of lead time, giving us the following revised formula for $s'_{d}$ of

$$ s'_{d} = (LT s'^{2}_{d} + d^{2}s_{LT}^{2})^{1/2}, $$

where $s_{LT}$ is the standard deviation of the lead time.

Combining demand and lead time variability in this way can greatly increase $s'_{d}$ and the resulting safety stock. In that manner, the equation may lead to an overstatement of $s'_{d}$ and the resulting amount of safety stock. Ballou (1999) suggests the following precise procedure for determining the standard deviation of demand during lead time: a lead time starts when the replenishment order is triggered. The end of lead time by definition is when the order is received. The year-to-date demand is then examined. The difference between the current demand year-to-date and the value when the order was released is precisely, by definition, the demand during the lead time (Ballou, 1999). The values of this variable can be forecast (usually with very simple forecast models), and the mean square error is the variance of demand during lead time, precisely the value being sought. Alternately and less precisely, the
longest lead time may be used as the average lead time with $S_{LT}$ set at zero (0). The standard deviations are then computed as $s’d = s_d(LT)^{1/2}$.

The following is an example of the EOQ and ROP out of Ballou (1999). Buyers Products Company distributes an item known as Tie Bar, which is a U-bolt used on truck equipment. The following data have been collected for this item.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly demand forecast, d</td>
<td>11,107 units</td>
</tr>
<tr>
<td>Std. error of forecast, sd</td>
<td>3,099 units</td>
</tr>
<tr>
<td>Replenishment lead time, LT</td>
<td>1.5 months</td>
</tr>
<tr>
<td>Item value, C</td>
<td>$0.11/unit</td>
</tr>
<tr>
<td>Cost for processing vendor order, S</td>
<td>$10/order</td>
</tr>
<tr>
<td>Carrying cost, I</td>
<td>20%/year</td>
</tr>
<tr>
<td>In-stock probability during lead time, P</td>
<td>75%</td>
</tr>
</tbody>
</table>

The reorder quantity is

$$Q = (2DS/IC)^{1/2} = [(2 \times 11,107 \times 10)/(0.20/12 \times 0.11)]^{1/2} = 11,008 \text{ units}.$$  

The reorder point is

$$\text{ROP} = d \times LT + z(s’d),$$  

where $s’d = s_d(LT)^{1/2} = 3,099(1.5)^{1/2} = 3795$ units. The area of the normal distribution being examined is 0.75; thus, using a statistical $z$ table, the $z$ value is equal to 0.67.

Thus,

$$\text{ROP} = 11,107 \times 1.5 + 0.67 \times 3795 = 19,203 \text{ units},$$  

so when the effective inventory level drops to 19,203 units, place a reorder for 11,008 units.

Ernest and Pyke (1992) develop a component part stocking model that minimizes the total inventory holding and ordering costs for all parts, subject to a fill rate constraint. They introduce $(Z_i)$ a random variable representing the undershoot of the reorder point for part $i$. This is the amount below the reorder point when an order
is placed. The distribution of \( Z \) is based on the excess random variable of a renewal process, and its expected value \( E(Z) \) is approximately \((\sigma^2 + \mu^2)/\mu - 1)/2\), where \( \sigma \) is the standard deviation of demand for the product per period and \( \mu \) is the mean demand for the product period.

**Supply Uncertainty**

Most of the traditional economic order quantity research focuses on demand variability. In contrast, the problem in this thesis is one of supply uncertainty. The variability’s of uncertainty in supply = \( f \)(quality, quantity, and price). Inventories are often used to protect the supply chain from the uncertainty, but these inventories at various points have different levels of responsiveness. Despite the sparse research on supply uncertainty, there are several useful articles that focus on the issue.

One of the most well-known simulation studies is that of Whybark and Williams (1976). They identify four types of uncertainty in a production system: in supply timing, in demand timing, in supply quantity, and in demand quantity. Supply timing is the uncertainty that can arise from variations in vendor lead times or flow rates. Supply quantity uncertainty arises when there are shortages. A shortage is when supplies are not able to meet demand. To simulate supply quantity uncertainty, Whybark and Williams (1976) assumed the actual quantity received was uniformly distributed around the quantity scheduled to be received for each order. “Lumpiness” is defined by Whybark and Williams (1976) as the change from period to period of the gross requirements. It is an important factor in material requirements planning (MRP) research and can be measured by the coefficient of variation (Cv). As the requirements become “lumpy,” the coefficient of variation increases. They then
show, using a simulation study of a single-item, single-stage system, that safety stocks are the best mechanism for protecting against uncertainty in the supply or demand quantity, while safety times are preferred for timing uncertainties in either the supply or demand process.

Lee and Billington (1993) use a stock keeping units (SKU) model to support the systems necessary to anticipate variability and optimize it. At a downstream site, the demand for SKU $i$ generates demand for input parts that are needed to manufacture the SKU. This, in turn, becomes the demand for parts at upstream sites that supply the current site. The demand transition process develops the first two moments of the demand on the SKUs at an upstream sight. Lee and Billington (1997) consider an upstream sight $g$ and multiple downstream sites that receive the part $j$ from this upstream site. Then,

\[
u_j(g) = \sum_{k \in K(g, j)} \sum_{i} v_{ij} u_i(k) b_{ij} p_j(g, k) \quad \text{and} \quad 3.10
\]

\[
s_j^2(g) = \sum_{k \in K(g, j)} \sum_{i} v_{ij} s_i^2(k) b_{ij}^2 p_j^2(g, k), \quad 3.11
\]

where

$k =$ the index of downstream sites;

$K(g, j) =$ the set of downstream sites that are supplied by this upstream site for part $j$;

$u_i(k) =$ the mean demand per week for SKU $i$ at downstream site $k$;

$s_i(k) =$ the standard deviation of demand per week for SKU $i$ at site $k$;

$p_j(g, k) =$ the proportion of the requirement of part $j$ at downstream site $k$ that is to be sourced from the current upstream site $g$;

$u_j(g) =$ the mean demand per week for part $j$ at the upstream site $g$ (known as SKU there);

$s_j(g) =$ the standard deviation of demand per week for part $j$ at upstream site $g$ (known as SKU there);

$v_{ij} =$ the indicator variable: equals 1 if part $j$ is needed for SKU $i$, 0 otherwise; and

$b_{ij} =$ the number of units of part $j$ needed in one unit of SKU $i$. 


Using the model, suppose that input material at a site $k$ is sourced from upstream site $g$. Then, the immediate availability level of the material at site $k$ is the fill rate of the material at site $g$, where the material is known as an SKU. Delays due to material shortage at site $g$ are the response time of the materials at site $g$ given that the SKU is out of stock there.

Anupindi and Akella (1993) assume that demand is stochastic and continuously distributed with a known distribution, but they have uncertainties in supply that arise from delivery contracts, suppliers, quantity, higher priced suppliers, etc. They state the optimal policy with the following structure: there exist two order points, $\bar{u}^n$ and $\bar{o}^n$, and the on-hand inventory is denoted with $x$, such that when on-hand inventory $x < \bar{o}^n$, then both suppliers may be used; when $\bar{o}^n \leq x < \bar{u}^n$, only the cheaper supplier will be used. Finally, no order will be placed when $x \geq \bar{u}^n$. They assume in the model that the supply process is such that from facility $i$ with probability $\beta_i$, all of the order quantity is supplied, and with probability $1 - \beta_i$, nothing is supplied. The expected cost of ordering from two facilities can be written as follows:

$$3.12 \quad M(w_1, w_2, x) = c_i \beta_1 w_1 + c_2 \beta_2 w_2 + h \beta_1 \beta_2 \cdot \int_{\min(w_1, w_2, x)}^{\max(w_1, w_2, x)} (w_1 + w_2 + x - \xi) f(\xi) d\xi$$

$$+ h \beta_2 \int_{0}^{\min(w_1, w_2, x)} (w_1 + x - \xi) f(\xi) d\xi + h \beta_1 \int_{\min(w_1, w_2, x)}^{\max(w_1, w_2, x)} (w_2 + x - \xi) f(\xi) d\xi$$

$$+ \pi \beta_1 \beta_2 \int_{\min(w_1, w_2, x)}^{\infty} (\xi - w_1 - x) f(\xi) d\xi + \pi \beta_2 \beta_1 \int_{\min(w_1, w_2, x)}^{\infty} (\xi - w_2 - x) f(\xi) d\xi$$

$$+ \pi \beta_2 \beta_1 \int_{x}^{\infty} (\xi - x) f(\xi) d\xi \beta_1,$$

where

$$w_i = \text{order quantity from facility } i.$$
\( x = \) inventory on hand,
\( c_i = \) per unit ordering cost from facility \( i \),
\( h = \) holding cost per unit per period,
\( \pi = \) penalty cost per unit per period,
\( \beta_i = \) probability that all of the orders from facility \( i \) will be delivered,
\( \bar{\beta}_i = 1 - \beta_i \),
\( \xi = \) total requirement (random variable),
\( f(\cdot) = \) density of total requirement,
\( F(\cdot) = \) distribution of total requirement,
\( \overline{F}(\cdot) = \) complementary distribution function,
\( M(x,w_1,w_2) = \) single-period cost function,
\( J_1(x) = \) optimal cost function, and
\( E(\cdot) = \) the expectation operator.

The first two terms represent the expected ordering cost on the amount delivered, the next three terms represent the expected holding costs, and finally, we have the expected penalty costs. Thus, the problem can be formulated as

\[
J_1(x) = \min_{w_1, w_2 \geq 0} M(w_1, w_2, x). \tag{3.13}
\]

This is the single-period cost function for the inventory policy. It is evident from the nature of the optimal policy that the ordering depends upon the on-hand inventory.

There could be a modification to the model which assumes a fixed minimum order quantity that is always ordered from both suppliers. The suppliers in return promise delivery of the fixed quantity every period; the uncertainty in the supply process does not affect the delivery of the fixed quantity.

**Total Relevant Costs**

The total relevant cost is useful for comparing alternative inventory policies or determining the impact of deviations from optimal policies. The total relevant cost function can be expressed as Total cost = order cost + carrying cost (regular cost) + carrying cost (safety stock) + stock-out cost:

\[
TC = \frac{D}{Q} x S + IC x \frac{Q}{2} + ICzs' d + D/Qks' d E(z), \tag{3.14}
\]
where $k$ is the out-of-stock cost per unit. Stock-out costs consist of two elements. First, the combined term of $s'dE(z)$ represents the expected number of units out of stock during an order cycle. $E(z)$ is called the unit normal loss integral whose values are tabled as a function of the normal deviate $z$. Second, the term $D/Q$ is the number of order cycles per period of time, usually a year. Hence, the number of order cycles times the expected number of units out of stock during each order cycle gives the total expected number of units of stock for the entire period. Multiplying the total expected number of units of stock by the out-of-stock costs yields the total period cost.

McClelland (1960) also notes that it is helpful to divide up stockholding costs into two separate components. McClelland (1960) divides up the stockholding costs into value of the stockholding and physical storage cost. Value stockholding costs include costs such as interest, insurance, and risk; the total cost of each varies with the purchase price of the goods. Guerrero et al. (1986) note that inventory cost can be lowered by shifting safety stock to lower (less costly) levels of the product structure.

**Risk Assessment**

Most of the classical EOQ inventory models do not account for economic and financial risks associated with carrying inventory. Inflation and time value are modeled into the EOQ by Ray and Chaubhuri (1997). In their model, they use an item having an inventory-level-dependent demand rate at any instant. Shortages are allowed, and deterioration of the inventory over time is not taken into account, nor is the time value of money and different inflation rates for various costs associated with the inventory system.
Two distinctly different price discovery mechanisms are used in the agribusiness market. The first is spot price, and the second is forward prices; differences in these two pricing mechanisms must be understood as pricing with forward prices will have varying degrees of uncertainty depending on the basis, liquidity, and other arbitrage opportunities. Spot prices are prices set for immediate delivery that day, whereas forward prices are contracts to buy or sell an asset at a certain time in the future for a certain price (Hull, 2000). Most of the forward contracts are done over the counter, whereas futures contracts are traded on exchanges.

In years of shortages in supply and lack of quality, prices may be relatively high and heterogeneous, whereas years with a surplus in supply and good quality, the price may be relatively low and homogenous. Forward prices can provide a way to help eliminate the risk because of uncertain forward prices of commodities. Once a forward contract is agreed upon, it has to be stored from the time it was harvested until the contract delivery date. This is a role of either the supplier or the buyer. The risks associated with the future commodity prices are a clear role of the forward contracts.

**Contingent Claims**

**Introduction**

The EOQ model and capital asset pricing (CAPM) models have been used in the past, but there currently is a much more sophisticated way to model inventory payoffs using contingent claims and real options.
Contingent claims analysis is well suited to evaluate inventory decisions under uncertainty because the payoff from the firm’s inventory decisions is contingent with its value dependent upon uncertain supply or demand in the future. Payoffs can then be replicated to build a portfolio of options deriving the value of an inventory policy.

Cox et al. (1979) showed that the value of a financial call option can be computed by the construction of a portfolio that replicates the return of the option under any state of nature. An option on a stock can be replicated by borrowing funds and buying some of the stock (for a call option) or by lending funds and selling some of the stock (for a put option). Using arbitrage, risk-neutral probabilities can be derived. This approach is called contingent claims because the valuation of the option is contingent upon the value of the underlying asset.

To apply contingent claims evaluation to real investment analysis, it is necessary to find an asset or groups of assets that are correlated with the net return of the investment to be analyzed. This correlated asset is called a “twin asset” by Tauer (2000). Tauer (2000) states that the value of the twin asset is known at the beginning of the investment period, but the value of the investment is not known at the beginning since that is the value being estimated. Information about changes in the value of the twin asset is used to determine the value of the investment. This process of determining the value of the investment is much easier when the financial options are used as derivatives of an underlying asset, but it is much harder to find a “twin asset” to model the net returns of an investment. In the case of agriculture, however, there are strong correlations between the price of an underlying commodity and the net returns.
A major advantage of the discrete time option pricing model is the absence of the assumption of the ability to form a riskless portfolio with both the contingent claim and the underlying asset (Chung, 1990). Such a portfolio can only be formed under strong assumptions that the underlying assets and the contingent claim be not only traded as assets but may also be purchased and sold continuously in any proportions. The discrete time model does not impose such requirements and thus extends the applicability of the option pricing model.

**Financial Options**

There are two basic types of financial options. A call option is defined by Hull (2002) as an option to buy an asset at a certain price by a certain date. A put option is defined by Hull (2002) as an option to sell an asset for a certain price by a certain date. In the case of this study, the call option will be used as the agribusiness is demanding commodities to be supplied.

There is one major difference between financial traded assets and real assets, and that is that in financial traded assets, the settlement is always in cash rather than delivering the portfolio underlying the index, but in the case of real options, the underlying assets are real assets in that they have an underlying value.

The Black-Scholes model for valuing stock options has one major underlying assumption, and that is that stock prices are normally distributed in the short period. They have a normally distributed mean and standard deviation. Several other assumptions of the Black-Scholes model as noted by Hull (2002) are that there are not transactions costs or taxes; there are not dividends on the stock during the life of the option; there are no riskless arbitrage opportunities; security trading is continuous;
investors can borrow or lend at the same risk-free rate of interest; and the short-term, risk-free rate of interest is constant. Some of the assumptions have been relaxed by other researchers, but these were the initial assumptions when the model was developed.

Forward contract prices can be derived using the Black-Scholes model. From Hull (2002), the long forward contract that matures at time $T$ with delivery price $K$ has a value at maturity equal to

$$S_T - K$$

because it enables an asset worth $S_T$ to be purchased for $K$. In a risk-neutral world, the expected value of $S_T = S_0e^{uT}$. In the variable $S_0e^{uT}$, $S_0$ is the price of the asset underlying the forward or futures contract (in years), $u$ is the storage costs per annum as a proportion of the spot price, and $T$ is time until delivery date in a forward or futures contract (in years). The expected payoff from the contract at maturity in a risk-neutral world is therefore

$$S_0e^{uT} - K.$$ \hspace{1cm} (3.15)

Discounting at the risk-free rate $r$ for time $T$ gives the value $f$ of the forward contract today as

$$F = e^{-rT} (S_0e^{uT} - K) = S_T - K e^{-rT}.$$ \hspace{1cm} (3.17)

The expected return on the stock is not in the Black-Scholes equation. There is a general principal known as risk-neutral valuation which states that a security dependent on other traded securities can be valued on the assumption that the world is risk neutral (Hull, 2002). In the risk-neutral world, the expected return from all
securities is the risk-free interest rate, and the correct discount rate for expected cash flows is also the risk-free interest rate.

An implied volatility substituted into the Black-Scholes equation gives the market price of the option. Empirical results show that the volatility of a stock is much higher when the exchange is open than when it is closed (Hull 2002). The Black-Scholes equation is well suited for simple real options, those with a single source of uncertainty and a single decision date (Amram and Kulatilaka, 1999), but this is not always the case when making inventory management decisions, so a more complex model that handles multiple sources of uncertainty and differing decision dates is necessary.

The Black-Scholes equation cannot be used to value options when there is a seasonal or time-varying pattern to the convenience yield (Amram and Kulatilaka, 1999). A time-varying pattern can be accommodated in the binomial options valuation model by calculating separate values of $p$ for each node based on the changing values of $\delta$. The lattice of the binomial tree will remain intact because the rate of return to the underlying asset is independent of its value.

