VALUE AT RISK: AGRICULTURAL PROCESSOR

PROCUREMENT AND HEDGING STRATEGIES

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ABSTRACT

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Value at Risk (VaR) is a relatively new methodology used to quantify risk exposure. Although widely used in the financial and energy sectors of the economy, VaR has yet to gain the same acceptance in the field of agriculture. This thesis provides an introduction to Value at Risk and explains both its strengths and weaknesses. Empirical case studies are developed, and VaR calculation is shown for the unique portfolios of three different agricultural processor situations.

The procurement division of a domestic bread baking company is used to empirically demonstrate how VaR could be implemented to evaluate the price risk associated with both the ingredient and energy inputs. A second case considers the same input portfolio; however, the analysis is expanded to include output price risk and show how considering input and output risk simultaneously impacts the risk-reducing effects of numerous hedging strategies. The third case introduces foreign currency exchange risk as VaR is computed for the portfolio of a Mexican flour milling company that purchases its inputs in a foreign currency.

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CHAPTER I. INTRODUCTION

Price risk management is a crucial function in the overall success of many different types of businesses. The field of agribusiness is no exception. Agribusiness firms involved in production, trading, and processing all realize that the market prices of the commodities inherently involved with their businesses will fluctuate. The risk associated with this price fluctuation is one of the most obvious and well-studied aspects of price risk management. Considering only commodity price risk may be sufficient for some agribusinesses. However, the agricultural processor’s hedging decision is complicated by several facets, including the fact that these firms are exposed to the price risks of both inputs and outputs.

Researchers have addressed the subjects of price risk management and hedging in the past. The literature contains a significant number of hedging studies from the perspective of producers and grain traders, describing optimal strategies for dealing with a single source of price risk. Relatively few studies have considered the topic from the processor’s position, incorporating the various aspects of risk that these firms must face. Of the studies that have been done from the processor’s position, most have used some form of the mean variance framework as the analytical tool.

The focus of this thesis is to address the problem of risk management for agricultural processors. Instead of using the traditional mean variance framework, this thesis will use Value at Risk (VaR) as the measure of price risk. Value at Risk offers a unique advantage over other methods of analysis in the fact that Value at Risk is able to separate the potential of large profits from the risk of large losses. The traditional mean variance framework is not able to make this distinction and characterizes all deviations from expected return, positive or negative, as risk. Since managers and decision makers do not consider the potential of realizing large profits as true risk, Value at Risk is considered by many to be much more intuitive than traditional risk measures.

Problem Statement

The extreme importance of price risk management for most firms, especially those in the agriculture industry, has caused the subject of hedging to be studied extensively. To date, however, most of the research has considered agribusiness firms that have long cash positions offset by short futures positions. This type of portfolio relates well to the positions of agribusinesses such as producers and traders. These studies consider commodity price risk as the main source of risk and provide few answers for those whose portfolios differ drastically.

The portfolio of the agriculture processor differs from that of traders and producers in that processors tend to hold short cash positions offset by long futures positions. For instance, bread baking firms tend to have short positions in flour and other commodity inputs, which can be hedged with long positions in futures contracts. This position is the opposite of that held by the previously mentioned agribusinesses. Although the producer and trader scenarios have received much attention from researchers, the situation of the agricultural processor has received little attention.

Not only does the agricultural processor hold a much different portfolio than those which have been well studied, but there are also other complicating factors that weigh into the hedging decision for processors. Input price risk for the processor is equivalent to the output commodity price risk faced by many other agribusinesses. However, the need for numerous inputs makes the matter much more complex than that of the producer or trader who deals only with a limited number of commodities.

Output price risk is the risk associated with the price that the firm will receive for its finished products. As with its inputs, most processors also face uncertain markets for a number of different outputs. Even consumer goods firms that seem to produce only one refined output may generate several by-products that must be sold in competitive markets.

The typical bread baking firm, in essence, has an infinitely large short flour position. The decision of how far in the future this agricultural processor should hedge its flour needs is the concept known as the hedge horizon. Although the firm's short flour position is infinitely large, it is not likely that this company would hedge its needs years into the future. The optimal hedge horizon for a firm is determined by a number of factors, including the frequency with which the firm's outputs can effectively be repriced and the hedging strategies implemented by the firm's competitors. It is also important to note that the optimal hedge horizon may not be equivalent across all inputs.

Another component in the hedging strategy that must be considered is the correlation between input and output prices. If a processor's input prices have very little effect on, or correlation to, the prices that can be charged for the outputs, the processor is under considerable price risk. If input prices rise suddenly and output prices remain constant, the profit margin between input and output prices could quickly disappear. Alternatively, if a firm's input and output price correlations are close to one, the output prices may effectively serve as a hedge against adverse movements in the input prices. In this case, if input prices rise, the output prices that could be charged by this firm would also increase, which to some extent would maintain the original profit margin. In situations where inputs and outputs effectively hedge each other, Hull (2000) describes how traditional hedging can actually increase the variability of profit margins. In these situations, offsetting cash input positions with derivative instruments would actually increase the risks, which would defeat the purpose of hedging altogether. For this reason, it is extremely important to consider input/output correlations in the hedging decision.

The event where output prices are highly correlated with input prices, which provides some hedging protection, brings up another complicating factor for the agricultural processor. Although the correlated inputs and outputs may effectively hedge each other, the time lag between input purchase and output sale may significantly reduce the hedging protection. The time lag is the amount of time that passes between the purchase of inputs and the sale of outputs. If input purchasing takes place a month or more before the output sales price is determined, the hedging protection offered by strong contemporaneous correlations can be drastically less than if the time lag is only a few days. Therefore, the magnitude of the correlation is often a function of the time lag.

Another factor that can have an important effect on the correlation of prices has to do with the degree of value added by the firm. As the degree of value that is added to an input increases, the correlation between the input and output prices tends to decrease. The less the finished product resembles the original input, the lower the price correlation will likely be.

The competitive situation of the output market also has a significant effect on an agricultural processor's hedging strategy. The optimal hedge horizon, input/output price correlation, and output price risk are all affected significantly by this factor. In industries where competition is mild and firms do not reprice products frequently, changes in input prices would have little effect on output prices, and hedging can be crucial. In a fiercely competitive industry where small cost advantages or disadvantages could strongly affect a firm's profitability, close attention must also be paid to hedging and procurement strategies. In these situations, Hull (2000) explains that it is important not only to consider the firm's actions, but also the actions of the competition. If all of a firm's major competitors employ extensive hedging strategies, it could be very risky for the firm not to use a similar hedging strategy. The opposite case is also true, where competitors avoid using any form of hedging. Breaking from industry standards will almost always cause either price advantages or disadvantages. If the firm gains the advantage, the strategy could be very profitable. If the strategy causes a disadvantage, the results could be disastrous.