With a constant $\delta$, the expected capital gains return to a futures contract is $(r-\delta)$, where $\delta$ is the net of storage costs (Amram and Kulatilaka, 1999). The relationship between the futures price and the spot price is

$$\text{Futures price} = \text{Spot Price} \times e^{(r-\delta)T},$$

where $T$ is the time to maturity of the futures contract. Because the futures price, the spot price, and the risk-free rate of return are observable, the convenience yield (net of storage costs) can be calculated using this equation.
Pricing options in a risk-free environment is relatively simple using a one step binomial model. This is possible because the option values are independent of risk preferences, and the same valuations will be obtained even when it is assumed that everyone is indifferent to risk or risk neutral (Amram and Kulatilaka, 1999). It is argued that with the assumption that no arbitrage opportunities exist, the return on the portfolio of options must be equal to the risk-free interest rate. This enables the determination of the cost of setting up the portfolio and therefore the options price. The probability of an upward movement in a risk-neutral world is not the same as the probability of an upward movement in the real world. The probability of an up movement in the real world is q (Hull, 2002).

A one-step binomial model can be extended to a two-step binomial model to show the option price at the initial node of the tree. The option prices at the final nodes of the tree are calculated. The stock price is initially $S_0$. During each time step, it either moves up to $u$ times its initial value or down to $d$ times its initial value. The notation of the value of the option is shown on the tree. The risk-free interest rate is $r$, and the length of the time step is $\delta t$ years. Thus,

$$f_u = e^{-r\delta t}[pf_{uu} = (1 - p)f_{ud}], \quad 3.19$$

$$f_d = e^{-r\delta t}[pf_{ud} = (1 - p)f_{dd}], \quad 3.20$$

$$f = e^{-r\delta t}[pf_u = (1 - p)f_d]. \quad 3.21$$

Then substituting the above equations, the below equation is derived:

$$f = e^{-2r\delta t}[p^2 f_{uu} + 2p(1 - p)f_{ud} + (1 - p)^2 f_{dd}]. \quad 3.22$$

Figure 3.2 is a stock and option prices model in the general two-step tree.
This is consistent with the risk-neutral principle of risk-neutral valuation. The variables \( p^2, 2p(1-p), \) and \( (1-p)^2 \) are the probability that the upper, middle, and lower final nodes will be reached. The option price is equal to its expected payoff in a risk-neutral world discounted at the risk-free rate. This options approach is a tool that can be used in managing inventories.

Simulation models can be used to perform the Monte Carlo technique. The path of the stochastic variable is simulated over a large number of random paths. This results in a sample mean that is close to the actual mean. The optimal investment strategy at the end of each path is determined, and the payoff is calculated. The current value of the option is then found by averaging the payoffs and then discounting the average back to the present.

The Monte Carlo simulation method can handle many aspects of the real-world applications, including decision rules and relationships between the option value and the underlying asset (Amram and Kulatilaka, 1999). Uncertainty can be added with much less burden using the simulation model compared to other numerical models.
Simulation models also can solve path-dependent options in which the value of the option depends not only on the value of the underlying asset but also on the particular path of value followed by the underlying asset. This approach has the additional benefit of providing an estimate of the error by giving the standard error of the sample mean (Benninga et al. 1996).

**Real Options**

An option is the right, but not the obligation, to take an action in the future. Options are valuable where there is uncertainty. For example, one option contract traded on the financial exchanges gives the buyer the opportunity to buy stock at a specified price on a specified date and will be exercised only if the price of the stock on the date exceeds the specified price. Many strategic investments create subsequent opportunities that may be taken, so the investment opportunity can be viewed as a stream of cash flow plus a set of options.

The real options approach is the extension of financial option theory to options on real (non-financial) assets (Amarm and Kulatilaka, 1999). While financial options are detailed in the contract, real options embedded in strategic investments must be identified and specified. Moving from financial options to real options requires a change in the way of thinking, one that brings the discipline of the financial markets to internal strategic investment decisions.

Real options have three components that are useful to managers. The first is that options are contingent decisions, meaning an option is the opportunity to make a decision after you see how events unfold. On the decision date, if events have turned out well, a different decision will be made rather than if the events turned out poorly.
This implies that the payoff to an option is nonlinear, meaning it changes with the decision made. Second, option valuations are aligned with financial market valuations. The real options approach uses financial market inputs and concepts to value complex payoffs across all types of real assets. The result is a comparison of managerial options, financial market alternatives, and internal investment opportunities and transaction opportunities. The last component is that options can be used to design and manage strategic investments proactively. The nonlinear payoffs can also be a design tool. Managers must identify and value the options in a strategic investment, then redesign the investment to better use the options, and manage the investment proactively though the options created.

The real options approach gives managers a decision-making and valuation tool that reflects good project management, ensuring that these decisions lead to the highest market valuation of corporate strategy.

**Contingent Claims and Inventory**

Traditional valuation tools use forecasts of outcomes (sales, profits, and so on) of future investments and of terminal values to arrive at seemingly precise numbers, but that imposes a fixed path on a business’s future developments. It is well known that a fixed path view is artificial, and in reality, pursuing a new business opportunity may result in a variety of outcomes that call for a wide range of strategic responses. This wide range of responses is called the cone of uncertainty (Amram and Kulatilaka, 1999). The real options approach recognizes this uncertainty, and it sees what managers see: an investment today creates opportunities or options to change operating and investment decisions later depending on the actual outcome.
The real options model must be adjusted for leakage in values arising from cash flows or a convenience yield\textsuperscript{1} that accrues between decision points. Leakage arises because only the holder of the underlying asset obtains the cash flow and or convenience yield from the underlying asset. Amram and Kulatilaka (1999) state that sources of leakage include positive cash flows (dividend, rental, interest, license, royalty income), explicit negative cash flows (storage costs, taxes, licensing/royalty fees, insurance costs, loss from perishable damage), and implicit benefits (convenience yields). Convenience value arises from the ability to store a commodity and to sell it at will in the spot market. Owners of commodities obtain benefits that are not obtained by owing a contract on a commodity. Most agricultural commodities will have a convenience yield just prior to harvest, when it is most likely that shortages will occur.

Inventories can be thought of as insurance investments in that they reduce exposure to uncertainty. Investing in excess capacity ensures against running out of product if demand surges, but with a cost which is the incremental costs of construction and carrying capacity built beyond immediate needs. This insurance pays off only in a few outcomes. This is where managers use real options to value investment and also check to see whether the value exceeds the cost.

Real options have recently applied to inventory analysis. As managers are searching for a more quantitative tool that can be used to guide decisions about inventories and assess the impacts of inventories on these decisions, they are looking to real options. Real options help solve this problem of correctly handling all risks

\textsuperscript{1} The smaller the level of stocks, the greater is the convenience yield for holding an additional unit (Leuthold et al., 1989). There is long standing literature on convenience yields in agricultural commodities due to the cyclical pattern of harvest.
associated with carrying inventory that the net present value analysis overlooks. The objective of the real options approach is to maximize the value of an investment using a portfolio of assets which are highly correlated with the investment. If both upside and downside risk are not properly addressed, managers have difficulty properly identifying the correct inventory investments. The real options analysis to invest under uncertainty and in the determination of optimal investment rules depends on the stochastic process for the underlying asset.

An inventory can be viewed as a call option by mapping between the characteristics of the inventory and the determinants of the call option value. A call option is the right to buy an asset at a set price. Inventory is similar to a call option in that if it is bought at a lower price and stored until processed with the market price increasing, the processor has the ability to use inventories that were purchased at a lower price. Given this ability, processors with inventory have the potential to sell or use inventory, depending on market conditions. The decision makes the payoff of the inventory nonlinear. An inventory decision involves making expenditures to buy the commodity asset. This is similar to exercising an option: the amount expended is the exercise price (X), and the value of the asset acquired is the stock price (S). The length of time the company can wait without losing the opportunity is the time to expiration (t), and the risk of the project is reflected in the standard deviation of returns on the asset (σ). The time value is given by the risk-free rate (r) (Luehrman, 1995).

A call option should be exercised if the stock price exceeds the exercise price at expiration. In this case, the stock price (S) corresponds to present value (expected...
net cash flows), and the exercise price (X) corresponds to present value (capital expenditure). Thus, for a call option, \( \text{NPV}_q = S / \text{PV}(X) \) (Luehrman, 1995). If \( \text{NPV}_q \) exceeds 1, the options should be exercised, but if \( \text{NPV}_q \) is less than 1, the option is “out of the money” and should not be exercised.

The variability per unit of time of returns on the project is measured by the variance of the returns, \( \sigma^2 \). Multiplying the variance per unit time by the amount of time remaining gives cumulative variance, \( \sigma^2 t \) (Luehrman, 1995). Cumulative variance is a measure of how much things could change before time runs out and a decision must be made. The more cumulative variance an option has, the more valuable the option. Thus, if the option value is ignored and the variance “uncertainty” is not taken into account, there is a tendency to undervalue inventories. If there is greater uncertainty in the market place in that stock-outs might occur, the option value will increase and the managers will see the full potential of the shortages on firm.

**Real Options and Inventories**

There are three good examples of how option pricing approaches can be applied to the inventory problem. Stowe and Gehr (1991) provide two financial approaches to handling inventory. The first is a replicating portfolio approach comparing the value of a portfolio of securities, and the second is the option pricing solution to the discrete sales problem. Stowe and Gehr’s (1991) payoffs replicate those of the inventory policy compared to the value of a portfolio of securities. Chung (1990) shows another option pricing model. In Chung’s model, the inventory payoffs are a linear function of demand with demand and the investors’ discount rate
jointly, log-normally distributed. The inventory reorder point is solved using a binomial option pricing approach by Becker (1994).

These studies did not take into account that managers can make decisions after certain payoffs are known; thus the payoffs become nonlinear or contingent claims. The real options approach utilizes contingent claims in trying to find the profit-maximizing inventory level given the circumstances that are developed. Real options can be used to value inventories as an insurance policy in that they are an option on future sales.² If the inventory is depleted, the sales are lost. If the inventory is in excess, the profit declines because of excess inventory carrying costs.

Real options also take into consideration uncertainty in a more “respectful way,” thus allowing for more flexibility and giving more value to uncertainty. Convenience yields are also a large part of agricultural commodity markets, and the real options approach accounts for this and maximizes the profit from and inventory with convenience yields.

Inventory can be defined in the rectangle defined by Luehrman (1998) as option space. Option space is defined by two-option value metrics, each of which captures a different part of the value associated with being able to defer an investment. Option space can help address the issue of whether to invest in an inventory or not and what to do in the meantime. The first metric contains all of the usual data captured in NPV but adds the time value of being able to defer the investment. Luehrman (1998, p91) defines the metric $NPV_q$ as the value of the

² The option on future sales is the excess capacity real option. This option is also a contingent claim in that the future sales depend upon the underlying asset investments value. This real option is the decision to build excess capacity and value the shortages. In this study, net present value (NPV) was used to place a value on the option.
underlying assets intended to acquire divided by the present value of the expenditure required to buy them. This is simply defined at the value-to-cost ratio. When the ratio is between zero and one, the inventory is worth less than it costs. When the metric is greater than one, the inventory is worth more than the present value of what it costs.

Volatility, the second metric defined by Luehrman (1998, p91) measures how much things can change before an inventory investment decision must finally be made. That depends on how uncertain the future value of the assets in question is and on how long we can defer a decision.

The two-option value metrics figure below (Figure 3.3) shows that moving to the right or downward corresponds to the higher option value.

![Figure 3.3. Value-to-cost ratio (Luehrman, 1998, p91).](image)

If the value-to-cost ratio is greater than one and the NPV>0, the investment should be exercised immediately; thus, it is a timing option. If NPV<0, the projects are out of the money but have a value-to-cost metric greater than one which makes
them promising in the future. If the volatility is in the lower region and the value-to-cost ratio is greater than one, the investment should be exercised.

### Contingent Claims and Discrete Demand

The conventional approach to inventory stocking decisions relies on a criteria that maximizes expected profits given a distribution of demand for the inventory item. The contingent claims approach presented maps the demand and payoffs from an inventory policy onto the price of an underlying state variable, builds a portfolio of options that replicates payoffs, and then derives the value of an inventory policy using option pricing models in which the value of the policy is contingent on the financial characteristics of the underlying asset.

The basic distribution of discrete demand does not have probabilities attached to each demand level. Demand in the conventional approach is assigned a probability, thus limiting the results. The discrete demand with contingent claims has various demand levels that are associated with price ranges of the underlying state variables. The underlying asset can be the value of a foreign currency, fixed income indexes, individual securities, and or commodities.

The payoffs according to Stowe and Su (1997) depend on the level of demand in the state \(D_i\) and the choice of inventory stocking level \(Q_k\). The investment is

\[
Payoff_{ik} = P \min(Q_k, D_i) + E \max(Q_k - D_i, 0),
\]

where

- \(P\) = Selling price per unit,
- \(Q_k\) = Choice of inventory stocking level in that state,
- \(D_i\) = Level of demand in that state,
- \(E\) = Excess quantity selling price per unit, and
- \(C\) = Cost per unit.
Stowe and Su (1997) state that the net present value (NPV) of a particular inventory policy is equal to the present value of the payoffs minus the outlay:

\[ \text{NPV}_k = \sum_{i=1}^{n} \text{Payoff}_{i,k} p_i - \text{Outlay}_k. \]  

The net present value of the futures payoffs is the sum of the products of the cash flow in each state ($\text{Payoff}_{i,k}$) and its respective state price ($p_i$) (Stowe and Su, 1997). The state price according to Stowe and Su (1997) is the value of a claim that pays off $1.00 in that state of nature and zero otherwise. These above state prices are the basic element to the contingent claims approach to inventory analysis. The state prices in this contingent claim model are similar to the element of probabilities in conventional inventory models.

The state price using the Black-Scholes option pricing model is as follows:

\[ p(Y_1, Y_2) = e^{-rT} \{ N(d_2(X = Y_1)) - N(d_2(X = Y_2)) \}, \]

where

- $p(Y_1, Y_2) =$ state price for a claim that pays $1.00 if $Y_1 < S_0 < Y_2$,
- $e^{-rT} =$ present value factor for risk-free rate $r$ and time period $T$,
- $N(d) =$ value of cumulative normal for $d$,
- $d_2 = \left[ \ln(\frac{S_0}{X}) + (r + \sigma^2/2)T \right]/(\sigma \sqrt{T})$,
- $S_0 =$ current price of underlying state variable,
- $X =$ exercise price of the option,
- $t =$ time until maturity of the option,
- $r =$ riskless rate of return, and
- $\sigma =$ volatility of the underlying state variable.

These calculated state prices account for the likelihood that each state of nature will occur. This method of determining state prices is obtained through intuition rather than reasoning, as variable $N(d_2)$ represents the risk-neutral probably of an option expiring in the money (Stowe and Su, 1997). The state price is then found by multiplying the discount factor for a riskless rate by the difference between
two risk-neutral probabilities. Time value of money and risk are both reflected upon in these probable state prices.

The optimal inventory policy using contingent claims and this discrete demand case depend on the shape of the payoff function, market conditions, and the characteristics of the underlying assets. Changing one or all of these will result in a change in the NPV of an inventory policy in somewhat predictable ways.

Contingent Claims and Continuous Demand

Inventory policies having continuous demand can also be analyzed using contingent claims in a similar fashion as discrete demand. There are three steps in finding the optimal inventory level. First, the payoffs have to be mapped from an inventory policy onto the price of the underlying asset. Second, a portfolio of options has to be constructed that replicates the payoffs, and last, the NPV has to be obtained from determining the value of the portfolio and then subtracting the investment costs.

A base case as shown in Stowe and Su (1997) has no salvage value for excess inventory and no penalties for shortages. It is assumed that sales are tied to an underlying asset with a current price of $50, standard deviation of 0.35, an interest rate of 0.10 percent, and a payoff maturity of 0.5 years. Sales will be zero if the asset price is below $30 but will increase by ten units per $1 above this price. The investment per unit is $6, the price per unit sold is $10, and the salvage value is zero. An inventory stocking policy with 300 units was mapped out. The payoff is zero if the asset price is below $30, increases with a slope of 100 for $>30, and reaches its maximum of $3,000 when the asset price is $60. The option strategy is thus a vertical spread with a portfolio of options that replicates a payoff of a long position of 100
call options with an exercise price of $30 and a short position of 100 call options with an exercise price of $60. The NPV of the inventory policy under which the firm stocks 300 units can be calculated by subtracting the initial outlay from the value of the call options. NPV = 100C(X=30) – 100C(X=60) – 6(300). Inventory carrying costs can also be built into the option pricing model by treating them as a dividend yield, which is used to adjust the stock price before it is input into the Black-Scholes option pricing model (Stowe and Su, 1997):

$$C = Se^{-\delta t} N(d_1) - Xe^{-rt} N(d_2),$$

where

$$d_1 = \frac{\ln(S/X) + (r - \delta + \sigma^2/2)t}{\sigma \sqrt{t}},$$
$$d_2 = d - \sigma \sqrt{t},$$
$$\delta = \text{annualized dividend yield}.$$ 

The NPV is not the optimal inventory policy because at the margin, adding an additional unit to the inventory stock increases the investment by $6 and raises the potential payoff by $10; the additional $10 payoff is achieved by increasing the exercise price of the short position in 100 call options by $0.10. The NPV is

$$NPV = 100C(X=30) - 100C(X=30+0.10Q) - 6Q.$$  

This option strategy above is representative of a vertical spread in which the optimal inventory policy can be found by taking the derivative of the NPV equation with respect to Q, which results in

$$\frac{\partial NPV}{\partial Q} = -100 \frac{\partial C(X = 30 + 0.10Q)}{\partial Q} - 6 = 0 \text{ and}$$

$$\frac{\partial NPV}{\partial Q} = -100 \frac{\partial C(X = 30 + 0.10Q)}{\partial X} \frac{\partial X}{\partial Q} - 6 = 0.$$  

Since $\partial X/\partial Q = 0.10$ in this problem, the optimal state price occurs where
\[-\frac{\partial C(X = 30 + 0.10Q)}{\partial X} = 0.6.\]

Since \( \partial C/\partial X = -e^{-rtN(d_2)} \), Q and X are found by iterating until \( \partial C/\partial X \) has the desired level, and in this case, it happens when \( Q = 169.35 \) units and \( X = $46.94 \).