The use of options as risk management tools should also be considered when determining optimal hedging strategies. One of the main advantages that options offer over the traditional futures contracts has to do with the concept of demand uncertainty. Since options involve the exchange of premiums, their use in risk management strategies tends to increase the costs of implementation. For this reason, it is usually not advantageous to hedge a firm's entire commodity needs with option contracts. However, when demand for output is uncertain, and therefore demand for inputs is uncertain, it may be desirable for a firm to hedge a portion of its input needs with option contracts. If the entire position was hedged with futures contracts and output demand decreased, a portion of the hedge would essentially behave as a speculative position, which would increase risks. Alternatively, if the portion of demand that was uncertain was hedged with options, and prices moved against the options contracts, the options would expire worthless. This scenario would result in a loss of only the option premium, instead of the much larger loss that would be realized from the comparable futures position.

Finally, processors also have other sources of input price risk which are less obvious, such as transportation and energy prices, and for international firms, foreign currency exchange rates. When all of the facets incorporated in the hedging decision of agricultural processors are accounted for, the complexity of the problem becomes evident. Considering and accounting for each of the sources of uncertainty mentioned in this section can be extremely difficult.

While the management of price risk is an extremely important function for an agribusiness, it is important to realize that firms are by no means attempting to minimize risk. The trade off involved in reducing risk through hedging requires that a portion of expected return must be foregone. Instead, the objective of most firms is to choose the risk versus expected return combination that best suites the mission and goals of the company and prevents scenarios that would cause undue stress. For this reason, the term managing risk is much more appropriate than minimizing risk when describing an agricultural processor's goals for procurement and hedging activities.

Need for the Study

The unique procurement and hedging situations that agricultural processors must face have not been a major focus of academic researchers. Alternatively, the positions of crop and livestock producers, as well as the positions of grain merchants, have been analyzed intensely. The single source of uncertainty usually considered is that of the output commodity price risk. Although important, addressing only this one source of risk does not sufficiently capture the needs of agricultural processors dealing in consumer goods markets.

In Johnson's (1982) discussion on the use of futures contracts in consumer goods industries, he touches on several areas. He explains that the firm's pricing strategy and lags inherent in the production process are two crucial factors which determine the optimal hedge horizon for the firm. His largest area of emphasis is the strategic aspect of hedging, where he discusses how a firm's size and market share can be extremely important. Smaller firms may be able to extract larger benefits from hedging when competitors deal only in the cash market. However, firms with a larger market share may be vulnerable to competitors if they do not conform to industry standards concerning hedging. Johnson also discusses the differences between traders and end users and outlines how processors are able to differentiate their products, which tends to reduce the input/output correlation. An example of how M&M/Mars may have anchored their marketing strategies around futures market activities shows the important implications of hedging for end users.

Jackson (1980) elaborates on how intermediate industries, though not specifically agricultural, can often benefit from certain risk-reducing measures that are not as practical for those in consumer goods industries. The use of adders, which vary the output price in contracts with customers in response to price changes in important inputs, and other formulas that attempt to divide price risk between producer and processor cannot be employed as easily in end user situations. In Jackson's (1980) discussion of the importance of timing, she includes examples of how the timing of pricing can cause significant differences in acceptable prices. Although timing of output pricing is her emphasis, the discussion is synonymous with that of production time lags in an end user situation. Using research and development to develop production methods allowing reduced usage or substitution of less volatile inputs is another option that many companies have employed in the past, but may not be practical for agricultural processors.

Research, such as that of Jackson (1980) and Johnson (1982), that incorporates more levels of complexity into the hedging decision still does not consider all aspects of the agricultural processor's hedging decision discussed earlier. A majority of the studies focusing on these sources of risk have also tended to use the traditional mean variance framework in their analysis.

Value at Risk (VaR) has acquired an ever-increasing number of advocates and practitioners in both the financial and energy sectors of the economy. These users apply VaR for internal risk management and employ it as a tool for reporting risks to government regulators when required. The agricultural sector, however, has lagged behind the financial and energy sectors in the adoption of this relatively new risk measurement methodology. Currently, only a few of the largest agricultural conglomerates use VaR in their risk management and reporting divisions. The use of VaR, by the few agricultural firms that do employ the tool, has primarily resulted from a crossover of techniques utilized by the companies' financial and energy desks. While the largest agricultural companies likely have the most to gain from using VaR, the potential benefits of applying VaR in other mid- to large-sized agricultural firms are promising.

Only a limited number of studies have been done incorporating VaR into the context of agricultural hedging strategies. Using VaR in this application would lead to two distinct advantages. First, VaR would allow the processor's risks to be expressed to management and decision makers as a single, summary statistic which is more easily understandable than the output of other risk measurement methodologies. Value at Risk is also able to separate the potential of large profits from the risk of large losses. The traditional mean variance framework, as well as other well-used analytical tools, cannot make that distinction and express all deviation from the expected return as risk.

Incorporating the use of both futures contracts and options is another area where research is needed. Most previous studies have focused on either options or futures, but not the use of both simultaneously. Value at Risk allows for the truncated payoffs associated with options much more efficiently than the mean variance framework.

While the focus of this study will be centered around the unique factors affecting the agricultural processors' hedging and procurement decisions, the benefits will not be limited to this sector. Many of the same price risk variables that must be considered for processors also affect firms in other segments of the economy. Therefore, various firms with complex procurement and hedging needs are able to apply the results to their own individual situation.

Description of the Study

In this study, Value at Risk will be used to provide a measure of corporate price risk exposure for an agricultural processing firm. VaR and its distinct advantages offer an alternative to the traditional mean variance framework. The implementation of VaR allows for the construction of a model that accurately measures the price risk exposure of agricultural processors.

The model will be constructed in the @Risk™ software package using stochastic simulation. The model will include risk management instruments including cash forward, futures, and options contracts. Case studies empirically demonstrating and analyzing the model will consider a hypothetical domestic bread baking firm and a foreign flour milling company.

Study Objectives

The main objective of this thesis is to develop a Value at Risk model that incorporates the various aspects of corporate price risk management and to illustrate the uses of VaR in the context of agricultural processors. The model includes common risk management instruments and allows decision makers to compare the risk-reducing effects of different hedging and procurement strategies used to manage a firm's price risk exposure.

More specifically, the first objective includes reviewing Value at Risk and comparing and contrasting it to other, more traditional methods of modeling price risk management and procurement decisions. The second objective involves constructing a Value at Risk model in the @Risk™ software package that incorporates forward, futures, and options contracts. The third objective is to apply the model to two domestic bread baking company situations and then to the case of a Mexican flour milling company.

Thesis Outline

The second chapter of this thesis contains a review of background information and previous literature. Areas of focus include types of hedging instruments, optimal hedge ratios, and a discussion of both the mean variance framework and Value at Risk. Chapter III consists of an in-depth discussion of the theoretical aspects of Value at Risk, including the derivations and assumptions of the three VaR computation methodologies. Detailed model specifications are explained in Chapter IV, and the data and their characteristics used to illustrate the case studies examined in this thesis are presented. Chapter V discusses the results of each of the three case studies and illustrates how decision makers could use the VaR statistics. Conclusions, limitations, and implications for further study are presented in Chapter VI.

CHAPTER II. LITERATURE REVIEW

When the term, risk, is used in a statistical sense, it refers to any deviation from the expected value, whether positive or negative. For this reason, risk is most commonly quantitatively measured in terms of standard deviations from the expected return. Risk is an inherent reality for all businesses and individuals. As Jorion (2001) points out, the goal of these various entities is not the minimization of risk. Instead, the goal is to monitor and manage risk in order to achieve the best possible balance of risk versus expected return. Risk comes in many different forms. Some risks must be assumed in order for a business to operate, and others can be diversified away. The type of risk that is focused on in this thesis is that of market, or price risk. Firms that deal with agricultural commodities can be especially vulnerable to price risk. The constant changes in the prices of these commodities, whether inputs or outputs for a specific firm, can have very significant effects on profitability. Therefore, risk management proves to be a critical function for these businesses.

This chapter begins with a description of the most common hedging tools available. A discussion of various hedging models and the evolution of thinking on this topic follows. Finally, Value at Risk is introduced as an alternative risk measurement tool. The advantages and disadvantages of Value at Risk (VaR) are presented as the three different types of VaR calculations are explained. The final section of the chapter is a summary of the current status of Value at Risk in agriculture and a description of some of the areas where VaR could be applied to the area of agricultural procurement.

Hedging Instruments

While many of the most complex hedging tools have been developed only in the last few decades, the basic concepts of risk management can be traced back to biblical times. Melamed (1994) explains that the plan Joseph designed for the Pharaoh, in which food was stored during the seven years of bounty in Egypt and used to sustain the people through the following seven years of famine, was perhaps the earliest recorded example of risk management. This strategy is consistent with the American Heritage Dictionary's (Houghton Mifflin Company, 2000) definition of hedging, which is "to take compensatory measures so as to counterbalance possible loss."

Melamed (1994) also describes accounts of 16th century trade fair agreements for the sale of goods still at sea. These agreements are essentially early variants of today's forward contracts. The trading of call and put options can also be traced back well into the historical archives.

Hedging instruments have evolved immensely from their relatively simple beginnings and vary drastically in their complexity. Forwards, futures, and options are the most common and traditional hedging tools. In recent years, however, more complex derivatives of the traditional hedging tools have emerged, such as swaps, exotic options, real options, and credit derivatives. The traditional futures and options are currently traded around the world through organized exchanges, whether the traders are physically at the exchange itself or are trading electronically over the internet. Over-the-counter operations have emerged as the primary source of the more complex derivatives. Over-the-counter trading can occur between any two parties for essentially any good or service without using an organized exchange as an intermediary. However, some sort of financial intermediary is usually involved. The following three sections give detailed explanations of the most common types of derivatives, forwards, futures, and options and describe how they are commonly used to manage price risk.

Forward Contracts

Forward contracts are the most basic of the derivative assets used for hedging. These contracts are agreements between two parties, the buyer (long) and seller (short), that obligate both parties to engage in a transaction to be executed at a future date. The buyer agrees to purchase an asset from the seller at the predetermined future date for a specific price (Hull, 2000).

Forward contracts allow the parties to customize the terms of the agreement to meet their unique objectives precisely. These contracts can be initiated directly between two individuals or businesses and can also be traded in over-the-counter markets between financial institutions and their clients. Instead of being traded on organized exchanges with standardized terms, forward contracts are privately negotiated and can be tailored to the specific needs of the parties involved. A contract can be entered into for any particular asset, quality, quantity, delivery date, and delivery location, and any other relevant term can be specified. They allow a hedger to lock in an absolute price without incurring basis risk, brokerage fees, and margin calls. However, this flexibility does not come without a cost. Negotiating details for each contract takes time and, due to the extreme variation of terms in forward contracts, they are very illiquid. Once the position has been entered, the only way for either a long or short to relieve themselves of their obligation to deliver or accept delivery of the asset is to transfer the obligation to another party and exchange the current value of the contract. Identifying another party willing to agree to the exact terms of the initial agreement can be difficult and costly.

Forward contracts are used for a wide range of assets by a variety of hedgers. Country elevators offer forward contracts to commodity producers seeking to reduce price risk exposure. These contracts obligate producers to deliver a specific volume of a commodity of a predetermined quality to a specific location at a future date for a predetermined price. The party holding the opposite position, the country elevator, would be obligated to accept delivery and pay the predetermined price. Price adjustment terms are also included in these agreements to allow for quality differences between the commodity delivered and that agreed upon in the contract. Large commercial banks also enter forward contracts with corporations seeking to hedge foreign currency exchange risks.

Futures Contracts

Futures contracts are similar to forward contracts in that they are agreements between two parties to exchange an asset on a future date. However, futures contracts differ in most other areas. Futures contracts are traded on exchanges and cannot be bought or sold over-the-counter (Hull, 2000). They are highly standardized agreements that specify exact quantity, quality, delivery periods, and delivery location for an asset. By standardizing most of the factors that must be negotiated in forward contracts, the only aspect of the futures contract to be negotiated is the price (Burns, 1979). This allows for very methodical purchase and sale agreements on exchange floors and eliminates much of the expense associated with negotiating the terms of forward contracts.

Futures contracts are commonly traded for several different delivery months throughout the year. Unlike forward contracts, delivery is not normally specified for a specific date, but rather a delivery period within the delivery month (Hull, 2000). Futures contracts offer flexibility in terms of closing out a position. While fulfillment of the delivery obligation is mandatory for both futures and forward contracts, this obligation is much easier to transfer to another market participant, or close out, in the case of futures contracts. Due to the high liquidity offered by the complete standardization, traders are able to trade in and out of positions at will, which allows investors who do not wish to make or take delivery to participate as both hedgers and speculators in futures markets.

Futures markets offer advantages over forward contracts, but there are also many disadvantages. First, entering into a futures contract requires a deposit by the investor into a margin account. A minimum margin must be maintained and, since futures contracts are marked-to-market daily, when prices move against a position, the margin accounts must be replenished or the position may be liquidated. Another cost inherent to futures trading is brokerage fees. The relatively large number of units represented by each futures contract brings up indivisibility aspects. It is often not possible to cover an entire cash position in the futures market without being over hedged. For example, a trader with 8,000 bushels of wheat will either be over or under hedged when using 5,000-bushel wheat futures contracts. For large traders, indivisibility is rather insignificant; however, smaller traders may find it difficult to cover their positions adequately. Large traders may also be affected by the maximum position limits set by exchanges. However, true hedgers are exempt from these limits, since they only apply to speculative positions.