In most cases, the above inventory situation will not exist, in part because most firms will encounter penalties for shortages and have salvage values for excess inventories. If demand exceeds quantity stocked, the company will incur stock-out costs on the inventory they are short. These stock-out costs are conventionally looked at as a loss of profit on the sales. Penalties are also considered as part of the stock-out costs. The profit lost on the sale due to a stock-out is captured. Stock-outs can also have additional costs to the extent that it injures the customer, thus causing the customer switching costs or losing the customer. The customer could also impose a stock-out cost on the supplier by paying a lower price for the good.

Using the above example to imply a salvage value and stock-out penalties, the problem becomes more realistic. The salvage value limits the risk of carrying excess inventory so that the stocks have a positive value. Stock-out penalties will cause the payoff to decline when the inventory is depleted. Figure 3.4 shows the payoff with a positive salvage value and a penalty for stock-outs.

This nonlinear payoff function in the figure above is due to the contingent claim that managers have the decision to invest or not, and that decision will affect the payoff function.
The real options approach looks at inventories as a type of insurance investment because of the reduced exposure to uncertainty. This insurance pays only in a few outcomes, so traditional tools that show the same payoffs in all outcomes cannot value insurance. This is where managers use real options to value investment and also check to see whether the value exceeds the cost, so the call option value derived above can be seen as an insurance premium for carrying the excess inventory. Inventory viewed as a call option, when a call option is the right to buy, has large benefits when supply is uncertain. Market price fluctuations that are greater than the call option premium will yield a larger payoff than if the options were not exercised because it gives the managers the right to buy inventories at a lower cost and/or hold until processed, thus possibly also experiencing a convenience yield.
Conceptual Relationship

The conventional approach of inventory analyzes the optimal stock-out probability by deriving the expected benefit of stocking an additional unit with the expected cost of stocking that additional unit (Stowe and Su, 1997). The following equation shows expected revenue from selling the unit of $p\pi$; the expected salvage value is $(1-\pi)s$, and the expected stock-out penalty avoided is $n\pi$ (Stowe and Su, 1997):

$$p\pi + (1-\pi)s + n\pi = c$$

3.31

$$\pi = \frac{c-s}{p-s+n},$$

3.32

where

- $\pi = \text{the stock-out probability}$,
- $p = \text{the selling unit price per unit}$,
- $c = \text{the cost per unit}$, and
- $n = \text{penalty for a stock-out}$.

Stowe and Su (1997) alter the conventional equations above to be consistent with the contingent claims approach by replacing objective probabilities with risk-neutral probabilities. The expected benefits also have to be discounted by the riskless rate. With $N(d_2)$ as the risk-neutral probability of a stock-out, the new equation is as follows. This equation sets the present value of the expected benefit of stocking an additional unit of inventory equal to the investment in one unit of inventory (Stowe and Su, 1997):

$$e^{-\alpha}[pN(d_2) + s(1-N(d_2)) + nN(d_2)] = c.$$  

3.33

Using the above equation and rewriting it to get the equation in terms of the state price of the contingent claim yields the following equation (Stowe and Su, 1997):
The equation \( \frac{\partial C}{\partial X} = e^{-rt} N(d_2) = \frac{c - se^{-rt}}{p - s + n} \) is the state price of the contingent claim. The state price of the contingent claims is the value of a $1.00 payoff if the underlying asset price is above the exercise price. The state price is a calculation of two factors, the first factor being the present value and the second being \( N(d_2) \), which is the risk-neutral probability of a stock-out. According to Stowe and Su (1997), \( N(d_2) \) is the risk-neutral probability of the underlying asset price exceeding the exercise price, thus meaning the option is in the money. Using this state price is more economically sound because it includes the time value of money and the effects of risk. This advantage is the fundamental difference between the conventional approach and contingent claims and is why the latter is a more appropriate way to manage inventory.

Conventional and contingent claims forecast sales in two different ways. Conventional inventory management tools rely on an estimation of the distribution of future demand. Contingent claims use the underlying variables to forecast sales. These same variables are then used to value a portfolio of options. The payoff functions calculated with the contingent claims are based on the underlying assets value in the marketplace; once the payoffs are calculated, the optimal inventory policy is determined using the payoffs.

**Summary of Variables**

This section defines the important variables that must be considered when analyzing inventory strategies. All of the variables will have an effect on the inventory policy both negatively and positively, but they have to be understood in order to find the optimal level.
Procurement costs that are associated with the ordering cost, shortage cost, and temporal prices are among some of the factors that force agribusiness managers to carry inventory (Polataglu and Sahin, 2000). These costs will all go down with increased lot size and less reordering. Holding costs have an opposite effect; hence, as the lot size increases, holding costs will also increase (Polataglu and Sahin, 2000). In order to determine the optimal inventory level, a balance has to be found between the two types of cost. This balance will depend on demand forecasts, cost structures, and the operating conditions of the business.

Uncertainty of supply is an important factor when determining the optimal inventory policy. Uncertainty in quantity, price, quality, and other variables will have a large effect on how much inventory will be carried. Uncertainty of supply forecasting will have an impact on the supply prices, thus the need for accurate forecasting methods. The amount of a forecasting error will determine if a business is penalized with shortage costs or with excess inventory carrying costs. The amount of safety inventory carried by a business is also reflective of the accuracy of their forecasting. Thus, if the forecasting methods are not accurate, higher safety inventories must be carried. Hoshino (1996), states that the level of safety stock can be lowered by adopting the fixed interval ordering policy when the forecasting error is small.

Shortage costs and service levels are also related to supply forecasting in that if supply forecast is accurate, there would be no need to take these variables into consideration. Since there is always some sector of supply that is uncertain, shortage costs and service levels have to be incorporated into inventory management. In order
to meet a required service level in most cases, some level of safety inventory is required to prevent stock-outs which lower the service level of the firm. Shortage costs can be very costly to the firm if the buyer encounters switching costs and or decides to buy from another firm.

The salvage value of the inventory plays a vital role in the overall expected profit function of a firm. If the inventory can be sold at a reduced price other than zero, there is a greater chance the firm will hold inventories to reduce procurement costs as well as shortage costs and to increase the service levels.

The traditional purchasing approach (Goffin et al., 1997) shows that there are several key factors that influence purchasing decisions for the buyers. They are unit price, quality conformance, and speed of delivery. Goffin et al. (1997) found the unit price to be the main emphasis of purchasing, but the others were also taken into consideration when buying.

**Summary**

There is an apparent increasing awareness of real options and contingent claims in the management sector of business. Two factors to consider when managing inventory include the time value of money and the risk level associated with the particular industry. These two factors are not accounted for in the conventional inventory management approach. The ability of managers to conceptually see how inventory levels affect their profit will greatly influence how inventory is managed.

The EOQ model was an acceptable way of managing inventory in the past and still is used by many businesses today, but its value is limited in situations with higher
risk. It does not take into account the market value of the product which in today’s agribusiness is a very large part of risk, as commodity quality and quantity are sometimes limited.

The contingent claims approach is not new, but it is just starting be researched and adapted to more and more managerial tools. Its adds flexibility to the business as well as increases the NPV by finding the optimal inventory level that maximizes profits. Using contingent claims for inventory analysis can greatly increase profits once recognized and portfolios are built. There are a few basic limitations to contingent claims as listed in Chapter 2, but the benefits far outweigh the limitations when compared to conventional methods of inventory management.
CHAPTER 4
EMPIRICAL MODEL

Introduction

Wheat, durum, and oats are three commodities that pose unique problems in the marketplace. Fundamental characteristics in these markets create uncertainties that result in large volatility in the prices, quantity, and quality. It is difficult to correctly address uncertainty in conventional inventory models such as the economic order quantity model (EOQ). Until the recent introduction of contingent claims in inventory strategy analysis, there was not a financially sound method to address the issue of uncertainty.

Inventories can be interpreted as a real option. Specifically, they are similar to a call option. A bull spread is the type of call spread used in this study. It is created by buying a call option on a stock with a certain strike price and then selling a call option on the same stock with a higher strike price. This option created by call requires an initial investment which is the inventory. Inventories are a form of real options called the option on future sales; as such, there are two critical issues. One is to determine the optimal level of stocks, given the option value. The second is to determine the option value of inventories held in stocks.

This chapter describes the development of a contingent claims model that maps the payoffs of the inventory strategy onto the price of the underlying asset. The second step of the real options analysis is to develop a portfolio of options that replicates those payoffs. Then the final step is to place a value on the portfolio by subtracting the investment to obtain the net present value.
The purpose of this chapter is to describe the analytical model of the contingent claims approach. It starts with a brief model overview before proceeding into the mathematical models. The contingent claims models are then defined for three cases. The first case does not have a salvage value or shortage penalty, the second case has a salvage value but no shortage penalty, and the third case has a salvage value and shortage penalty. The model variables are then described and defined for all three cases and commodities. The economic order quantity (EOQ) model follows with explanations of the variables. A data section follows indicating where the information was gathered. It is followed by a chapter summary.

**Model Overview**

The contingent claims methodology is used to model commodity inventories. Sales of the finished product, in this case flour, are mapped onto the underlying commodity that is carried in inventory. Northern spring wheat 14+ percent protein, hard amber durum, and milling quality oats were used as the underlying commodities to be milled into flour or various other products. There are several differences among the wheat, durum, and oat commodities in this study. Wheat can be hedged, but the basis risk for higher protein may induce stockholding. Hard amber durum and milling oats cannot be easily directly hedged and thus have to be cross-hedged, which introduces additional risk. Overall quality of the commodities cannot be hedged, thus introducing a shortage risk for meeting requirements for end-use.

Demand for the finished product follows a continuous distribution. In this model, demand and inventory policy are determined annually. Using a continuous
distribution of demand focuses on supply risk and the effects it has on overall inventory implication of the raw commodity.

Three specific cases are modeled in this study. The first, the base case, does not have a salvage value or a shortage penalty, the second case encompasses a salvage value but has no shortage penalty, and the third case considers the salvage value along with a shortage penalty. The first model should result in the least amount of inventory due to less risk associated with running short, and also, it does not have a floor in place to which the company can liquidate storage and still at least gain the salvage value of the asset. The NPV of the inventory policies should also decline from Case 1 to Case 3 due to the increased costs of carrying higher inventory at a greater risk.

The EOQ is compared to the contingent claims inventory analysis in this study. The EOQ model is from Ballou (1999, p325). The EOQ model at its basic formulation is a close approximation to the base case of the contingent claims model. The more complex EOQ model takes into consideration lead times, probability of product being in stock, and standard error of forecast and is closely related to the complex contingent claims Case 3. The EOQ determines the optimal order quantity subject to assumptions where the contingent claim determines the value of an inventory carrying strategy.

**Mathematical Model Description**

The mathematical model accounts for the primary inventory costs and revenues. Costs include interest, storage, shipping, commodity, and a shortage penalty. Revenues include the price per unit sold and the salvage value. The
mathematical models in this study are adopted from Stowe and Su (1997). Derivations of the optimal stock-out probability calculations given the three cases are partially related to their work.

**Determining the Optimal Stock-Out Probability**

The optimal stock-out probability is an important parameter to determine and is one of the major differences between the conventional and contingent claims inventory approaches. The optimal stock-out probability is the probability of demand exceeding quantity (Stowe and Su, 1997). Conventional approaches that use the EOQ choose the optimal stock-out probability by equating the expected benefit of stocking an additional unit with the expected cost of stocking that additional unit; thus, payoff can be defined as

\[ p\pi + (1 - \pi)s + n\pi = c, \]

where

\( \pi = \) the stock-out probability, 
\( p = \) the selling price per unit, 
\( c = \) the cost per unit, 
\( s = \) the salvage value per unit, and 
\( n = \) the penalty for a stock-out.

Optimizing 4.1 can determine the conventional optimal stock-out probability which is

\[ \pi = \frac{c - s}{p - s + n}. \]

Selling price per unit is the value of what the good can be sold for. It is revenue that is flowing into the business from a sale of the finished product. The selling price is the flour price in this study. Cost per unit is the cost to buy the assets required to produce one unit of output. In this study, the cost per unit is the underlying commodity price. Salvage value is what the asset can be liquidated for; it
is the minimum value which the firm would see from holding or producing an asset. The salvage value used in this study is the price paid for the commodity plus ten cents, which implies normal market returns to storage with storage and shipping subtracted out. This is because the storage fee is lost, and the business has to pay to ship out the commodity. The stock-out penalty is a cost to the business if demand requirements are not met. Stock-out penalties in this study are calculated to be the margin on flour production. Finally, the stock-out probability is defined as the point at which demand exceeds quantity; this point is calculated in Equation 4.2.

The conventional approach to choosing stocking levels can be used in all three cases of this study. The numerator in Equation 4.2 is considered to be the carrying cost of an additional unit of inventory. The benefit of selling an additional unit of inventory is the denominator.

The contingent claims solution of the optimal stock-out probability replaces objective probabilities with risk-neutral probabilities by discounting the expected benefits by the riskless rate which generates the optimal state price. The optimal state price replaces the objective probabilities in the conventional approach with risk-neutral probabilities. The salvage value is an expected benefit that, if used, has to be discounted by the riskless rate. Using Equations 4.1 and 4.2 and transforming them into contingent claims formulation yields the optimal state price used in this study. The optimal state price is the probability of demand exceeding quantity that will maximize the NPV of the firm. The NPV calculations for Cases 1 through 3 can be seen in Equations 4.4, 4.8, and 4.12, respectively. The optimal state price is

$$-\frac{\partial C}{\partial X} = e^{-rt} N(d_2) = \frac{c - se^{-rt}}{p - s + n},$$

4.3
where

\[ N(d_2) = \text{Risk-neutral probability of a stock-out}, \]
\[ r = \text{Interest rate}, \]
\[ t = \text{Time}. \]

This formulation was adopted from Stowe and Su (1997). \( N(d_2) \) is the risk-neutral probability of the underlying asset price exceeding the exercise price, and the option is in the money. \( \partial C/\partial X \) is the state price which is the value of a \$1.00 payoff if the underlying asset price is above the exercise price. The exercise price is a price at which the underlying asset may be bought or sold in an option contract. This state price is one of the Black-Scholes option Greeks called delta. Delta is the first partial derivative of a portfolio’s value with respect to the value of the underlying asset.

Delta approximation is defined where a small change in the underlying asset’s current value equals the corresponding change in the portfolio’s current value (Contingency Analysis, 1996-current). Using the optimal state price and the delta approximation, the new exercise price of the option in the portfolio can be determined.

### Case 1: Base Case

Table 4.1 defines variables used in the mathematical equations.

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>Net present value of the inventory carrying policy</td>
</tr>
<tr>
<td>L</td>
<td>Number of long call options</td>
</tr>
<tr>
<td>S</td>
<td>Number of short call options</td>
</tr>
<tr>
<td>C(X=Min)</td>
<td>Value of the call option using the minimum exercise price</td>
</tr>
<tr>
<td>M</td>
<td>Additional payoff of increasing the exercise price of the short position</td>
</tr>
<tr>
<td>Q</td>
<td>Number of units stocked</td>
</tr>
<tr>
<td>C</td>
<td>Cost per unit stocked</td>
</tr>
<tr>
<td>V</td>
<td>Salvage value of commodity</td>
</tr>
<tr>
<td>X</td>
<td>Effect of adding an additional unit of inventory at the margin</td>
</tr>
<tr>
<td>( \partial C/\partial X(X=0) )</td>
<td>Time value of money with an exercise price of zero</td>
</tr>
</tbody>
</table>
The inventory strategy defined in the base case has no salvage value for excess inventories or no penalties for shortages. Sales are tied to the underlying commodity with a current price and standard deviation. The mathematical formulation of the NPV of the base case is

$$NPV = LC(X = MIN) - SC(X = MIN + MQ) - CQ.$$  \hspace{1cm} 4.4

The optimal inventory is the derivative of the NPV with respect to $Q$, and then setting the derivative equal to zero. Equation 4.5 is the derivative of Equation 4.4 with respect to $Q$ and setting it equal to zero.

$$\frac{\partial NPV}{\partial Q} = -S \frac{\partial C(X = MIN + MQ)}{\partial Q} - C = 0.$$ \hspace{1cm} 4.5

Equation 4.5 can be reduced to add in the marginal effect of adding another unit of inventory on the payoff, thus yielding

$$(4.6) \quad \frac{\partial NPV}{\partial Q} = -S \frac{\partial C(X = MIN + MQ)}{\partial X} \frac{\partial X}{\partial Q} - C = 0.$$ \hspace{1cm} 4.5

$\partial X/\partial Q$ is defined at the margin as the effect of adding an additional unit to inventory NPV. The additional unit of inventory increases the payoff by $X$ at the margin, and the $Q$ is the number of short call options. This number is calculated for the base case and is the same for all cases. The optimal state price is a risk-neutral probability level that is a balance between the costs and revenues of inventory that maximizes the total expected profits. The optimal state price occurs where

$$\frac{\partial C(X = MIN + MQ)}{\partial X} = C / (S * \frac{\partial X}{\partial Q}).$$ \hspace{1cm} 4.7

Using the $\partial C/\partial X$ in Equation 4.3, the optimal quantity and price can be found. The quantity found is the amount that should be held in inventory, and the price is the
value of that particular option of carrying that inventory on future sales. The optimal inventory suggested by the contingent claims model maximizes NPV. Figure 4.1 shows the payoff of the base case wheat model.