While hedging with futures contracts relieves a hedger of a majority of the price risk to which he is exposed, the risk of basis changes remains. Since most traders offset their positions prior to maturity instead of making or taking delivery, futures contracts do not lock in an absolute, fixed price. The basis is the difference between the futures market price and the cash price where the physical commodity is actually purchased or sold. Since futures and spot prices rarely move in perfect synchrony, the risk of basis changes is real. However, basis risk is usually significantly less than the full price risk associated with an assets, and therefore, futures markets provide significant hedging protection.

The delivery process of futures markets has the effect of forcing convergence of the spot and futures prices as delivery approaches (Hull, 2000). If these prices are not similar, arbitrage opportunities exist, and risk-free profits can be captured until the difference between spot and futures prices narrow. The recent trend toward cash settlement of futures contracts, which replaces the delivery mechanism, also results in convergence. Here, convergence is explicitly guaranteed, and contracts are settled in cash according to spot prices by a method specified in the terms of the contract (Minneapolis Grain Exchange, 2001).

In many cases, futures contracts for the exact asset that a firm wishes to hedge do not exist. This dilemma leads to the concept of cross hedging. Since no futures contract exists for flour, a miller or baker wishing to hedge its production or inventory must do so in a related asset, the price of which is correlated to that of the asset to be hedged. This correlation can be positive or negative. The level of risk reduction, or hedging efficiency, depends on how highly the cash and futures prices are correlated. High correlations result in high hedging efficiency. As price correlation declines, so does the effectiveness of the hedge; however, any correlation other than zero offers some level of risk reduction (Anderson, 1981).

Hedging cash positions with futures contracts when prices between the two are not perfectly correlated results in basis risk and the absence of convergence. The concept of basis risk is consistent, whether it arises in hedges where the physical asset and the asset underlying the futures contract are the same, or in cases of cross hedging. Therefore, the key in choosing futures contracts for hedging is focusing on the actual correlations between cash and futures prices.

Options on Futures Contracts

An option is the right, but not the obligation, to buy or sell a particular asset on or before a specific date for a specific price. The two basic types of options are calls and puts. A call option gives the holder the right to buy the particular underlying asset. A put option gives the holder the right to sell the underlying asset (Bittman, 2001). Options exist for many different underlying assets; however, this discussion will focus on options on commodity futures contracts, since these are the main type used by agricultural end user hedgers.

Options offer many advantages over futures and forward contracts. While futures and forward contracts represent the obligation to transact on a future date, options provide the right to buy or sell an asset, but do not impose any obligation on the long position holder. Perhaps the best way to envision option contracts is as insurance policies. An investor taking a long position in a put option would be equivalent to the purchaser of the insurance, and the short would essentially be the insurance provider. Unlike the other hedging tools, option contracts require an initial exchange of funds, called the premium, between the long and the short investors when a position is opened. The long is required to pay the short a premium in exchange for the protection that the option provides. An option allows the holder, or long, to establish a price ceiling or floor for an asset, so the option holder has, in essence, purchased price insurance from the short. This position protects the long from any adverse market movements while still allowing the investor to take advantage of favorable price movements.

The flexibility offered by options contracts comes at the price of the premium. Like an insurance premium, once the option is purchased and the premium is paid, the premium itself cannot be recovered. If prices move in favor of a put option holder, and the individual chooses to exercise the option, the short is obligated to purchase the underlying asset at the strike price. When the holder of a call option chooses to exercise, the short must furnish the long with the underlying asset. Since the long position has the right but not the obligation to either purchase or sell an asset, the short has the obligation to fulfill the terms of the option should the long choose to exercise. Therefore, long position holders have limited potential losses but have unlimited potential gains. Alternatively, short option traders limit their potential gains but, in exchange for the premium received, have exposed themselves to potentially unlimited losses.

Hedging vs. Speculation

Futures, forwards, and options contracts are valuable hedging tools when used for this purpose. However, it is important to draw a distinction between hedging and speculative activities. When used for hedging, these mechanisms reduce risk exposure very effectively. When used for speculation, the opposite occurs and risks are dramatically increased.

Although speculation receives much negative press, it serves an important function. Speculators increase the liquidity of markets and accept the market risk that hedgers are not willing to bear (Leuthold et al., 1989). Overall efficiency of an economy is therefore increased by transferring risk from those in the economy who are least willing and able to bear them to those who are most willing and able.

Speculation only becomes a problem when traders, with the ultimate goal of reducing risk, use derivative assets to increase their risk exposure. In some cases, this situation is truly accidental, resulting from a lack of basic understanding of derivatives. In others, traders have intentionally taken speculative positions attempting to increase returns to their companies and receive personal bonuses. The danger is that the difference between hedging and speculating cannot be observed by looking only at the positions taken in derivatives. Instead, these positions must be compared to the positions held in the underlying assets to determine the actual affect on risk exposure. For this reason, it is easy for those who are believed to be hedging to shift into speculative activities.

Hedging Models

Throughout the years, numerous attempts have been made to devise a hedging model that accurately reflects the observed hedging behavior of agricultural processors, traders, and producers. Collins (1997) explains that each of the various models presented is able to predict the observed actions of some of the market participants. However, none of the models to date are able to capture the various attitudes toward hedging that exist between the three main categories of potential hedgers.

Prior to the 1950s, hedging was viewed as an activity used for the sole purpose of reducing price risk exposure (Blank et al., 1991). This end was accomplished by taking a position in futures contracts that was equal and opposite to the position held in the cash market. This strategy is relatively easy to implement and offers significant risk reduction in cases where spot and futures prices exhibit high correlations.

In the 1950s and early 1960s, Working (1962) challenged this naive view of hedging, arguing that pure risk avoidance is only one of several legitimate economic reasons for hedging. He separated hedging activities into three broad categories. The first is arbitrage hedging, where a trader takes both spot and futures positions to take advantage of anticipated basis changes resulting from convergence of spot and futures prices as the futures contract maturity nears. In this case, traders are arbitraging the basis, which can be thought of as capturing a storage fee, or carrying charge. Operational hedging is commonly described from the perspective of a flour miller. The miller uses futures as a substitute for concurrent cash transactions. This practice facilitates day-to-day business operations by temporarily protecting unsold inventories and forward contracts. The third category, anticipatory hedging, is used by producers and end users in anticipation of cash transactions. Producers can hedge growing crops not yet ready for sale, and end users can cover future requirements of raw materials. These transactions are made not to offset a position in the physical commodity, but in anticipation of a merchandizing contract to be made in the future.

In the past 50 years, researchers have developed hedging models that provide risk reduction superior to that of the naive equal and opposite model. These models come closer to mirroring the observed actions of hedgers in the market. The following discussion describes some of the most popular methods and models developed since 1950 and follows the progression in the search for a model that accurately reflects the observed hedging behavior of all market participants.