The payoff below the minimum exercise price of the underlying asset is zero and increases until it reaches a peak at the maximum exercise price of the underlying asset. The payoff increases at a slope equal to that of the selling price times the increase in sales as the underlying asset value increases.

The behavior of the payoff function occurs for several different reasons. The first is that the underlying asset price in wheat has a historical high and low. The historical low is $2.70/bu, and the historical high is $5.07/bu. At a price below the minimum price, the mill will not process any grain, and at a price above the maximum, it will operate at capacity. These two points are the reason for the long
and short calls. The long call replicates the minimum payoff of the inventory carrying policy, while the short calls replicate the maximum payoff of the inventory carrying policy. A linear line drawn between the two suggests the payoff may fall anywhere between the two points.

A call option is the right to buy an asset at a certain price by a certain date. The long call option means the business is purchasing the right to own an asset at a certain price by a certain date. The short call option means the business is selling the right to own an asset at a certain price by a certain date. Thus, the long call option replicates buying the inventory, and the short call options replicate selling in the inventory.

This base case is representative of an agribusiness that does not have a salvage value on excess inventory items stocked and also will not incur a shortage penalty if demand exceeds quantity stocked.

**Case 2: Salvage Value for Excess Inventories**

This second case assumes excess inventory can be liquidated for a predetermined salvage value. In the base case, salvage value is equal to the price paid per bushel plus ten cents to imply a normal market, minus storage and shipping costs. The salvage value in the grain market varies. If the market is inverted, the salvage value is less than the price paid for the commodity when it was bought. If the market is normal, the salvage value is greater than the price at which the asset was bought. This has a direct link on the optimal inventory level as it creates a floor in which to limit potential losses from the inventory carrying policy. Thus, the NPV is

\[
NPV = VQ \cdot \frac{\partial C}{\partial X}(X = 0) + LC(X = MIN) - SC(X = \text{MIN} + MQ) - CQ.
\]

4.8
The derivative of the above NPV with respect to $Q$ and setting it equal to zero yields

$$\frac{\partial \text{NPV}}{\partial Q} = V \frac{\partial C}{\partial X}(X = 0) - L \frac{\partial C(X = \text{MIN} + MQ)}{\partial Q} - C = 0. \tag{4.9}$$

Simplifying Equation 4.9 by adding in the marginal effects of adding an additional unit of inventory yields the following:

$$\frac{\partial \text{NPV}}{\partial Q} = -L \frac{\partial C(X = \text{MIN} + MQ)}{\partial X} \frac{\partial X}{\partial Q} - (C - V e^{-rt}) = 0. \tag{4.10}$$

Equation 4.10 can then be broken down to the optimal state price where

$$- \frac{\partial C(X = \text{MIN} + MQ)}{\partial X} = \frac{(C - V e^{-rt})}{S \ast \frac{\partial X}{\partial Q}}. \tag{4.11}$$

Using the $\partial C/\partial X$ as defined above, the optimal quantity and price can be found by deriving the optimum. The numerator in Equation 4.11 is the cost per unit stocked minus the salvage value per unit with the time value of money. This is a major difference between the conventional approach, and this contingent claims approach is the time value of money associated with the salvage value. The time value of money adds depth to the model because it takes into consideration the cost of having money tied up in inventory. The denominator of Equation 4.11 is the number of short call options times the marginal effect to the NPV of adding an additional unit of inventory. The quantity is optimal stock level and is the amount that should be held in inventory, and the price is the value of that particular option of carrying that inventory on future sales.

Figure 4.2 shows the payoff associated with the base case in wheat.

The payoff now has a different slope than the previous figure (Figure 4.1). The payoff changes because the inventory can now be sold at a salvage value, thus
changing the minimum payoff from zero. The minimum payoff using the strategy is the optimal stock multiplied by the salvage value. This is similar to imposing a price floor on the inventory payoff.

Figure 4.2. Wheat payoff with salvage value.

The long position in Figure 4.2 is shifted upwards compared to the base case (Figure 4.1), while the short position stays the same level. This upward shift in the payoff function represents the salvage value payoff if that option is exercised. With a salvage value, the firm has the option to liquidate the inventory. The slope of the line is not as steep as in Figure 4.1, meaning that an increase in the underling asset price does not have as much of an affect on the payoff as it did before in Case 1.

This second case is representative of an agribusiness that has a salvage value on excess inventory items stocked but will not incur a shortage penalty if demand exceeds quantity stocked.
Case 3: Salvage Value and Stock-Out Penalty

Case 3 deals with a common inventory problem of demand exceeding the quantity carried on inventory. When this happens, the business generates a stock-out penalty. If the business has a shortage, it implies a shortage penalty. A salvage value is also allowed in this third case. With both the salvage value and the shortage penalty, the case replicates what a typical agribusiness would face. The NPV for the inventory carrying policy is

$$NPV = VQ \frac{\partial C}{\partial X}(X = 0) + LC(X = MIN) - SC(X = MIN + MQ) - CQ.$$  \hspace{1cm} 4.12

Taking the derivative of the NPV with respect to Q and setting it equal to zero yields the following:

$$\frac{\partial NPV}{\partial Q} = V \left( \frac{\partial C}{\partial X}(X = 0) - L \frac{\partial C(X = MIN + MQ)}{\partial Q} \right) - C = 0.$$

Simplifying the above equation by adding in the marginal effects of adding an additional unit of inventory along with the shortage penalty if demand exceeds quantity stocked yields the equation below:

$$\frac{\partial NPV}{\partial Q} = L \frac{\partial C(X = MIN + MQ)}{\partial X} \frac{\partial X}{\partial Q} - (C - Ve^{-r_t}) = 0.$$

Equation 4.14 can be broken down to derive the optimal state price. The optimal state price is equal to the equation below:

$$\frac{\partial C(X = MIN + MQ)}{\partial X} = \frac{(C - Ve^{-r_t})}{S * \frac{\partial X}{\partial Q}}.$$  \hspace{1cm} 4.15

Using the $\partial C/\partial X$ formula, Equation 4.3, the optimal quantity and price can be found by iterating until it has the desired value. Equation 4.15 is similar to Equation 4.11 but has a different number of short call options. This is because the short call
options now allow for a shortage penalty. The shortage penalty increases the
denominator; thus, the optimal state price is a smaller number than found in Equation 4.11. The optimal quantity derived is the amount that should be held in inventory, and the price is the value of that particular option of carrying that inventory on future sales. Figure 4.3 shows the payoff associated with Case 3.

![Figure 4.3. Wheat payoff with salvage value and shortage penalty.](image)

Note the payoff change between Figures 4.3 and 4.2 when a penalty is implied for shortages. The payoff changes because the firm can lose sales and revenue if the commodity is not available when needed. The short call option position now becomes a negative slope instead of a constant.

This third base case is representative of an agribusiness that has a salvage value on excess inventory items stocked and will also incur a shortage penalty if demand exceeds quantity stocked.
All three cases are combined in Figure 4.4 to illustrate differences among the three case payoffs.

Figure 4.4. Combined wheat payoffs of all cases.

Case 1 shows the payoffs if there is no salvage value or shortage penalty. Case 2 shows the payoffs of an inventory strategy with a salvage value but no shortage penalty. Payoffs for Case 3 with a salvage value and a shortage penalty are also shown. This figure is interesting as it plots all the wheat payoff functions combined into one figure.

**Number of Calls Calculations**

The following sections define how the number of long and short calls is determined in the different cases. The contingent claims approach ties future sales to the underlying asset price. Thus, an increase in the asset price yields an increase in sales. This number stays the same throughout all the cases.
In this study, calls are used to replicate the payoffs of the inventory policy. Both long and short call positions are used to map the inventory policy onto the underlying commodity given the associated risks in price. A call option is the right to buy an asset at a certain price by a certain date. The long call options modeled means the business is purchasing the right to own an asset at a certain price by a certain date. The short call options modeled means the business is selling the right to own an asset at a certain price by a certain date. Thus, the long call options replicate buying the inventory, and the short call options replicate selling or liquidating the inventory. This strategy of two call options is created by buying a call option on the underlying asset stock with a certain strike price and selling a call option on the same stock with a higher strike price. In financial terms, this is called a bull spread. A call price always decreases as the strike price increases; thus, the value of the option sold is always less than the value of the option bought (Hull, 2002). This strategy requires an initial investment because it is created from calls.

**Case 1**

Table 4.2 defines the variables used in defining the mathematical equations for calculating the number of call options.

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Number of long calls</td>
</tr>
<tr>
<td>S</td>
<td>Number of short calls</td>
</tr>
<tr>
<td>P</td>
<td>Selling price per unit</td>
</tr>
<tr>
<td>V</td>
<td>Salvage value per unit</td>
</tr>
<tr>
<td>H</td>
<td>Shortage penalty</td>
</tr>
<tr>
<td>IS</td>
<td>Increase in sales as asset price increases</td>
</tr>
</tbody>
</table>
The number of long and short call options in the base case is equal. The number of long call options is derived by multiplying the increase in sales contingent on the underlying asset price by the price per unit sold:

\[ L = P \times IS. \]  \hspace{1cm} 4.16

An example from Stowe and Su (1997, p47) states “Sales will be zero if asset price is below $30, but will increase by ten units per $1 increase above this price.” That describes the increase in sales contingent on the underlying asset price. Once the numbers of long call options are calculated, the same number can be used for the number of short call options in this case because these are offsetting options. The long call option is the value of the underlying asset or product, and the short position is the value of future sales of that asset.

**Case 2**

The numbers of long and short call options for Case 2 are also equal. The difference between the calculations in Case 1 is the number of call options and Case 2 is the sales price or net revenue changes due to the fact of having a salvage value. The number of long call options in Case 2 is derived by subtracting the salvage value from the price per unit sold and then multiplying it by the increase in sales contingent on the underlying asset price:

\[ L = (P - V) \times IS. \]  \hspace{1cm} 4.17

This yields the number of long call options which is equal to the number of short call options.
Case 3

The numbers of long and short call options in Case 3 with a salvage value and shortage penalty are not equal. The numbers of long call options are the same as Case 2, but the number or short call options changes to reflect the shortage penalty. The number or short call options now becomes the shortage penalty multiplied by the increase in sales contingent on the underlying asset price, then adding it to the number of long call options:

\[ S = H \times IS + L. \]

4.18

The number of short call options is increased due to the shortage penalty if the business is not able to meet demand in the specified time period. This is the downward slope to the payoff function in Case 3 that can be seen in Figure 4.4.

Detailed Description of Model

This section provides a more detailed description of the model. The flow of the inventory and how various costs are calculated are included in the following sections. A table at the end of the section shows the values used in the following calculations. The following section describes data sources.

Calculations

Conversions

With the inventory payoff being modeled to the underlying asset, it is necessary to convert underlying asset input prices into output prices (e.g., flour equivalents). These numbers were obtained from contacts within each of the industries.
Underlying Asset Details

The model reacts to changes in the price of the underlying asset and its variability. The mill draws grain from the local market. In this case, Minneapolis prices were used for NO. 1 dark northern spring (14%+Protein), NO. 1 hard amber durum, and milling quality oats. The standard deviation for the prices was obtained from the years 1990-02. Yearly prices were used because annual stock-use ratio was also used in this study.

Investment per Unit

Investment per unit is a calculated value based on the price of the commodity bought and placed in storage and storage costs. Actual price of the commodity in a given time period was used. The implied storage cost was four cents per bushel, close to commercial storage costs typically paid by agribusiness. Investment per unit is

\[
I = (PB + SC) * M
\]

where
\[
I = \text{Investment per unit } $/\text{cwt},
PB = \text{Price paid per bushel } $/\text{bu},
SC = \text{Storage cost per bushel } $/\text{bu}, \text{ and}
M = \text{Multiplier for converting bushels to hundredweight.}
\]

Adding the price of the commodity per bushel and the storage costs yields the investment per unit.

Price per Unit Sold

Price per unit sold is calculated by adding the revenues from the finished product, to mill feed values. The product price was used for the finished product, and that price was obtained from the same time period as the commodity price. Mill feed
values were obtained from the same time period and also added into the price per unit sold as they are a valuable byproduct of milling:

\[ PS = PP + MF \]

where

\[ PS = \text{Price per unit sold } \$/\text{cwt}, \]
\[ PP = \text{Product price } \$/\text{cwt}, \]
\[ MF = \text{Mill feed price } \$/\text{cwt}. \]

Adding the product price to the mill feed price calculates the product price that was used in this study. The prices are in hundredweights.

**Salvage Value**

The salvage value is an important calculation in Cases 2 and 3. It is calculated by subtracting the storage cost and shipping cost on a per-bushel basis from the commodity price used in the investment per unit and then adding ten cents to account for a normal market with positive carry. This salvage value is then converted into cwt:

\[ SV = (CP - ST - SP + B) * M \]

where

\[ SV = \text{Salvage value } \$/\text{cwt}, \]
\[ CP = \text{Commodity price } \$/\text{bu}, \]
\[ ST = \text{Storage cost } \$/\text{bu}, \]
\[ SP = \text{Shipping cost } \$/\text{bu}, \]
\[ B = \text{Normal market carry } \$/\text{bu}, \]
\[ M = \text{Multiplier for converting bushels to hundredweight}. \]

A normal market is used in this study on all the grains. The variable B in Equation 4.21 is ten cents in this study. That implies that the market price for the grain went up ten cents from the time the grain is bought to the time the salvage value is calculated. This is a normal assumption as the market is normal and will see an increase in the salvage price due to normal market conditions.
Inverted markets can be added into this model using the salvage value. The value of the underlying asset can be adjusted to show inverted or normal market trends. For example, if the base price is $4.00/bu, an inverted market would be when the commodity value twelve months from now is $3.80/bu, which provides a disincentive to carry stocks, but the reverse is true if the market price in twelve months is $4.20/bu; this provides an incentive to carry greater stocks. This is an added benefit of using the contingent claims model.

Using the salvage value creates revenue in the option that has to be discounted by the time value of money which, in this case, is the annual interest rate. This creates a floor for the option given that a particular inventory position is at least guaranteed the salvage value per unit.

**Shortage Penalty**

A shortage penalty is used in the third case and has an impact on how much inventory to carry and the option value. The shortage penalty in Case 3 is the lost milling margin from not running the raw commodities through the mill. This is much like a lost sale because it has a negative effect on the revenue flows of a business. If demand exceeds the quantity available in storage, the mill loses revenue because of the lost sales. This does not take into consideration lost or disgruntled customers due to the fact that in the industry, there are many substitutes. Milling margin values used for the shortage penalty were obtained from industry contacts.
**Base Case Model Values**

Table 4.3 shows the values used in the three base cases. Variables in the below table are representative of what a typical agribusiness would face in the milling industry. The sources of data used in Table 4.3 are described in the section below.

<table>
<thead>
<tr>
<th></th>
<th>Wheat</th>
<th>Durum</th>
<th>Oats</th>
</tr>
</thead>
<tbody>
<tr>
<td>lb/bu</td>
<td>60</td>
<td>60</td>
<td>32</td>
</tr>
<tr>
<td>Mill Extraction</td>
<td>0.720</td>
<td>0.706</td>
<td>0.600</td>
</tr>
<tr>
<td>Price $/bu</td>
<td>4.49</td>
<td>4.90</td>
<td>2.43</td>
</tr>
<tr>
<td>Normal Market Carry $/bu</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
</tr>
<tr>
<td>St. Dev. Price</td>
<td>0.5587</td>
<td>1.0949</td>
<td>0.3534</td>
</tr>
<tr>
<td>Interest Rate %</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Storage $/bu</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Shipping $/bu</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Flour $/cwt</td>
<td>10.85</td>
<td>12.85</td>
<td>18.75</td>
</tr>
<tr>
<td>Mill Feed $/cwt</td>
<td>3.95</td>
<td>3.95</td>
<td>0.75</td>
</tr>
<tr>
<td>Grain Input Value $/cwt</td>
<td>10.37</td>
<td>11.56</td>
<td>12.66</td>
</tr>
<tr>
<td>Shortage Penalty $/cwt</td>
<td>1.2</td>
<td>1.2</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**Parameters and Assumptions**

There are several technical parameters that exist for each industry. These include capacity, extraction rates, operation rate, and storage capacity. The average capacity in cwt/day was taken along with the average storage capacity of the firms. These data are used to help break down overall industry usage into a prototypical agribusiness milling firm.

**Milling Margins and Increases**

The model requires a payoff from the inventory strategy to be mapped on the underling asset. In order to map the payoff onto the commodity, several industry characteristics are considered on an industry basis and then broken down to a firm
level. The industry characteristics were flour grind, commodity production, and percentage of acceptable quality.

**Effect of Quality and Quantity Shortfalls on Inventory Value**

An important variable motivating the contingent claims theory in modeling commodity inventory carrying is the effect on the underlying asset price on the future sales of a product. This has a large impact on stability and reliability of the model. To pinpoint this number along with adding quality and quantity variables into the model, the following steps were used.

Stowe and Su (1997, p47) use the following as their contingent claim: “Sales will be zero if the asset price is below $30, but will increase by ten units per $1.00 increase above this price.” This is a large assumption in the contingent claims model as it is one of the main numbers behind mapping the payoff of the future sales on the underlying commodity. Similar data and relationships do not exist in the grain processing sector, though if they could be derived for individual firms, they would be highly autonomous. Thus, in order to represent how much storage would be used in the grain industry, the following steps were taken to simulate typical industry storage demands.

First the following annual data were collected: flour grind, commodity production, beginning stocks, imports, domestic use, exports, price, and quality. The data were all converted into similar units. A representative annual usage was calculated using historical industry data:

\[
BS + PD + I - DU - X = ES \\
SUR = ES / (DU + X)
\]

4.22
where
- BS = Beginning stocks,
- PD = Production,
- I = Imports,
- DU = Domestic use,
- X = Exports,
- ES = Ending stocks, and
- SUR = Stocks to use ratio.

Beginning stock is the average of the years 1998-2002 beginning stock. Production was assumed to be the average of the years 1998-2002. Imports were the average of the past five years. Total supply is found by summing up beginning stocks, production, and imports. Domestic use was assumed to be the average of the past five years with dependence on the correlation between total domestic use and commodity grind. Exports are the average of the past five years. Ending stocks are then total supply minus domestic use minus exports. The stocks-to-use ratio was calculated by adding the domestic use and exports and then dividing the ending stocks by that total usage.

A projected price is estimated using historical annual stock to use ratio and commodity prices. The regression was run using the years 1990-2002. A shorter time period was used to maintain a more accurate forecast due in part to changes in the industries. Price is a function of the stocks-to-use ratio of that given year. A regression was also run using the inverse stocks-to-use ratio, thus taking away the linear assumption. The two regressions were used to forecast a price of the above forecasted year using the above stocks-to-use ratio for the forecasted year:

\[ P = \alpha + \beta \cdot SUR + E, \]

where

\[ tESURP = 4.23. \]
where
\[ P = \text{Price}, \]
\[ \alpha = \text{Intercept}, \]
\[ \beta_1 = \text{Estimated parameter, and} \]
\[ E_t = \text{Error term from regression}. \]

The effect of the stocks-to-use ratio on the price of the commodity was then estimated. The more accurate forecast using the inverse stocks-to-use ratio was used in this study.

Quality of the commodity has an important effect on how much inventory agribusiness firms carry. The following equation was used to determine if an agribusiness would be able to process and sell the commodity stored:

\[ PD \cdot PM \cdot PA - D = LS, \]

where
\[ PD = \text{Production}, \]
\[ PM = \text{Average percent milled}, \]
\[ PA = \text{Percent acceptable}, \]
\[ D = \text{Demand}, \text{ and} \]
\[ LS = \text{Long/short}. \]

Production was represented as a risk-normal function with dependency on the correlation between commodity grind and commodity production. Average percentage of milled commodity is a calculated value that takes the acceptable production divided by the commodity grind. The acceptable production is found by taking production and multiplying it by percent acceptable. Percent acceptable is estimated using quality data. Yearly percent acceptable was best fit using the past ten years’ quality data. Quantity acceptable is found by multiplying production by average percent milled by percent acceptable. Demand is the risk normal of the past five years and is the independent variable. The quantity of storage used is found by
subtracting demand from the quantity acceptable. Quantity of storage used is also added as an output along with the two prices.

Quantity of storage used and prices were then simulated as a function of the inverse stocks-to-use ratio. The output data were gathered, and all negative data were censored at zero. Negative quantities were also set to zero to account for the resulting inability to sell. Figure 4.5 shows wheat supply and usage plotted against price with censored data.

Figure 4.5. Wheat supply and usage plotted against price.

The simulated prices and quantities were then plotted (Figure 4.5) for all the commodities. The figure shows the simulated industry selling pattern as prices increase. The high and low underlying commodity prices were input into the equation. This industry equation was then interpreted to be representative of a firm-
level decision. Table 4.4 shows the variables from the regression along with the variables used in breaking down the overall industry to a plant level.

<table>
<thead>
<tr>
<th></th>
<th>Wheat</th>
<th>Durum</th>
<th>Oats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>7.89</td>
<td>1.25</td>
<td>0.07</td>
</tr>
<tr>
<td>t-stat</td>
<td>5.0639</td>
<td>3.7690</td>
<td>1.5733</td>
</tr>
<tr>
<td>Error</td>
<td>166.81</td>
<td>2.56</td>
<td>12.69</td>
</tr>
<tr>
<td>t-stat</td>
<td>7.6555</td>
<td>2.0198</td>
<td>2.2601</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.8648</td>
<td>0.2042</td>
<td>0.2550</td>
</tr>
<tr>
<td>Minimum Price (bu)</td>
<td>2.70</td>
<td>3.48</td>
<td>118.93</td>
</tr>
<tr>
<td>Maximum Price (bu)</td>
<td>5.03</td>
<td>7.00</td>
<td>236.28</td>
</tr>
<tr>
<td>Industry Capacity Daily (cwt)</td>
<td>1,531,035</td>
<td>153,093</td>
<td>57,582</td>
</tr>
<tr>
<td>Plant Capacity Daily (cwt)</td>
<td>7,500</td>
<td>6,379</td>
<td>4,429</td>
</tr>
<tr>
<td>Number of Mills</td>
<td>204</td>
<td>24</td>
<td>13</td>
</tr>
<tr>
<td>Mill Slope</td>
<td>38,988</td>
<td>77,870</td>
<td>123,011</td>
</tr>
</tbody>
</table>

The result was a slope behavior that has quality and quantity risk. This number was adjusted so that it was between the ranges of historical prices. In the wheat market, the historical low was $2.70/bu, with a high of $5.03. The following formula derives the contingency used in this study:

$$CS = MS \times PP / HD,$$

where

- $CS =$ Contingent sales if the underlying asset increases by $1.00,
- $MS =$ Mill slope,
- $PP =$ Product price $/cwt, and
- $HD =$ Historical difference between minimum and maximum underlying asset price.

For example, the mill slope value of 38,988 is interpreted as the increase in the use of storage over the range of historical prices. The contingent sales value is then used as the contingent claim stating that flour production will increase at a given amount with an increase in the underlying commodity. This calculated number is
how much sales from inventory will increase when the asset price increases by a dollar.

**EOQ Model**

The conventional approach to inventory stockholding decisions using the EOQ model was also included in this study. It was compared to the contingent claims approach to evaluate if the contingent claims approach places more value on the future sales of the underlying asset and thus results in higher inventory levels. The values listed in Table 4.5 are those input into the EOQ model as defined in Chapter 4.

<table>
<thead>
<tr>
<th></th>
<th>Wheat</th>
<th>Durum</th>
<th>Oats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Demand</td>
<td>3,750,000</td>
<td>3,252,748</td>
<td>7,312,500</td>
</tr>
<tr>
<td>St. Deviation of Annual Demand</td>
<td>94,125</td>
<td>233,616</td>
<td>620,684</td>
</tr>
<tr>
<td>Procurement Costs, $/order</td>
<td>75</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>Value of the Item Carried in Inventory, $/unit</td>
<td>4.49</td>
<td>4.90</td>
<td>2.43</td>
</tr>
<tr>
<td>Storage Cost, $/unit</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Lead Time (year)</td>
<td>0.023</td>
<td>0.023</td>
<td>0.023</td>
</tr>
<tr>
<td>Selling Price, $/unit</td>
<td>6.394</td>
<td>7.116</td>
<td>3.744</td>
</tr>
<tr>
<td>Salvage Value, $/unit</td>
<td>4.35</td>
<td>4.76</td>
<td>2.29</td>
</tr>
<tr>
<td>Shortage Penalty, $/unit</td>
<td>0.518</td>
<td>0.508</td>
<td>0.480</td>
</tr>
<tr>
<td>In-Stock Probability</td>
<td>0.716</td>
<td>0.911</td>
<td>0.949</td>
</tr>
</tbody>
</table>

Standard deviation of annual demand is derived using industry usage data from the years 1990-2002. The industry data were then divided by the number of mills to get the average mill standard deviation. This value is then input into the EOQ model. Table 4.5 shows the values necessary to compute the EOQ on an average mill scale. The same values were used for both the EOQ and contingent claims model, but the value for the EOQ was broken down into bushels verses the contingent claims model using hundredweight.
Data Sources

Data used in this research was obtained from the following sources (Table 4.6).

Data Behavior

Stochastic and strategy variables are identified and included in the model. The stochastic variables in the model introduce quality and quantity risk in addition to the option valuation model to determine the optimal inventory carrying level.

The strategy variables in projecting the production and usage in a year include production, domestic use, percent acceptable, and demand. These variables are stochastic with the commands for @Risk shown in Table 4.7 for the base case. Variations were made for the different commodities as needed.

Summary

Contingent claims models can be used where the optimal level of inventory using options and the option value of carrying inventory are desired. When using contingent claims as an inventory management tool, managers have to carefully address the input values. The contingent claims inventory model has many different variables that have a significant impact on the model. The values determine the optimal state price which is more robust as it impounds the effects of risk and the time value of money. The conventional stock-out probability uses an objective function that is not risk neutral, and it also does not take into consideration the time value of money. Values that are used for the variables in this study were all explained earlier in this chapter.
Table 4.6. Data Sources

<table>
<thead>
<tr>
<th>Model Component</th>
<th>Data Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat Price</td>
<td>2004 USDA Wheat Year Book*</td>
</tr>
<tr>
<td></td>
<td>2004 USDA Wheat Year Book, Quarterly Wheat</td>
</tr>
<tr>
<td>Durum Price</td>
<td>Outlook*</td>
</tr>
<tr>
<td>Oat Price</td>
<td>MGEX Milling Quality Cash Price*</td>
</tr>
<tr>
<td>Flour Prices</td>
<td>Ingredient Week, Nov 4, 2003*</td>
</tr>
<tr>
<td>Mill Feed Prices</td>
<td>Ingredient Week, Nov 4, 2003*</td>
</tr>
<tr>
<td>Milling Margins</td>
<td>Industry Contacts</td>
</tr>
<tr>
<td>Storage Cost</td>
<td>Industry Contacts</td>
</tr>
<tr>
<td>Shipping Cost</td>
<td>Industry Contacts</td>
</tr>
<tr>
<td>Wheat Quality</td>
<td>US Hard Red Spring Wheat*</td>
</tr>
<tr>
<td>US Durum Quality</td>
<td>US Northern Grown Durum Wheat Report*</td>
</tr>
<tr>
<td>CN Durum Quality</td>
<td>Canadian Wheat Board*</td>
</tr>
<tr>
<td>Oats Quality</td>
<td>Saskatchewan Agriculture Statistics Handbook*</td>
</tr>
<tr>
<td>Wheat Grind and Durum Grind</td>
<td>2004 Grain &amp; Milling Annual, Milling &amp; Baking</td>
</tr>
<tr>
<td>Oat Grind</td>
<td>USDA Feed Grains Report*</td>
</tr>
<tr>
<td>US Wheat Supply and</td>
<td>2002 USDA Wheat Yearbook*</td>
</tr>
<tr>
<td>Disappearance</td>
<td>2002 USDA Wheat Yearbook*</td>
</tr>
<tr>
<td>US Durum Supply and</td>
<td>Canada: Grains and Oilseeds Supply and Disposition*</td>
</tr>
<tr>
<td>Disappearance</td>
<td>2004 Grain &amp; Milling Annual, Milling &amp; Baking</td>
</tr>
<tr>
<td>CN Oat Supply and</td>
<td>USDA Feed Grains Report, Feed Grains Delivery</td>
</tr>
<tr>
<td>Disappearance</td>
<td>Canada: Grains and Oilseeds Supply and Disposition*</td>
</tr>
<tr>
<td>Wheat Mill Characteristics</td>
<td>2004 Grain &amp; Milling Annual, Milling &amp; Baking</td>
</tr>
<tr>
<td>Durum Mill Characteristics</td>
<td>Wilson and Dahl, The Logistical Costs of Marketing</td>
</tr>
<tr>
<td>Oat Mill Characteristics</td>
<td>USDA Feed Grains Report, Feed Grains Delivery</td>
</tr>
<tr>
<td>EOQ Procurement Costs</td>
<td>Identity Preserved Wheat*</td>
</tr>
<tr>
<td>EOQ Lead Time</td>
<td>Industry Contacts</td>
</tr>
</tbody>
</table>

*Full reference found in References
Table 4.7. Case Variables and Distributions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>Normal Distribution, RiskDepC(&quot;demand&quot;,correl(Grind:Production))</td>
<td></td>
</tr>
<tr>
<td>Domestic</td>
<td>Normal Distribution, RiskDepC(&quot;demand&quot;,correl(Domestic Use:Grind))</td>
<td></td>
</tr>
<tr>
<td>Use</td>
<td>Percent</td>
<td>Acceptable Normal Distribution</td>
</tr>
<tr>
<td>Demand</td>
<td>Normal Distribution, RiskIndepC(&quot;demand&quot;)</td>
<td></td>
</tr>
</tbody>
</table>

The parameters and distributions form the base case models on which sensitivities are conducted to determine the impact of a variable. Assumptions are made where necessary for calculation simplicity. It is difficult to portray all factors that are included in an inventory; however, the most important aspects captured by this model include the inventory option value, salvage value, shortage penalty, revenue, and embedding the contingency of future sales to the underlying asset price. In addition, distributions are estimated for variables to make the model more realistic in the sense that many factors are not known with certainty.

The EOQ model parameters for the various commodities are also defined in this chapter. Historically, the EOQ model is very successful in finding an optimal inventory level, but it has several uncertain areas when trying to address the issue of risk. It is not easy to address uncertainty in the EOQ model; thus, this creates a pitfall in which the contingent claims method is superior.
CHAPTER 5
RESULTS AND SENSITIVITIES

Introduction

Inventory can be viewed as a real option, specifically an option on future sales. Real options create value in uncertain conditions and, using contingent claims payoffs from inventory positions, can be mapped onto the underlying asset. Contingent claims determine the optimal quantity using the state price and also create an option on the inventory. Contingent claims are a valuable inventory management tool. As there becomes more uncertainty in the commodity markets, it is creating higher volatility, and to address this, it is necessary to use financial instruments that account for the increased risk. Chapter 4 builds up the contingent claims model used in this study, and this chapter will explain the results of the model.

This chapter includes the results from the base case models and a presentation of the impacts that various strategy variables have on the optimal state price that has a direct effect on inventory holding. Section one is a definition of the three different commodity models for wheat, durum, and oats. The second section defines the results of the economic order quantity (EOQ) model on each of the three commodity models. The third section consists of sensitivities to the Case 3 model. A summary of the results is presented in the last section. The base case models are designed to represent a typical inventory situation in the various commodity milling industries analyzed.

Base Case Model Definition and Results

Each of the models contains three separate cases. The first case does not contain a salvage value or shortage penalty. The second contains a salvage value but
does not have a shortage penalty. Salvage value and shortage penalties are both included in the third case.

**Wheat Base Case**

The grain in the wheat model is a northern spring wheat with 14+ percentage protein. Since wheat can be directly hedged, the relevant risk is that of the basis risk. Basis is defined as the difference between the current cash price and the futures price of the same commodity (Chicago Board of Trade, 1989). The basis in the northern spring wheat with 14+ percent protein market fluctuates, thus imposing uniqueness in this grade of wheat.

In presenting the results, there are a number of important values that are derived and interpreted. They were derived in the previous chapters and are defined here prior to presenting the results. The optimal state price is the risk-neutral probability of the underlying asset price exceeding the exercise price and the option in the money. This optimal state price replaces the objective probability in the conventional optimal stock-out probability and is more economically feasible because it discounts for the time value of money. In option valuation, the optimal state price is similar to the Greek delta in the Black-Scholes option pricing model. The exercise price for the short call options is determined using the optimal state price representing the exercise price for the short call option that maximizes the NPV of the inventory storage function. When the short exercise price derived by the contingent claims model is greater than the exercise price of the long position, a positive inventory is suggested, else a short position is suggested to maximize the NPV. In some cases, the optimal inventory level is positive; in others it is negative, but by definition, it is the
value that maximizes NPV. The NPV is the option value and is shown on a per-bushel and hundredweight basis. The interpretation of the option value is the value that a firm can add back into the revenue flows from that specific inventory storage function. Finally, the results are used to derive the optimal storage level as a percent of milling capacity. This number is calculated by dividing the optimal inventory quantity suggested by the contingent claims model by the annual requirements.

Table 5.1 shows the results of base case in wheat. These results were derived using values defined in Chapter 4. Given the optimal state price, the optimal quantity is then calculated using the formulas given in Chapter 4. The larger inventory affects the option value as shown in Table 5.1.