Portfolio Theory

Portfolio theory is an investment strategy first introduced by Markowitz in 1952. The strategy counteracts the problems associated with investment, or price, risk. The underlying observation which Markowitz (1991) felt necessitated portfolio theory is relatively basic. Earlier investment theories indicated that profit maximization was the ultimate goal of investors. Markowitz (1952) believed that investors' decisions are based not only on expected return, which is viewed as a desirable thing, but also on the variance of expected return, which is viewed as an undesirable thing.

He supports this argument by indicating that if maximizing expected returns was the ultimate goal of investors, there would be no logical argument for investing in more than one asset. An investor would simply identify the asset with the highest expected return and invest all their funds in that one asset. Markowitz (1952) explains that under the expected return maximization goal, no diversified portfolio would ever be preferred to an undiversified portfolio. Diversification, however, is a common place in the market and has been observed throughout history, indicating the presence of another objective.

The concept that investors like returns and dislike risks is followed by two other essential assumptions. These are that investors act rationally when making investment decisions, and that these decisions are based on maximizing the expected return for the level of risk accepted. Rather than assessing the risk of owning an individual asset, portfolio theory suggests that the effect of owning an asset on the investor's total portfolio risk should be the focus. By considering all possible combinations of assets, where each combination has an expected return and variance, an investor can then identify an efficient (E-V) frontier, indicating the profit-maximizing level of returns for each level of risk accepted. The investor then selects the "efficient" portfolio that maximizes expected utility, which depends on the investor's level of risk aversion.

The first economists to apply portfolio theory to the hedging decision were Johnson (1960) and Stein (1961). They indicated that the motives for hedging are equivalent to any other investment decision. The objective is to obtain the optimum risk and expected return combination. Previous theories could justify both the completely hedged and the completely unhedged positions. However, before portfolio theory was applied to hedging, the observed decision to partially hedge one's cash position could not be explained. Ederington (1979) explains one difference between investment and hedging applications of portfolio theory. Investment assets tend to be viewed as substitutes for one another, where spot and futures positions in commodity markets are not. He explains that spot market positions tend to be considered fixed, and the percentage of this position to hedge is the decision variable.

Minimum-Risk Hedge Ratio

There are two main approaches taken by researchers when working with more complicated hedging models. The first approach is that of minimizing the risk associated with a cash position. The result, the minimum-variance hedge ratio, is the ratio of futures contracts to the cash position that minimizes the variance of income (Lence and Hayes, 1994). The equation that represents this risk-minimizing ratio is given as $H^* = -\sigma_{sf} / \sigma_f^2$. In this equation, H^* represents the risk-minimizing hedge ratio, σ_{sf} denotes the covariance between futures and cash prices, and σ_f^2 denotes the variance of futures prices.

At first glance, this risk-minimizing hedge ratio seems to have logical appeal. The fact that the main reason for hedging is the reduction of risk would lead to the notion that achieving the absolute minimum risk should be the objective of a hedger. The fact that minimizing the risk also minimizes the expected return is not the only adverse outcome of this theory. Collins (1997) points out that this approach usually leads to hedge ratios that are very close to the traditional, equal and opposite hedge ratio of one. He notes that the minimum risk hedge ratio is consistent with the observed behavior of traders and arbitrageurs, which usually cover nearly their entire position. However, he rejects this methodology as an overall hedging model because it does not capture the actions of all participants. The only way to achieve the no-hedge action, taken by most producers, is to observe a covariance between futures and cash prices of exactly zero, which is essentially never the case.

It is also interesting to note that the risk-minimizing hedge ratio can also be found through linear regression by regressing the spot price against the futures price. The linear regression model is represented by $c_t = \alpha + \beta f_t + \varepsilon$, where c_t is the cash price at time *t*, f_t is the futures price at time *t*, α is the vertical intercept, β is the slope coefficient, and ε is a random error term. Since β is equal to the covariance of c_t and f_t divided by the variance of f_t, $-\beta$ is therefore equivalent to $H*$, which is the risk-minimizing hedge ratio (Blank et al., 1991). The level of hedging effectiveness achieved can be taken from the regression in the form of the R^2 parameter, which ranges from 0 to 1.

There are several estimators that are used in these regression models. The first method, the price-level model, is used in the previous description. Here, the absolute cash price is regressed on the absolute futures price. The second model is referred to as the price change model, which regresses the change in cash price on the change in futures price. The third model, called the percentage price change model, regresses the percentage change in the cash price against the percentage change in the futures price. These different estimation methods all produce valuable hedge ratios; however, there is disagreement as to which produces the best results. In fact, arguments have been made that the best choice of estimation method depends on the type of hedge in question.

Expected Utility-Maximization

The second main approach to hedging models is that of expected utility-maximization. This concept was first introduced by Stein (1961) and Johnson (1960), who reasoned that an entity chooses to hold its spot position hedged, unhedged, or partially hedged in an attempt to maximize expected utility. Variants of the expected utility-maximization model have been used by others such as Sakong et al. (1993), Lapan et al. (1991), Rolfo (1980), and Haigh and Holt (1999). These models identify two components of demand for futures contracts. The first component is the speculative demand, determined by future price expectations and the risk aversion level of the entity. Hedging demand, the second component of demand for futures contracts, consists of the risk-minimizing hedge ratio (Collins, 1997). In the equation,

$$H^* = \frac{E(f_1) - f_0}{2\lambda\sigma_f^2} - \frac{\sigma_{sf}}{\sigma_f^2},$$

$E(f_1) - f_0$ represents the entity's expectation for upcoming futures price movements. If the entity expects end-of-the-period futures prices (f_1) to be greater than current prices (f_0), speculative demand will be positive. If end-of-period futures prices are expected to be below current futures prices, the opposite is true. Instances where the entity believes futures prices are unbiased result in a speculative demand of 0, indicating the utility-maximizing hedge is equal to the risk-minimizing hedge. The entities' risk aversion parameter (λ) governs the magnitude of the speculative position taken to exploit the perceived bias of current futures prices.

Wilson and Wagner (2002) modify the utility-maximizing hedge ratio to account for the relationship between input and output prices. This version of the model results in a third term being added to the hedging equation, which is then represented as

$$H^* = \frac{\sigma_{sf}}{\sigma_f^2} + \frac{E(f_1) - f_0}{Q_i\lambda\sigma_f^2} - \frac{Q_o\sigma_{fo}}{Q_i\sigma_f^2}.$$

The first two components of this hedge ratio represent the hedging and speculative demand for futures contracts. The third component of this hedge ratio is referred to as the strategic demand. In the numerator, Q_o represents the quantity of outputs, and σ_{fo} is the covariance between the futures price and the price of the output. In the denominator, Q_i represents the quantity of inputs, and σ_f^2 is the variance of the futures price.