The exercise price variable in Table 5.1 is the exercise price chosen by the model through an analytical solution.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal State Price</td>
<td>0.85</td>
<td>0.59</td>
<td>0.39</td>
</tr>
<tr>
<td>Exercise Price</td>
<td>$1.28</td>
<td>$2.90</td>
<td>$4.18</td>
</tr>
<tr>
<td>Quantity (cwt)</td>
<td>(24,355)</td>
<td>3,325</td>
<td>25,315</td>
</tr>
<tr>
<td>Quantity (bu)</td>
<td>(56,379)</td>
<td>7,698</td>
<td>58,601</td>
</tr>
<tr>
<td>% Mill Cap. Store</td>
<td>-1.50%</td>
<td>0.21%</td>
<td>1.56%</td>
</tr>
<tr>
<td>NPV</td>
<td>$(502,197)</td>
<td>$809</td>
<td>$(11,979)</td>
</tr>
<tr>
<td>NPV ($/bu)</td>
<td>$(8.91)</td>
<td>$0.11</td>
<td>$(0.20)</td>
</tr>
<tr>
<td>NPV ($/cwt)</td>
<td>$(20.62)</td>
<td>$0.24</td>
<td>$(0.47)</td>
</tr>
</tbody>
</table>

The percent of stored milling capacity increases from Case 1 to Case 3. The pattern among the cases is to be expected as the first case does not have a salvage value or shortage penalty. Case 2 has a salvage penalty, thus increasing the amount stored due to an implied price floor. Storage is the greatest in Case 3 due to the salvage value and shortage penalty implied.
The NPV is negative along with a negative storage quantity in Case 1. This is not abnormal due to the lower profit margins in the industry and is due to the volatility in prices. The model is calculating that the optimal position is to be short in Case 1. With a short inventory position, the model speculates that the prices are too high and carrying inventory is risky without a salvage value. Case 2 with the salvage penalty has a positive NPV and quantity stored. Case 3 shows a negative NPV but a positive quantity stored. This can be related to option theory in which it maximizes NPV under current conditions. The negative NPV in Case 3 shows that when a mill stores wheat, it will lose money, but losses are minimized.

The negative quantity suggested by the contingent claims model is similar to a short inventory position. Price volatility is one of the main reasons the model suggests a short position in Case 1 and long positions in both Cases 2 and 3. The historical price range used in wheat in the years 1990-2002 had a minimum price of $2.70/bu and a maximum of $5.03/bu. Standard deviation of the historical price ranges is 0.56. This standard deviation is similar to oats, but both are much less than the standard deviation of durum.

Figure 5.1 illustrates the NPV of the three different cases along with the percentage stored. The x-axis in Figure 5.1 is case. Case 1 is the number on the axis and so on. A peak can be seen in the NPV in Case 2, but the percent stored is increasing linearly. The peak in the NPV in the second case is due to the salvage value with no shortage penalty. When implying a shortage penalty, it drops the NPV to a negative value, but it is still maximizing the NPV given the conditions. The contingent claims model maximized the NPV of an inventory storage function; thus,
for example, using Case 3, the NPV per bushel is negative, but it is still at the maximum. If a different level of inventory was carried other than the optimal quantity suggested by the contingent claims model, the losses would be greater.

Figure 5.1. NPV and wheat stored as a percentage of milling capacity.

There are three payoff functions that are generated when using the three cases. Case 1 without a salvage value and shortage penalty, Case 2 with a salvage value but not shortage penalty, and Case 3 with a salvage value and a shortage penalty all have different payoff functions. Figure 5.2 illustrates the payoff function for each. As seen in Chapter 4, the payoff of Case 1 at a price below $2.70/bu is zero but increases until it reaches a peak at a price of $5.03/bu and flattens out. The payoff function for Case 2 using a salvage value creates a price floor in which at least the salvage value is guaranteed as revenue from storing wheat and increases to a price of
$5.03/bu and then flattens out like Case 1. Case 3 then implies a shortage penalty on top of Case 2, and the negative slope occurs due to the negative affect of the shortage penalty on the payoff.

Figure 5.2. Combined wheat payoffs all cases.

**Durum Base Case**

The specification in the durum model is No.1 hard amber durum. Semolina is the durum milling output that is mapped onto the underlying commodity. Durum cannot be directly hedged in the commodity market. Hence, volatility and quality risk exist in the durum industry, thus making it interesting to analyze. The data used in this model are defined in Chapter 4. One specific variable that has an impact is the volatility. The drastic volatility in this case causes the model to show negative optimal storage levels suggesting a short inventory position. In this low margin, industry volatility has a large impact on the overall results of the model.
The optimal state price decreases when moving from Case 1 to Case 3. When the optimal state price decreases, it increases the exercise price of the short call option, thus calculating an increased optimal quantity stored. The increased quantity stored has a positive effect on the NPV as it brings it closer to zero. Table 5.2 shows the results of the base case durum model.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal State Price</td>
<td>0.84</td>
<td>0.58</td>
<td>0.39</td>
</tr>
<tr>
<td>Price</td>
<td>$0.04</td>
<td>$1.54</td>
<td>$3.06</td>
</tr>
<tr>
<td>Quantity (cwt)</td>
<td>(76,030)</td>
<td>(43,039)</td>
<td>(9,244)</td>
</tr>
<tr>
<td>Quantity (bu)</td>
<td>(179,487)</td>
<td>(101,602)</td>
<td>(21,823)</td>
</tr>
<tr>
<td>% Mill Cap. Store</td>
<td>-5.52%</td>
<td>-3.12%</td>
<td>-0.67%</td>
</tr>
<tr>
<td>NPV</td>
<td>$(1,557,154)</td>
<td>$(1,078,766)</td>
<td>$(258,230)</td>
</tr>
<tr>
<td>NPV ($/bu)</td>
<td>$(8.68)</td>
<td>$(10.62)</td>
<td>$(11.83)</td>
</tr>
<tr>
<td>NPV ($/cwt)</td>
<td>$(20.48)</td>
<td>$(25.07)</td>
<td>$(27.94)</td>
</tr>
</tbody>
</table>

The model suggests holding negative inventories, thus suggesting a short inventory position. The short inventory position is due in part to the normal distribution of prices along with very high volatility in the market, and the current price of durum is very high. With that the model is showing that the potential to lose money in durum is very large with high prices and high volatility. The historical prices from the years 1990-2002 had a minimum price of $3.48/bu and a maximum of $7.00/bu. The standard deviation of the historical prices was 1.0959. This standard deviation creates high volatility in the durum model when compared to the volatility of the wheat model of 0.5587. This greater volatility is one of the reasons the model speculates a short inventory position.

The model is calculating a short position, but when moving from Case 1 to Case 3, the size of the short position decreases. This is also due to the assumptions.
among the three cases of salvage in Case 2 and salvage value and shortage penalty in Case 3, but Case 3 has the most negative NPV per bushel. The more negative NPV per bushel is because the quantity short is getting closer to a positive storage quantity along with the NPV. Once again the NPV could be more negative if any position is taken other than the short inventory. Figure 5.3 shows the NPV of the durum storage and the percent stored of milling capacity.

Figure 5.3. NPV and durum stored as a percentage of milling capacity.

The visual effects of the cases are shown in Figure 5.3. This shows that decreasing the short inventory position decreases the NPV per bushel, but notice that it does not fall as fast going from Case 2 to Case 3 as it did from Case 1 to Case 2. The short inventory position is due to high volatility, current price, and low margin in the industry. A later section in this chapter will show what affect lowering the underling durum price has on the storing pattern.
Oats Base Case

The specification in the oats model is No. 2 heavy Minneapolis milling oats. This is an interesting market with higher margins than both wheat and durum, but it also has similar characteristics to durum in the fact that it cannot be directly hedged with high volatility. The high margin in this industry leads to some very interesting inventory storing results. Table 5.3 shows the oat model base case results.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal State Price</td>
<td>0.66</td>
<td>0.28</td>
<td>0.21</td>
</tr>
<tr>
<td>Price</td>
<td>$1.51</td>
<td>$2.55</td>
<td>$2.82</td>
</tr>
<tr>
<td>Quantity (cwt)</td>
<td>58,960</td>
<td>205,811</td>
<td>243,308</td>
</tr>
<tr>
<td>Quantity (bu)</td>
<td>307,086</td>
<td>1,071,931</td>
<td>1,267,227</td>
</tr>
<tr>
<td>% Mill Cap. Store</td>
<td>4.20%</td>
<td>14.66%</td>
<td>17.33%</td>
</tr>
<tr>
<td>NPV ($)</td>
<td>$241,576</td>
<td>$677,041</td>
<td>$(1,275,399)</td>
</tr>
<tr>
<td>NPV ($/bu)</td>
<td>$0.79</td>
<td>$0.63</td>
<td>$(1.01)</td>
</tr>
<tr>
<td>NPV ($/cwt)</td>
<td>$4.10</td>
<td>$3.29</td>
<td>$(5.24)</td>
</tr>
</tbody>
</table>

The optimal state price for the oats model is much less than the optimal state prices for wheat and durum. The state prices are calculated in Chapter 4. With the lower state prices, the model uses a higher short option exercise price; thus, it calculates a higher optimal quantity. This higher quantity increases the NPV but decreases the NPV per bushel, from Case 1 to Case 2. Case 3 has a negative NPV and a negative NPV per bushel, but that is the calculated maximum given the shortage value under Case 3.

As expected in a higher margin industry such as oats, there is a greater incentive to store more of the raw commodity. Moving from Case 1 to Case 3, the percentage stored of milling capacity increases, but it is interesting to note how much more of the commodity is stored in this model versus the other wheat and durum
models. Once again, like all of the other models, Case 3 in this oats model also has a negative NPV; this can be attributed to the shortage penalty in which, if there was no inventory carried, the losses would be much greater. Figure 5.4 illustrates the NPV of storing oats as well as the oats stored as a percentage of milling capacity.

![Figure 5.4. NPV and oats stored as a percentage of milling capacity.](image)

The visual effects of the cases can be seen in Figure 5.4. One interesting thing to notice is how fast the percentage stored increases from Case 1 to Case 2 and slowly increases to Case 3. While the percentage stored increases rapidly, the NPV per bushel gradually falls, and when the percentage stored increases slowly from Case 2 to Case 3, the NPV per bushel falls drastically.

The drastic fall in the NPV can be associated with a greater shortage penalty in oats than the implied penalty for wheat and durum. This higher margin industry has greater risk of falling short because of the already limited number of suppliers.
The oats industry has different characteristics than the wheat and durum markets, such as many suppliers and highly concentrated industry.

**Base Case Comparison**

The base cases are all different, and all three grains have characteristics that created the differing inventory strategies. Wheat can be hedged but has high basis risk, durum cannot be hedged but has high volatility, and oats cannot be hedged and is a high margin industry. Table 5.4 shows the optimal state price, NPV per bushel, and the percent store of milling capacity. The percent store of milling capacity is found by dividing the optimal quantity by the annual requirements of the mill.

The optimal state price decreases when moving from Case 1 to Case 3 in all three grains. In wheat and durum, the optimal state prices are similar in all three cases, but when looking at oats, the optimal state prices are much less in all three cases. The decrease in optimal state price is because of the value used for the variables as shown in Chapter 4. The lower the optimal state price, the higher the percent stored of milling capacity, or in durum, it is a smaller short inventory position. The NPV per bushel in Case 3 is less than Case 2. This is due to the shortage penalty in the model in which if the firm runs out of the grain, it loses revenue, in which case the model is speculating to store more inventory to compensate. The NPV per bushel shows the maximum option value that a firm can gain using the contingent claim model. If the option value is negative, it simply implies that storing is costing the firm money, but at a minimized amount when compared to a non-optimal quantity.
Table 5.4. Contingent Claims Results

<table>
<thead>
<tr>
<th></th>
<th>Optimal State Price</th>
<th>NPV ($/bu)</th>
<th>% Store of Milling Capacity (bu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>0.85</td>
<td>$(8.91)</td>
<td>-1.5</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.59</td>
<td>$0.11</td>
<td>0.2</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.39</td>
<td>$(0.20)</td>
<td>1.6</td>
</tr>
<tr>
<td>Durum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>0.84</td>
<td>$(8.68)</td>
<td>-5.5</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.58</td>
<td>$(10.62)</td>
<td>-3.1</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.39</td>
<td>$(11.83)</td>
<td>-0.7</td>
</tr>
<tr>
<td>Oats</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>0.66</td>
<td>$0.79</td>
<td>4.2</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.28</td>
<td>$0.63</td>
<td>14.7</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.21</td>
<td>$(1.01)</td>
<td>17.3</td>
</tr>
</tbody>
</table>

**Economic Order Quantity Model**

The base case wheat model is compared to the EOQ model in this section. One of the objectives of this study is to see if the contingent claims methodology actually implies that the agribusinesses should be carrying larger quantities of inventory. There are two different types of EOQ models used in this study; the first is the very basic model, and the second has a lead time and shortage penalties. The basic EOQ model is similar to Case 1 in all three models, while the EOQ with lead time and shortage penalties is similar to Case 3 of the contingent claims models in this study. A point to consider about the differences between the EOQ in the contingent claims is that the EOQ seeks to minimize total cost while contingent claims seeks to maximize NPV.

**Wheat**

Inputs for the wheat EOQ are in Chapter 4. Using those inputs, the EOQ was calculated for a wheat mill. The same prices were used in both models. Table 5.5
shows the results of the base and advanced EOQ along with Case 1 and Case 3 contingent models for comparison.

<table>
<thead>
<tr>
<th>Contingent Claims</th>
<th>Percentage Stored of Milling Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>-1.5%</td>
</tr>
<tr>
<td>Case 3</td>
<td>1.6%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EOQ Results</th>
<th>Percentage Stored of Milling Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>0.9%</td>
</tr>
<tr>
<td>With Lead &amp; Shortage</td>
<td>1.0%</td>
</tr>
</tbody>
</table>

Table 5.5 shows that in the basic EOQ and contingent claims model, the EOQ is storing but the contingent claims model is suggesting a short inventory position. The contingent claims model is suggesting a short inventory position of 1.5 percent of milling capacity, while the EOQ model is storing 0.9 percent of milling capacity. Milling capacity is the optimal quantity derived by the models divided by annual milling demand, but when comparing Case 3 in the wheat model to the EOQ with lead time and shortage penalties, the contingent claims model suggests storing a greater quantity. In Case 3, the contingent claims model is storing 1.6 percent of milling capacity, while the EOQ is storing 1 percent. This is what was to be expected of the model because the contingent claims methodology includes the time value of money and the risk-neutral probability and the option value.

**Durum**

Inputs for the durum EOQ are in Chapter 4. Using those inputs, the EOQ was calculated for a durum mill. Prices remain the same in both models to give an
accurate comparison. Table 5.6 shows the storage values for both contingent claims Case 1 and Case 3 along with both the EOQ values.

<table>
<thead>
<tr>
<th>Table 5.6. Durum Model EOQ and Contingent Claims</th>
<th>Percent Stored of Milling Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contingent Claims</td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>-5.5%</td>
</tr>
<tr>
<td>Case 2</td>
<td>-0.7%</td>
</tr>
<tr>
<td>EOQ Results</td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>1.0%</td>
</tr>
<tr>
<td>With Lead &amp; Shortage</td>
<td>1.3%</td>
</tr>
</tbody>
</table>

Table 5.6 shows that the EOQ model suggests storing more in both cases. This is attributed to several different conditions within the durum market. The durum price is very high in the low margin industry, and the durum price has very high volatility. Given those two variables, the contingent claims model says that it is too risky to store inventory in any of the cases. The contingent claims model suggests a short inventory position, while the EOQ model suggests storing inventory. One of the major theory differences can be seen when looking at this durum model comparison, which is that contingent claims is risk neutral and maximizes NPV, while the EOQ minimized total cost and does not imply risk-neutral probabilities. Negative quantities of storage are not feasible, but the model is showing that there is great downside risk; thus, a firm should be very careful when carrying inventory when prices are near their industry highs with high volatility.

Oats

Inputs for the oats EOQ are in Chapter 4. The oats industry has high margins with volatile prices and acceptable quantity. Prices in both the contingent claims and
EOQ are the same to make an accurate comparison among the two models. The data Table 5.7 show the results of the EOQ along with Case 1 and Case 3 of the contingent claims.

<table>
<thead>
<tr>
<th>Contingent Claims</th>
<th>Percent Stored of Milling Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>4.2%</td>
</tr>
<tr>
<td>Case 2</td>
<td>15.9%</td>
</tr>
</tbody>
</table>

Table 5.7 shows that the EOQ model suggests storing much less than the contingent claims approach in both cases. This is much different than is seen in both the wheat and durum. In this higher margin industry, it is more valuable to store. With higher margins in the industry, there is much greater potential for losses and foregone earnings if a shortage occurs. The contingent claims model suggests storing more quantity due to a few different reasons. The first is that it is a high-profit industry with volatility, and if a shortage occurs, it has a great effect on profits; thus, storing may return a negative NPV, but it is more profitable than the next best option which is not to store. The second thing is that the shortage penalty in this market is much greater due to the fact the industry does not have as much excess production as can be seen in the wheat and durum industry.

**Sensitivities**

Stochastic variables are used to demonstrate risks that are present in carrying inventory. The main uncertainty is in the commodity markets. Sensitivities are
performed on the current price, volatility, salvage value, flour price, and marginal sales. The sensitivities were evaluated on all three commodities on Case 3 as this case is most representative of what a typical agribusiness would confront.

**Market Price**

The market price sensitivity consists of increasing and decreasing the price in increments of ten percent, thus calculating new prices that were within the historical prices seen in the time period 1990-2002. These prices were then used in the model, and new optimal stocking values were observed.

The range in wheat price seen was $2.70/bu to $5.03/bu with the market price in the model set at $4.49/bu. The price was changed in increments to its limits by units of ten percent, and the storage and NPV values are plotted in Figure 5.5.

Figure 5.5 shows that the optimal storage level rises and then peaks out at around $3.50/bu before falling rapidly. The optimal storage level is greatest at a price of $3.50/bu because it is a price at which the potential gain from carrying revenue is the greatest. With prices less than $3.50/bu, the contingent claims model suggests storing less due but to increase storage when price increases to hedge against price risk. With prices greater than $3.50/bu, the optimal storage is less because it is more efficient to buy wheat as needed on the open market rather than storing. The NPV per bushel is positive until a price of around $4.25/bu, and everything after that is negative. Carrying inventory at a price above $4.25/bu becomes too risky which is why the model returns a negative NPV. Losses due to market fluctuations escalate with the increasing market prices. At prices below $4.25/bu, the NPV per bushel remains positive as it suggests that carrying inventory has a positive option value.
Figure 5.5. The effect of increasing and decreasing current wheat prices on optimal storage and NPV.

The range in durum price was $3.48/bu to $7.00/bu, and the market price used in the model is $4.90/bu. Using Case 3, the market price was increased and decreased in increments of ten percent until the high and low prices were met. The prices along with the optimal storage level and NPV were plotted in Figure 5.6.

Figure 5.6 is much different than the results from wheat in that it shows a constant decrease in the optimal storage level as prices increase. This can be partially attributed to the high volatility in the durum market and the low profit margins associated with the market. The NPV peaks at a price of $4.41/bu and then falls rapidly with increasing durum prices. Thus, there are positive returns to storage at durum prices less than $4.41/bu, but for prices above that level, the NPV per bushel declines rapidly. Thus, the optimal storage pattern then is to be short inventory at a
durum price above $4.41/bu. Furthermore, the optimal storage level is positive so long as the prices are less than $4.41/bu, and then they decline. The reason for this is that at a price above $4.41/bu, the contingent claims model suggests it is too risky to store given the high volatility in the market and the low margins.

![Figure 5.6](image_url)  

**Figure 5.6.** The effect of increasing and decreasing current durum prices on optimal storage and NPV.

The observed range in oat price was $1.09/bu to $2.77/bu, and the market price in the base case was $2.43/bu. The optimal storage level and NPV are plotted in Figure 5.7 below.

The results show that optimal storage levels increase with increased oat prices. At a price below $1.90/bu, the contingent claims model suggests positive storage even though the returns on that storage are negative. This is because it is still beneficial to store oats because the risk of a shortage is too overwhelming, thus the
negative returns to storage. The cost of running short and losing revenue from flour sales is much greater than the cost of storage at a price below $1.90/bu. The increasing optimal storage peaks the NPV per bushel at an oat price of $2.44/bu, and the NPV per bushel drastically falls at any oat price greater than $2.44/bu. At a price greater than $2.44/bu, the contingent claims model suggests positive storage levels but yields a negative return to this storage. The increased storage is valued as price increase because the cost of falling short and loosing a sale outweighs the increased storage cost. This can be partially attributed to the characteristics of the oat milling industry in that it is a high margin business with a low number of millers.

Figure 5.7. The effect of increasing and decreasing current oat prices on optimal storage and NPV.
Price Volatility

The price volatility used in the contingent claims model is the standard deviation of the annual market price from the years 1990-2002. To see the impact of volatility on the optimal storage quantity, sensitivities were run with changes in volatility. Variance of the prices was decreased and increased by twenty percent in increments of ten percent, once again giving four new volatilities. The variances were then transformed back into standard deviations which were used to calculate the new optimal storage quantity. Figure 5.8 shows the results of the sensitivity.

Figure 5.8 shows that increasing the standard deviation of the market prices lowers the optimal storage quantity in both wheat and durum but increases the optimal storage level in oats until a point. This is very interesting to note as the durum market has a very high volatility which was used in the base case and yielded all negative storage quantities. Oat has a low standard deviation and when increased shows the same downward trend as durum and wheat. Results from this volatility simulation differ from the theory of real options in that increased uncertainty does not always add increased value to the option. This just points out that given the market characteristics in this model of a high current price and high volatility and a low margin, there is potential for great losses in storing in the durum model.

The increase in oat storage suggested by the contingent claims model as standard deviation increases can be attributed to the low standard deviation of the oats market, and increases in the standard deviation have the impact of increasing the optimal level of storage. In contrast, the volatility of wheat and durum is far greater,
and an increase in the standard deviation of these values has the impact of decreasing the optimal storage level.

![Figure 5.8. Effect of increasing and decreasing volatility on optimal storage.](image)

**Salvage Value**

The salvage value in the contingent claims model is important in that it reflects whether the commodity market is normal or inverted. If the market is normal, the price will be greater than the current price; if the market is inverted, the market price will be less than the current price. In the base case models, ten cents were added to the market price to determine the future market price. This implies a normal market in which the model was analyzed. In this section, sensitivities were run to analyze the effect salvage value has on the optimal storage quantity. Salvage values were decreased by twenty percent and increased by twenty percent in increments of ten percent, thus giving five different points when counting the base model.
The optimal storage quantity was solved using the new salvage value prices, and Figure 5.9 shows the effect of increasing and decreasing the salvage value.

Figure 5.9. Effect of increasing and decreasing the salvage value on optimal storage.

The wheat and durum models react similarly when analyzing the salvage value prices, but the salvage value has a large impact on the optimal storage level in the oats market. There are several factors in the oats model that are different than wheat and durum; thus, the salvage value affects the oats in a more positive way. The first factor is that oats have a higher margin, and there is more profit to be made in the oats industry and a higher shortage penalty; thus running out of the commodity will have a large negative impact on the NPV. The low volatility in the oats when compared to wheat and durum prices is the second reason. With the lower volatility, there is less price risk in the oats industry. At a salvage value of 120% of normal, all three grains have an optimal state price that is negative because the profit from selling
the grain is greater than the profit of selling the flour. Wheat has all positive optimal storage levels; when the salvage value is at 80% of normal, the optimal storage level almost reaches zero. When the salvage value of durum is 110% of normal, the contingent claims model suggests storing, but at any level less than that, the model suggests having a short inventory position. This short inventory position when the salvage value is decreased is because it becomes more risky to store durum due to the high volatility in prices. There would be potential to lose a lot of money in the durum market without the price floor imposed by the salvage value. Oats has a very positive and highly sloped line when the salvage value is analyzed. This is partially attributed to the oats market with its greater margins.

The optimal state price decreases as the salvage value increases. The optimal state price decreases because the salvage value is in the denominator of the optimal state price formula; thus, as the salvage value increases, the optimal state price declines, and the contingent claims model suggests storing a greater optimal quantity. Percent stored of milling capacity increases as the optimal state price decreases. Smaller optimal state prices in the Black-Scholes model suggest an exercise price of the short call option to increase. When the exercise price of the short call option increases relative to the exercise price of the long call option, a higher optimal quantity is suggested; this is part of the contingent claims formulation. As the salvage value increases, there is an increase in the option value or NPV per bushel. This increase in NPV is associated with the ability to liquidate the asset at a higher price due to the increase in salvage value. Thus, as the salvage value increases, the option value of carrying inventory increases. Increasing the salvage value decreases
the downside risk of carrying inventory; thus, the option value increases, and it becomes more profitable to carry inventory.

Table 5.8 shows the effect of changing the salvage value on the optimal state price, NPV per bushel, and the percent store of milling capacity.

<table>
<thead>
<tr>
<th></th>
<th>Optimal State Price</th>
<th>NPV ($/bu)</th>
<th>% Store Mill Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wheat</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80% Salvage Value</td>
<td>0.58</td>
<td>$(1.65)</td>
<td>0.3</td>
</tr>
<tr>
<td>90% Salvage Value</td>
<td>0.51</td>
<td>$(0.58)</td>
<td>0.8</td>
</tr>
<tr>
<td>100% Salvage Value</td>
<td>0.39</td>
<td>$(0.20)</td>
<td>1.6</td>
</tr>
<tr>
<td>110% Salvage Value</td>
<td>0.18</td>
<td>$0.02</td>
<td>3.7</td>
</tr>
<tr>
<td>120% Salvage Value</td>
<td>-0.30</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Durum</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80% Salvage Value</td>
<td>0.58</td>
<td>$(11.36)</td>
<td>-3.2</td>
</tr>
<tr>
<td>90% Salvage Value</td>
<td>0.51</td>
<td>$(11.15)</td>
<td>-2.4</td>
</tr>
<tr>
<td>100% Salvage Value</td>
<td>0.39</td>
<td>$(11.83)</td>
<td>-0.7</td>
</tr>
<tr>
<td>110% Salvage Value</td>
<td>0.18</td>
<td>$0.12</td>
<td>5.7</td>
</tr>
<tr>
<td>120% Salvage Value</td>
<td>-0.35</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Oats</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80% Salvage Value</td>
<td>0.34</td>
<td>$(3.04)</td>
<td>12.6</td>
</tr>
<tr>
<td>90% Salvage Value</td>
<td>0.28</td>
<td>$(1.99)</td>
<td>14.5</td>
</tr>
<tr>
<td>100% Salvage Value</td>
<td>0.21</td>
<td>$(1.01)</td>
<td>17.3</td>
</tr>
<tr>
<td>110% Salvage Value</td>
<td>0.11</td>
<td>$(0.14)</td>
<td>22.5</td>
</tr>
<tr>
<td>120% Salvage Value</td>
<td>-0.01</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Flour Price**

In the contingent claims inventory model, the flour price is the revenue associated with selling the underlying assets. With commodities like wheat and durum that have low margins, flour price sensitivity was used to illustrate how increased flour prices affect the optimal storage quantity. Flour price sensitivity was also used in oats to illustrate the effect of flour prices on the optimal storage quantity. Flour prices were analyzed with a decrease and increase in prices of twenty percent in
increments of ten percent. The flour prices along with the optimal storage quantities can be seen in Figure 5.10.

![Figure 5.10. The effect of flour price on optimal storage quantity.](image)

The optimal level of storage between wheat and durum is similar, but there is a great difference when compared to oats. Wheat and durum both have lower levels of optimal storage. The margin from flour sales in wheat and durum is less than that of the oats industry. Thus, the reason for the oats’ higher optimal storage levels is the higher margins. Wheat and durum experience the greatest effect of increased flour prices on the optimal storage quantity, while storage in oats remained rather flat. Storage becomes infeasible when the flour price is 80% of normal, but optimal storage increases as the flour price increases from that point in both the wheat and durum markets. At a flour price of 80% of normal, it is no longer profitable to carry inventory as the costs far outweigh the benefits. At a price at and above $16.80/cwt
of flour, the contingent claims model suggests a positive storage. Oat flour price increased the optimal storage level to a point and then caused the optimal storage level to decrease at a point of greater than normal flour price. The high flour price in relation to the underlying grain reaches a saturation point where there is no more added benefit to storing more oats even if the flour price is increasing. This is to be expected as oats already have a large implied margin; thus, increasing the flour price will not have the same effect as it would on durum and wheat. Wheat and durum optimal storage quantities increase fairly rapidly with durum seeing the greatest increase of the two.

**Marginal Sales**

Using contingent claims methodology to model the optimal inventory quantity requires a link between sales and the underlying asset. In this study, the output of flour was linked to the underlying input which is the raw commodity. This variable is the contingent claim as it shows how much sales would increase with an increase in the underlying asset price. This is the parameter that does not have an effect on the optimal state price, but it does have an effect on the optimal order quantity. In this case, this relationship was estimated, and the result is a variable that shows how storage needs increase as the underlying grain price increases, which is in contrast with Stowe and Su (1997) who simply assumed that sales will increase 10 units for every $1 increase in the underlying asset. Regression results were used to derive prototypical relationships. Wheat regression calculated an added increase in sales from inventory at 38,988 cwt, durum had an increase of 77,870 cwt, and oats had an increase in sales from inventory of 123,011 cwt. These values were then divided by
the historical price range to derive the effect of increasing the asset price by $1. In wheat, the effect of a $1 increase in the asset price increases the quantity sold by 5,043 cwt. In durum, the effect of a $1 increase in the asset price increases the quantity sold by 7,430 cwt. Oats saw a 346 cwt increase in sales with a $1 increase in the asset price. Durum experienced the greatest increase in sales when the asset price was increased, followed by wheat and then oats. To evaluate how this impacts the results, sensitivities used the same percentage decrease and increase procedure as previously defined. The results are plotted below in Figure 5.11.

![Figure 5.11. The effect of marginal sales on optimal storage quantity.](image)

The results show how increasing the marginal sales affect the optimal storage quantity. For wheat and oats, increasing the marginal sales increases the optimal storage quantity, but it decreases it for durum. It is to be expected that increasing the marginal sales would increase the optimal storage quantity, but for reasons related to
the parameters in the durum model, it returns the opposite results. The durum model has several unique features to it. It has a very high volatility and high market prices that are near the upper limits of the distribution of historical prices. This has an effect on why the durum has a negative slope.

Summary

Market trends that have increased volatility along with quality shortfalls have pressed the issue of storage as of late. There are several ways to address this issue of quality shortfalls, and one of the ways is through storage and carrying increased stocks. Conventional models have a hard time capturing the increased volatility and quality shortfalls that are seen in the market. This chapter gives the results of the contingent claims model outlined in Chapter 4. The contingent claims model places value on the uncertainty in the marketplace and seeks to maximize the NPV of the storage function.

There are several highly sensitive variables in the contingent claims model that have different effects on the various grains in this study. Wheat with its high basis risk, along with durum and oat which are commodities that cannot be directly hedged, were analyzed in this chapter. Sensitivities were performed on the variables to illustrate the effect they have on the optimal storage level and also the NPV per bushel.

There are many results and sensitivities in this chapter that are useful to understand and have varying impacts on the optimal storage strategy. The major results are
1. Wheat has positive storage values in all three cases, and Case 3 of the wheat contingent claims model suggests storing a greater amount than the EOQ model.

2. Durum with its high volatility in the market returns a short inventory position using the contingent claims but a positive position using the EOQ model.

3. Oats with is high margin and fairly low price volatility maintains a highly positive optimal storage level that is far greater in the contingent claims model than the EOQ model.

4. When market price is adjusted using a sensitivity, it shows that there is increasing returns to a point and then there are decreasing returns.

5. Price volatility has an overall negative effect on the optimal storage level except in the oats model where there is already low market volatility. Thus, increasing the volatility in the wheat and durum model decreases the optimal storage level suggested by the contingent claims model.

6. Increasing the salvage value has a positive effect on the optimal storage quantity. When the salvage value is increased, it suggests a positive return to storage; thus, the contingent claims model suggests storing a greater quantity.

7. Flour price has a positive effect on the overall optimal storage quantity. In both the wheat and durum markets, the optimal storage quantity can see a rapid increase with an increase in the flour price, but oats, on the other hand, shows a peak in the optimal order quantity.

The contingent claims model provides another tool in which managers can analyze their inventory strategy, but there are several key variables that have a substantial impact on the overall optimal inventory level suggested by the contingent claims model.
model. Sensitivities were used on these variables to see their effect on the optimal inventory level.
An increase in the complexity of the grain marketing system is occurring and may become further complicated with more frequent quantity and quality shortfalls. The complexity is due to several developments. First, there is increased pressure on farmers to grow crops that are more economically feasible, thus shifting the production of some grains to other countries and then having to import the commodities. In addition, slight changes in the overall weather conditions affect the quality of the commodity drastically, and millers seek the highest quality for food grade. Furthermore, commodities considered in this study are high risk. High-protein spring wheat can be hedged, but it has very high basis risk, and hard amber durum and milling oats cannot be directly hedged.

Commodity prices in the agricultural industry are very important due to their effect on not only producers but also processors. This research examined how the underlying commodity price and risk affect their optimal storage level along with the profitability of storage. Inventories can be regarded as real options and can be viewed as call options because they require an initial investment. This initial investment is an option on future sales. Using the contingent claims methodology, the optimal inventory level along with the net present value (NPV) or option value can be derived. The option value is the added benefit of carrying inventories that is added back into the revenue flows.

Contingent claims models were used in this study to value inventories and evaluate storage decisions. Contingent claims place value on uncertainty and map the
payoff from the futures sales onto the underlying asset. One of the main features that the continent claims models uses is the risk-neutral probability. It accounts for the time value of money using the optimal state price, where the traditional approach uses the optimal stock-out probability. The conventional economic order quantity (EOQ) model uses just an objective probability without the risk-neutral assumption.

**Review of Objectives**

An option pricing model was used in this study to determine optimal inventories and the option value. Option pricing models are useful when addressing problems for which conventional models are less appropriate. This study uses the real option called “option on future sales.” The objective is to determine the optimal level of inventory held by a prototypical agribusiness firm under varying types of demand uncertainty and procurement using the contingent claims approach. The objective states the level of inventory which is most valuable to the firm and places an option value on the inventory in net present value (NPV) per bushel.