As the correlation between input and output prices increases, the magnitude of this third term also increases, which reduces the magnitude of the utility-maximizing hedge ratio. A high correlation between input and output prices results in increases or decreases in input prices being offset by similar changes in output prices. As this input/output correlation converges to zero, the entire strategic component of the hedge ratio converges to zero as well. This relationship indicates that decreases in the correlation between input and output prices result in a decreased significance of the hedging effect between the variables.

Utility-maximization models address some of the problems associated with minimum-variance models. First, they realize that the ultimate goal of many hedgers is risk reduction, not risk-minimization. The speculative component allows entities to exploit their price expectations to an extent while still reducing overall price risk exposure. Utility-maximization models also capture the no-hedge choice of many producers better than the risk-minimization models. Here, if speculative demand exactly offsets hedging demand, the entity will choose not to hedge. While it is unlikely that the risk aversion parameter and future price expectations for each producer choosing not to hedge will result in a speculative demand exactly equal to the risk-minimizing hedge, the chances of this occurring are still more likely than the correlation of 0 between spot and futures prices necessary to yield the no-hedge response in risk-minimizing models.

Value at Risk

The concept of Value at Risk (VaR) can be traced to the late 1980s, when major financial firms began to adopt VaR as a measure of the risks inherent to their trading portfolios. The release of *RiskMetrics™* by the risk management group at J.P. Morgan in October of 1994 provided a catalyst to Value at Risk's growth by attempting to standardize the use of VaR throughout the industry (Linsmeier and Pearson, 2000). Value at Risk's popularity as a risk measurement tool has risen dramatically in the last decade to include firms from nearly every sector of the economy (Mina and Xiao, 2001). VaR has also received increasing literary attention from the areas of finance and agricultural economics (Manfredo and Leuthold, 2001a).

Value at Risk can be formally defined as a single, summary statistic that measures the worst expected losses during a given time period, with a specified level of confidence, under normal market conditions (Jorion, 2001). A common example of Value at Risk considers a portfolio with a VaR measure of $1 million during a holding period of one day at the 95% confidence level. This example states that the portfolio will not experience one-day losses exceeding $1 million more than 5% of the time under normal market conditions (Manfredo and Leuthold, 2001a).

Value at Risk offers an attractive alternative to traditional risk measurement tools, such as the traditional mean-variance framework and delta-gamma-vega analysis (Hull, 2000). First, Value at Risk summarizes portfolio risks in terms of potential dollar or percentage losses, as opposed to classifying risk with respect to standard deviations above or below the expected portfolio returns. Although measuring risk in terms of standard deviations may provide accurate estimates of risk exposure for normally distributed random outcomes, managers and decision makers think of risk in terms of dollars. Value at Risk provides managers who may not have a deep understanding of statistical analysis with a single, summary statistic that expresses risk in easily understood terms (Manfredo and Leuthold, 2001a). Second, Value at Risk is able to focus on true downside risk, as opposed to traditional risk measures that classify both upside and downside potential equally, considering all deviations from the expected return as risk. These traditional measures consider any deviation from the expected return as a contribution to risk. This concept is not logical, however, because the potential for increased revenue is not viewed as true risk in the eyes of management. Therefore, traditional risk measures that do not make this distinction can give distorted impressions of risk to those interpreting the figures.

Although realized only in the two full valuation VaR methodologies, a third advantage offered by VaR is the ability to capture the nonlinear payoffs of portfolios that contain options or option-like instruments. One of the fundamental assumptions of most traditional risk measures, including analytical VaR, is that returns of a given amount above or below expected returns occur with equal likelihood. This assumption can hold for portfolios that contain only physical assets, forward contracts, and futures contracts. The presence of options in a portfolio invalidates this assumption by introducing nonlinear payoffs. The ability to provide accurate estimates of risk exposure for portfolios that contain options gives Value at Risk a significant advantage over other measures.

Although Value at Risk appears to address many of the problems associated with other risk-measurement techniques, the literature contains countless warnings that VaR should not be construed as a panacea (Beder, 1995; Jorion, 2001; Duffie and Pan, 1997; Manfredo and Leuthold, 2001a; Linsmeier and Pearson, 2000; Odening and Hinrichs, 2002). VaR describes only the loss that will be exceeded with some level of confidence. However, it says nothing about the absolute worst possible losses. VaR also assumes the portfolio remains constant over the entire time horizon. As the composition of the portfolio changes due to normal trading activity within the time horizon that VaR is measured, the accuracy of the VaR estimate declines. Value at Risk relies on historical price data, and the price risk associated with assets for which historical data are not available is difficult to quantify with VaR. VaR position limits can also lead traders to "game" the system, trading in markets where the historical data resulting in low VaR estimates do not accurately represent the current situation (Jorion, 2001).

Therefore, Value at Risk is not a substitute for the various other risk measures. VaR calculation is not an exact science, and it is not perfect. Jorion (2001) explains that risk management is much more of an art than a science and stresses that, as with any risk-management tool, the VaR user must understand its limitations. VaR is most useful when used to compliment the traditional tools that risk managers and traders have and use. The only place where Value at Risk may be responsibly substituted for traditional measures is in the boardroom where VaR provides an intuitive, easily understandable summary of total risk exposure (Linsmeier and Pearson, 2000).

Value at Risk Methodologies

The objective of risk valuation is to provide a reasonably accurate estimate of market risk at an appropriate, reasonable expense. For this reason, several different methodologies have developed for computing Value at Risk. Manfredo and Leuthold (2001a) describe how these methods vary with respect to accuracy, ease of implementation, time requirements, and ease of explanation to management. The following sections provide an introduction to the three most widely used methods of VaR computation. This discussion will focus on the basic Value at Risk concepts, while the theory and mathematical derivation of the Value at Risk statistics will be explored in depth in the next chapter. The first of the three methodologies is the parametric method known as the variance/covariance approach. The other two methods are full-valuation procedures called historical simulation and Monte Carlo simulation.

Variance/Covariance

Jorion (2001) explains that the fundamental assumption of the variance/covariance approach is that the random outcomes, or asset prices, are normally distributed. Since a portfolio of assets with jointly normally distributed returns will yield a portfolio with normally distributed returns, the assumption simplifies the model significantly. Once the parameters of the returns distribution have been estimated for the entire portfolio, Linsmeier and Pearson (2000) state that the Value at Risk statistic can be found easily by determining the loss value that will be equaled or exceeded only a specified percentage of the time.