There are several specific objectives of the study. The relationship between price, quality, and quantity and how it affects the overall demand for storage was determined. The optimal inventory level was found using both the contingent claims model and the EOQ model in wheat, durum, and oats. Specifically considered is which model is most effective in predicting the profitability of inventory decisions, thus providing information on which inventory model should be used. The models were also compared and analyzed. The method used in this study was contingent claims along with the EOQ for comparison of the optimal inventory levels. The contingent claims model used in this study was extracted from Stowe and Su (1997).
Ballou (1999) was used for the EOQ model and its formulations. Both of the models used the same market data so that comparisons across the two models were not biased.

There is a specific relationship between price and quality of the attributes that millers seek in the different commodities. Usually what millers seek for food grade is higher quality that also comes at a price premium. The optimal state price was derived from the contingent claims model in all three grains. The optimal state price is the time-discounted risk-neutral optimal level. The input data were representative of a prototypical agribusiness firm that is in processing or marketing. The optimal inventory level determined using the EOQ was also evaluated to examine if the EOQ model carried more inventory than the contingent claims model.

The study provides a model for developing a better understanding of the information requirements necessary for agribusinesses to evaluate inventory management strategies. This information forms the basis for implementing the contingent claims approach using real options into the agribusinesses corporate management idea. This information would also help the firm to make more precise decisions when working with commodities that have great volatility in the marketplace.

**Review of Procedures**

**Model Description**

Contingent claims methodology was used to determine the optimal inventory level that a prototypical agribusiness would carry under given circumstances. In this fashion, the inventory output which is flour sales is directly linked to the underlying
asset which is the commodity. A forecasted demand for inventory using quality and quantity variables that are affected by the overall production is represented by a normal distribution. The model has both long and short calls that are used to offset risks associated with carrying inventory. In this study, calls are used to replicate the payoffs of the inventory policy. Both long and short call positions are used to map the inventory policy onto the underlying commodity given the associated risks in price. A call option is the right to buy an asset at a certain price by a certain date. The long call options modeled mean the business is purchasing the right to own an asset at a certain price by a certain date. The short call options modeled mean the business is selling the right to own an asset at a certain price by a certain date. Thus, the long call options replicate buying the inventory, and the short call options replicate selling or liquidating the inventory. Use of this strategy is created by buying a call option on the underlying asset stock with a certain strike price and selling a call option on the same stock with a higher strike price. In financial terms, this is called a bull spread. A call price always decreases as the strike price increases; thus, the value of the option sold is always less than the value of the option bought (Hull, 2002). This strategy requires an initial investment.

There are a number of important values that are derived and interpreted. They are defined here in the summary. The optimal state price is the risk-neutral probability of the underlying asset price exceeding the exercise price and the option in the money. This optimal state price replaces the objective probability in the conventional optimal stock-out probability. Risk-neutral probability is defined as the value of the option increasing at the risk-free interest rate. In option valuation, the
optimal state price is similar to the Greek delta in the Black-Scholes option pricing model. The exercise price for the short call options determined using the optimal state price represents the exercise price that maximizes the NPV of the inventory storage function. When the short exercise price derived by the contingent claims model is greater than the exercise price of the long position, a positive inventory is suggested. Otherwise, a short position is suggested to maximize the NPV. In some cases, the optimal inventory level is positive; in others, it is negative, but by definition, it is the value that maximizes NPV. The NPV is the option value and is shown on a per-bushel and hundredweight basis. The interpretation of the option value is the value that a firm can add back into the revenue flows from that specific inventory storage function. Finally, the results are used to derive the optimal storage level as a percent of milling capacity. This number is calculated by dividing the optimal inventory quantity suggested by the contingent claims model by the annual requirements.

From an economic standpoint, the contingent claims model addresses the areas of uncertainty in a more meaningful way than the EOQ. Supply is not known for certain; it is forecasted, and this forecasted supply has an effect on how much to store and also the net present value (NPV) of the storage function. The forecasted supply distribution is introduced to show that there will be years with high and low quality and supplies of commodities. The supply distribution looks at quality, production, percentage milled, exports, and imports, each contributing to annual usage.
Review of Results

The contingent claims model calculates the optimal state price which is interpreted as the value of a $1 payoff if the underlying asset price is above the exercise price. The optimal state price is derived from the current price of the underlying asset, flour price, salvage value, shortage penalty, interest, and time. This optimal state price is important to agribusiness firms because the optimal inventory quantity is calculated from it. Precise numbers are necessary as inputs to determine the optimal state price because they have a significant impact on the results of the contingent claims model. Once the optimal state price is determined, the optimal storing quantity is then determined. The calculated NPV per bushel is interpreted as the value of having a unit in inventory. In some cases, it is negative, reflecting that the option value of the inventory is not positive.

Various sensitivities were performed to determine how different variables react to changes in the model and how they affect the optimal storing quantity. Sensitivities were performed to determine their effect on optimal storage using the Case 3 situation in each of the commodities. The third case model takes into account both salvage value and a shortage penalty which is more representative of the industry. Current price of the underlying asset, price volatility of the underlying asset, salvage value, flour price, and the marginal increase in sales were the sensitivities.

Base Case Model Results

The base case models use prices based in a specific period. The prices of the underlying commodity are fairly high and on the top end of the distributions of prices over the historical period examined.
The optimal storage quantity increases with a decrease in the optimal state price. This is in part because the optimal state price is then used to determine the new exercise price of the short call options, and the higher the optimal state price, the lower the exercise price of the short call options. When using contingent claims, the quantity is determined by subtracting the new short call price from the long call exercise price. Thus, the greater the exercise price for the new short calls, the greater the optimal amount of storage.

The economic order quantity (EOQ) model is the conventional approach to modeling inventory strategy decisions. There were two different EOQ models used in this study. One was a limited basic model, and the other had lead time and shortage penalties applied. Models were used as a comparison to the contingent claims results. The basic EOQ was similar to Case 1 in the contingent claims, while the EOQ with lead time and shortage is similar to the contingent claims Case 3.

Table 6.1 provides the results for the base case contingent claims model.

<table>
<thead>
<tr>
<th></th>
<th>Optimal State Price</th>
<th>NPV ($/bu)</th>
<th>% Store of Milling Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>0.85</td>
<td>$(8.91)</td>
<td>-1.5</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.59</td>
<td>$0.11</td>
<td>0.2</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.39</td>
<td>$(0.20)</td>
<td>1.6</td>
</tr>
<tr>
<td>Durum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>0.84</td>
<td>$(8.68)</td>
<td>-5.5</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.58</td>
<td>$(10.62)</td>
<td>-3.1</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.39</td>
<td>$(11.83)</td>
<td>-0.7</td>
</tr>
<tr>
<td>Oats</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>0.66</td>
<td>$0.79</td>
<td>4.2</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.28</td>
<td>$0.63</td>
<td>14.7</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.21</td>
<td>$(1.01)</td>
<td>17.3</td>
</tr>
</tbody>
</table>
The optimal state price is the risk-neutral probability of the value of a $1.00 payoff if the underlying asset price is above the exercise price. This optimal state price replaces the optimal stock-out quantity in the conventional models. This optimal state price is more feasible as it adds the time value of money and it is a risk-neutral function instead of an objective function. Agribusinesses have to be aware of the time value of money and account for the option value of carrying inventories. This option value can either be positive or negative.

As the optimal state price decreases, there is an increase in the optimal inventory level. This increase in inventory level is associated with the increase in the exercise price of the short call option. The short call exercise price increases with a decrease in the optimal state price deriving an increase in the optimal quantity. The NPV per bushel is the value added by using the storage strategy. It is maximized in the contingent claims analysis. The percentage stored of milling capacity is the optimal quantity suggested by the contingent claims model divided by the annual milling usage. The three cases are all different. In Case 1, there is no salvage value or shortage penalty. Case 2 has a salvage value but no shortage penalty. Case 3 has a salvage value and a shortage penalty. Moving from Case 1 to Case 3, optimal state prices decrease and optimal inventory levels increase.

The contingent claims model shows a very high optimal storage quantity for the oats industry. This is due partially to the higher profit margins associated with the industry, which produce lower state prices.

The negative option values calculated by the contingent claims model along with carrying a positive inventory can be associated with the model’s formulation.
The model derives the optimal inventory level maximizing NPV. A negative NPV reflects that even at its maximum, it is going to cost the firm to store. This is still preferred to not storing or storing an excess quantity. A positive NPV reflects that the firm is adding value back into the business by storing the underlying asset. Case 3 in both the wheat and oats model show this in that they both have negative NPV per bushel but positive storage numbers, which means that the mill has even greater potential to lose money if that amount of inventory is not held.

The EOQ model yielded similar results in the wheat industry but different in the durum and oat industries. This is partially due to the characteristics of the markets. Results from the EOQ models are below in Table 6.2.

<table>
<thead>
<tr>
<th></th>
<th>Percentage Stored of Milling Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>0.9%</td>
</tr>
<tr>
<td>With Lead &amp; Shortage</td>
<td>1.0%</td>
</tr>
<tr>
<td>Durum</td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>1.0%</td>
</tr>
<tr>
<td>With Lead &amp; Shortage</td>
<td>1.3%</td>
</tr>
<tr>
<td>Oats</td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>0.7%</td>
</tr>
<tr>
<td>Lead Time &amp; Shortage</td>
<td>1.0%</td>
</tr>
</tbody>
</table>

The base case in Table 6.2 shows the results for the basic EOQ model, and the other variable takes into consideration lead time and shortage penalties. The basic number is similar in underlying parameters and assumptions to the contingent claims Case 1, and the more advanced EOQ is similar to the contingent claims Case 3. One of the large differences between the two models is that the contingent claims model
maximizes NPV while the EOQ model minimizes total cost. The risk-neutral approach to the contingent claims approach makes more sense.

**Sensitivities**

Sensitivities were performed to see how the model reacts to changes in the parameter variables. These variables have a direct effect on the optimal inventory level. All of the sensitivities discounted the present value by twenty percent in increments of ten percent and also increased the present value by twenty percent in increments of ten percent. This resulted in four new values plus the present value. These values were then examined to see the relationship they had on the optimal storage level. The major results are

1. Wheat has positive storage values in all three cases, and Case 3 of the wheat contingent claims model suggests storing a greater amount than the EOQ model.

2. Durum with its high volatility in the market returns a short inventory position using the contingent claims but a positive position using the EOQ model.

3. Oats with its high margin and fairly low price volatility maintain a highly positive optimal storage level that is far greater in the contingent claims model than the EOQ model.

4. When market price is adjusted using sensitivity, it shows that there are increasing returns to a point and thereafter decreasing returns.

5. Price volatility has an overall negative effect on the optimal storage level except in the oats model where there already is low market volatility, but once the oats model volatility is increased, it also showed that it has a negative affect on
optimal storage. Thus, increasing the volatility in the wheat and durum model decreases the optimal storage level suggested by the contingent claims model.

6. Increasing the salvage value has a positive effect on the optimal storage quantity. When the salvage value is increased, it suggests a positive return to storage; thus, the contingent claims model suggests storing a greater quantity.

7. Flour price has a positive effect on the overall optimal storage quantity. In both the wheat and durum markets, the optimal storage quantity can see a rapid increase with an increase in the flour price, but oats, on the other hand, see a peak in the optimal order quantity.

**Implications of Results**

Stocks are important in agricultural marketing/processing. Traditional inventory models are useful, but they are limited in their ability to properly address the issue of price volatility and risk in the marketplace. These marketing and processing industries could be interpreted as real options, more precisely the option on future sales. The underlying asset is the grain, while the future sale is the flour. Market price, commodity price volatility, flour price, salvage value, and shortage penalties all are important variables to consider when analyzing the optimal inventory level.

Commodity shortfalls leading to increased volatility and prices are adding to the complexity of the commodity markets. Divergent results obtained along with the sensitivities suggest that decisions regarding inventories are highly sensitive and impacted by many variables. Thus, in order to adopt the real options framework, firms and managers have to pay careful attention to the values of the input variables.
An increase in the overall price volatility adds to the risks of carrying inventory and results in a loss of NPV. This loss, however, is diminished as the volatility in the marketplace is decreased when the quality and quantity variables are stabilized. This can be seen when comparing the wheat results to the durum results, as they both have about the same profit margins, but durum has an increased amount of volatility.

Price volatility has a large impact on optimal storage level. With high volatility, firms are at great risk if they store excess quantities. The high volatility examined in the durum industry illustrated the risks of storing. The lower volatility in the wheat and oats markets suggested positive storage numbers using contingent claims. As a firm in a highly volatile industry, strategic inventory decisions must be carefully analyzed.

Flour price is the revenue gained from sales. If the flour price has a high built-in margin, the firm has more to lose if it incurs a shortage. Thus, flour price has a direct affect on inventory levels. An industry such as oats has a relatively high margin when compared to wheat and durum, suggesting an increased optimal storage level for oats.

The salvage value implies either a normal or inverted commodity market. If a firm is seeing downward prices in the future, the optimal inventory will be less than if the market was normal with prices in the future above current. Salvage value has a direct affect on the optimal inventory level; thus, sensitivities on the salvage value illustrate that as it increases, the optimal inventory level also increases. A firm is going to store a greater quantity in a market where there are positive returns to
storage. When markets are inverted and future salvage value is less than the market price, the firm still stores but at a decreased level. This phenomenon was illustrated in all three grains.

The shortage value also implies what a firm has to lose if demand is not met by inventory. This is a cost to the firm; thus, it has a direct affect on the optimal inventory level. All of these variables are linked together in the contingent claims model to provide the option value of carry inventory in a NPV per bushel. This calculation shows what revenue a storage function is adding or taking away from the firm. Although the storage function may not always return a positive NPV per bushel, it is still the maximum at which the firm can achieve and it limits the downside risk.

The introduction of contingent claims into strategic inventory planning impacts the overall optimal storage quantity, especially in high-margin industries such as oats. These higher margin industries show that carrying a greater amount of inventory may not yield a positive NPV, but it does limit the losses to do price volatility and other market changes that cannot be directly hedged.

The study also points out something that is very interesting in the theory of storage and that is the convenience yield. This is when a commodity is held in storage in an inverted market. As seen in the wheat and oat models, even as prices of the salvage value drop, positive inventory numbers are suggested. The positive optimal storage quantity shows that millers and businesses yield some benefits from having inventory readily available for access. This option value created by the
contingent claims model is similar to an insurance policy. The policy is written and held by the firm; by doing this, the firm has the right to the added value of the policy.

**Limitations of Study**

A major limitation of this study is the lack of knowledge about how the underlying asset price affects the future flour sales. This contingent claims value was simulated in @Risk to get an approximation. In addition, this model shows that as the asset price increases, so does the quantity milled. While this is true in most cases, it might not always be the correct assumption. Stowe and Su (1997) have a contingent claim in their model that illustrates that sales will increase by ten units with a $1 increase in the underlying asset price; this assumption, while valid for the specific case, is not always an easy number to calculate.

A critical variable in this study is the market price. The market prices used in this study were from November 2004 and were on the high end of the historical distribution of prices from the years 1990-2002. They have an impact on the results as shown in the market price simulations of the grains.

In most cases, the flour and commodity prices change together in the marketplace unless there is a contract to be fulfilled at a specific price, but in this study, the corresponding numbers do not change together as they do in the marketplace, thus partially limiting the results of the model.

**Need for Further Research**

Further research on how flour sales are affected by the increase and decrease of the underlying asset price is needed. This contingent claim is one of the reasons this methodology has not been adopted more readily within the agribusiness industry.
The issue of how flour price increases with an increase in the underlying asset price is also another area that could be explored in more depth.

Further research on how future sales are affected by the increase in the underlying asset price would greatly enhance this work. It is shown in the results that different contingent claims values produced different optimal storage quantities with differing NPV. Determining an equation that incorporates a variable for the unit of sales increase as an increase in asset prices would be a noteworthy accomplishment.

This study uses data for prices and revenues at one point in time. Adding variability to this to see its effect over time would add depth to the model. The model then would have a distribution of prices with a relationship to revenue in which to calculate the optimal inventory level.

Obtaining data on actual mill configurations including storage, milling capacity, and efficiency would greatly enhance this study. The values used in this study of are a representative firm. The firm data are calculated using the industry data divided by the firms. Currently, industry averages are used to determine the optimal storage level.

**Summary**

As agribusiness firms see increased volatility in the marketplace, there is a need for a more sophisticated inventory management tool. The conventional approach will always play a vital role in solving inventory decisions. There comes a need for a more efficient way to deal with volatility. Many different variables are combined in the optimal inventory strategy, some of which have a large impact on the overall optimal storage quantity. This study points out how using contingent claims
can benefit agribusinesses and explains some of the variables that have a large impact on the overall feasibility of the option model.

This study makes three major contributions to inventory management. First, the optimal inventory storage function is derived from the optimal state prices. This results in an inventory position that maximizes NPV while accounting for uncertainty in an economically feasible way and discounting for the time value of money. Second, the study shows the option value of the inventory storage function. The option value is the value that a company would expect to pay to insure their payoffs from inventory. This value, whether positive or negative, is maximized. Finally, this study shows that there are several key variables that have a significant impact on the optimal state price that derives the optimal inventory level. Managers no longer can estimate what values are input into inventory strategies; these variables need to be precise and accurate for the particular firm.
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