Risk mapping is an important step in the variance/covariance methodology. This step is the process where the actual financial instruments and cash positions are divided into an appropriate number of simpler, standardized instruments or positions (Linsmeier and Pearson, 2000). Each simplified position represents a separate price risk variable, and a covariance matrix for the random outcomes caused by each source of financial uncertainty is then derived. This basic price risk variable matrix can then be used to determine the covariance matrix for the various standardized positions. In turn, this matrix is used to calculate the standard deviation for a portfolio that contains any combination of assets affected by these basic price risk variables. Linsmeier and Pearson (2000) explain that in order to analyze any portfolio, one must first break the portfolio into standardized positions that have similar sensitivities to changes in the basic price risk variables as the original position. Therefore, the Value at Risk estimate's accuracy depends heavily on how effectively the standardized portfolio mirrors changes in the actual portfolio.

The assumption of normality of returns and the fact that the VaR is calculated only for the equivalent, simplified portfolio brings about the first main disadvantage of the variance/covariance method. A portfolio with significant options content can produce inaccurate VaR statistics. Other option-like instruments produce the same results due to the nonlinearity of expected returns (Jorion, 2001). Manfredo and Leuthold (2001a) point out that the variance/covariance method produces adequate VaR statistics for portfolios with moderate option content, as long as the holding period is very short. Inaccuracies due to option content are compounded, however, when the holding period is increased.

This methodology also has the tendency to produce larger than normal tails in the distribution of returns. Jorion (2001) explains that these *fat tails* are of concern due to the focus of VaR on the events occurring in the extreme left-hand tail of the distribution. *Fat tails* can cause underestimation of the true risk associated with a portfolio by assuming the smaller left-hand tail of the normal distribution.

Despite its weaknesses, parametric variance/covariance analysis can be extremely useful. For portfolios with relatively minimal option content, this method provides a Value at Risk statistic that is easy to implement, and computation can be accomplished relatively quickly. The method is also very flexible in that correlations and standard deviations can be varied easily to determine the effects of changes in relationships among price risk variables (Manfredo and Leuthold, 2001a).

Historical Simulation

The first of the two full-valuation techniques to be examined is historical simulation. In this methodology, a current portfolio of assets is exposed to actual changes in relevant market factors over a historical period. The need for complex statistical covariance matrices and distributions is thus eliminated. The fact that historical simulation exposes a current portfolio of assets to actual market factors of numerous historical periods is the distinguishing feature of this method (Linsmeier and Pearson, 2000).

The first step in this analysis is to determine all of the market factors relevant to the portfolio in question. Then, hypothetical daily mark-to-market values of the specific portfolio are calculated for each historical day or other desired time period. The changes in the mark-to-market values are then ranked in order of magnitude, and the loss that is equaled or exceeded only *X* percent of the time is selected as the Value at Risk statistic. Jorion (2001) notes that the weights for each daily change in mark-to-market value are kept equal, indicating that each historical set of daily market conditions is equally likely to occur in the coming day.

Historical simulation offers several advantages over the other Value at Risk methodologies. One of the most important advantages is that, by calculating the full-valuation for each period, historical simulation is able to account for options and other instruments with nonlinear payoffs. As mentioned earlier, this method also eliminates the need for complex covariance matrices, which makes explanation to management much simpler. The fact that actual past market movements are used also makes the methodology intuitive and robust (Jorion, 2001).

The problem of horizon choice is dealt with much better in this methodology than in the other Value at Risk methods. Scaling up VaR statistics to obtain values for holding periods longer than one day can increase the estimation error for other VaR methods. Since historical returns can be measured over any desired time period, historical simulation can calculate Value at Risk statistics for relatively long time horizons with increased accuracy.

Jorion (2001) indicates that although its ease of implementation, explanation, and computation has made historical simulation the most widely used VaR computation method, it does have many disadvantages. In order to obtain high confidence intervals for these VaR statistics, large amounts of historical data are necessary. If price level information has been collected for marking-to-market, this problem diminishes since the same date can be used for both daily portfolio valuation and VaR calculation. The model also assumes that past market movements provide good indications of future movement. Atypical historical periods are difficult to account for, and structural changes are slow to be incorporated into the data set (Linsmeier and Pearson, 2000).

Although large time series data sets tend to increase model accuracy, Manfredo and Leuthold (2001a) point out that large data sets are more likely to contain excess extreme market movements that do not reflect current market conditions. This can have the effect of creating an upward bias in the Value at Risk statistic. Historical simulation also ignores the time variance associated with the variance of the distribution by giving each historical time period an equal weight. Temporary periods of increased or decreased volatility will also be unaccounted for in this methodology (Jorion, 2001). These possibly predictable situations will not be reflected in the VaR measure when diluted in large time series data sets.

Monte Carlo Simulation

The second full-valuation methodology is known as Monte Carlo simulation. Monte Carlo simulation is based on the same principal as historical simulation in that portfolio returns are actually generated for numerous possible scenarios. However, instead of subjecting the current portfolio to actual historical price changes over the previous *N* time periods, Monte Carlo simulation requires the user to assign an appropriate statistical distribution to each price risk variable that adequately approximates its possible changes (Linsmeier and Pearson, 2000).

Once statistical distributions have been assigned to each price risk variable, pseudo-random values are generated for each, constructing *N* possible overall return values for the portfolio in question. Linsmeier and Pearson (2000) describe that *N* is a significantly large number greater than 1,000 and, in some cases, greater than 10,000. These *N* possible returns are then treated just as those in historical simulation. Each of the *N* simulated portfolio values is subtracted from the marked-to-market value of the actual current portfolio. These hypothetical profit and loss values are then ranked in order of magnitude, and the loss that is exceeded no more than *X* percent of the time is selected as the Value at Risk statistic.

Jorion (2001) summarized the relationship between Monte Carlo simulation and historical simulation by saying that hypothetical price changes used in Monte Carlo simulation "are created by random draws from a prespecified stochastic process instead of sampled from historical data" (p. 225).

Although various forms of the three previously described methodologies are used throughout the corporate world, Jorion (2001) stresses that Monte Carlo simulation is a much more powerful tool than the other Value at Risk methods. It offers much more flexibility and overcomes many of the problems associated with parametric methodologies and historical simulation. Instruments with nonlinear payoffs, such as options, do not present a problem for Monte Carlo simulation. The ability to vary parameter distributions and evaluate "what-if" scenarios offer another advantage (Linsmeier and Pearson, 2000). These aspects, coupled with Monte Carlo simulation's ability to incorporate *fat tails* in the distribution, unlikely extreme scenarios, and the passage of time lead Jorion (2001) to suggest its technical superiority over other methodologies.

The costs associated with this technical superiority are significant, however, which is likely the reason that historical simulation and parametric methodologies are also widely used. The relatively long computational times for large portfolios can make Monte Carlo simulation comparatively expensive to implement and are perhaps its most significant disadvantage. The actual implementation is not overwhelming when off-the-shelf software is available. Alternatively, under circumstances where the necessary software does not exist, developing Monte Carlo models from scratch can be very time consuming. Manfredo and Leuthold (2001a) also discuss how the valuable freedom to choose statistical distributions to represent each price risk variable can result in adverse affects. Distributions chosen by the designer of the model may not accurately represent the variability of each of the price risk variables, which increases model error.

Value at Risk in Agricultural Economics

Manfredo and Leuthold (2001a) discuss several agricultural areas in which Value at Risk could provide substantial benefits. They indicate that one of the main uses has to do with the fact that "Publicly traded agribusiness firms must comply with SEC regulations concerning the reporting of positions in highly market sensitive assets, including spot commodities, futures, and options positions" (p. 110). They also express that, had elevator managers and producers been using VaR, the hedge-to-arrive crisis of 1996 could possibly have been averted. Agricultural lenders could also apply Value at

Risk in credit scoring as well as use it to determine the magnitude of price risk they are exposed to indirectly through their borrowers.

Although the study and application of Value at Risk has received considerable attention in the financial literature, its implementation in the agricultural economics literature is limited. Potential agricultural applications of VaR are wide ranging; however, only three works were identified that applied VaR to an agriculture-based scenario. Manfredo and Leuthold (2001b) examined the relationship between the prices of fed cattle, which are a cattle feeders' output, and corn and feeder cattle prices, which are inputs for this type of firm. Sanders and Manfredo (1999) use a hypothetical food service company to demonstrate VaR implementation for a commodity end user, but consider only the risk of the commodity inputs in their analysis. Odening and Hinrichs (2002) consider hog production in Germany when contrasting extreme value theory with a variant of VaR, Cash Flow at Risk, as a means of quantifying market risk.

This thesis uses the portfolio of a hypothetical bread baking company and a flour milling firm to provide examples of Value at Risk in a commodity processor application. The scope of the basic commodity end user application is expanded in this thesis by including both input and output price risk. The scope is further complicated by the fact that prices of consumer goods, such as bread, are exposed to forces that differ significantly from those that affect prices of raw commodities. Another extension considers the effect of foreign exchange risk for a Mexican flour milling company that purchases its input in a foreign currency. These case studies use Value at Risk to quantify the price risk associated with the portfolios of hypothetical agricultural processing firms and evaluate the risk-reducing effects of various hedging strategies.

CHAPTER III. THEORETICAL MODELS OF VALUE AT RISK

The historical foundations of Value at Risk can be traced back half a century to the work of Harry Markowitz and Andrew Roy, who both published key findings in 1952. As described in the previous chapter, Markowitz (1952) was the first to formally define the trade-offs that investors faced between risk and expected returns, which explained the logic behind the practice of diversification. The mean-variance framework that he used, however, is only accurate when the portfolio returns are normally distributed or the utility function of the investor is quadratic.

Roy (1952) argued that the objectives behind portfolio selection and diversification are concerned much less with stabilizing expected returns than with avoiding economic disasters. His "Safety First" criterion indicates that the ultimate goal of portfolio selection is to minimize the probability that a disastrous loss will be incurred. The similarities between the ideas of Roy and Markowitz become apparent when Roy (1952) indicates that the definition of a disastrous loss is likely to vary as the expected return of an investment changes. In other words, low levels of expected return lead to relatively small losses being considered to be disastrous. As expected returns increase, he states that the investor's definition of a disastrous loss that must be guarded against likely increases as well.

The first support for a confidence-based risk measurement criterion comes from Baumol (1963). He describes a situation where some unacceptable portfolios can actually be found among the set of portfolios that Markowitz's selection criterion lists as efficient. He then points out that the absolute value of the risk measure, standard deviation, is much less important than the value of the standard deviation relative to the value of the expected return. A confidence-based selection criterion is then offered as a method of incorporating both risk and expected return into one number which captures the relationship between the two. This criterion is the fundamental basis for Value at Risk. The equation Baumol (1963) uses to represent the lower confidence limit (L) is $L = E - K\sigma$, where E is the expected portfolio return, σ is the standard deviation of portfolio returns, and K is the number of standard deviations from the expected return that corresponds to the desired confidence level. This equation is essentially equivalent to the equation for R^*, representing the cutoff return, used in the Value at Risk models explained in the following sections.

This chapter focuses on the theoretical model of Value at Risk and provides detailed derivations of the alternative Value at Risk methodologies. Various aspects of quantitative factor selection are discussed, and the three VaR methods are compared and contrasted. Model validation, also called back-testing, is described, and two common testing methods are shown. An explanation of stress testing and an evaluation of the similarities and differences between Value at Risk and portfolio theory conclude the chapter.

Steps and Decisions in Value at Risk Construction

Value at Risk (VaR) is formally defined as a single, summary statistic indicating the portfolio loss that will be exceeded only with a probability of $1-c$, during a given time period (t) under normal market conditions, where c is the specified confidence interval. Before the computation of VaR can actually begin, there are two steps that must be completed and several decisions that must be made.

The first step is to mark-to-market the current portfolio of assets. This figure, W_0, represents the current value of the portfolio and is commonly referred to as the initial portfolio value. The second step is that of collecting historical price data associated with each relevant price risk variable, which is used to determine the variability caused by each of the various factors or, in the case of historical simulation, is used to construct the actual simulations themselves.

The selection of the appropriate time horizon (t) is the first decision to be made. The second decision is in regard to the confidence interval (c) which will be used. The third decision is likely the most crucial and consists of choosing the methodology to employ from the three basic methods of computing Value at Risk. There is no obvious right or wrong choice for any of these three decisions. Situations and circumstances favoring certain time horizons and confidence intervals will be explained later in the chapter as well as the advantages and disadvantages of the three main VaR computational methods.

After these steps have been completed and the decisions have been made, Value at Risk computation can begin. Once completed, the final result can be reported as the loss that will be exceeded in the next t period with a probability of $1-c$.

Value at Risk Computation

The three different VaR approaches can be separated into two basic categories. Models in the general distributions category, consisting of the two full valuation approaches, use simulation techniques to calculate Value at Risk statistics for portfolios with returns that exhibit any distribution. The model in the alternative category uses an analytical approach. Also called the variance/covariance method, the analytical approach assumes that possible returns take the form of a parametric distribution. The theoretical model for both of the categories will be described in detail in the following sections.

General Distributions

Jorion (2001) begins his explanation of Value at Risk by defining W as the end-of-period portfolio value, W_0 as the initial portfolio value, and R as the rate of return on the portfolio such that $W = W_0(1+R)$. W^* is then defined as $W^* = W_0(1+R^*)$, or the portfolio value when R^*, the critical rate of return associated with the confidence level c, is realized. The confidence level c indicates that a return equal to, or lower than, the critical rate of return R^* is only expected to occur with a frequency of $1-c$ during normal market conditions. Jorion (2001) continues to explain that, in its broadest context, the Value at Risk statistic for a future portfolio value W with a probability distribution $f(W)$ can be derived from the integral equation,

$$1-c = \int_{-\infty}^{W^*} f(W)dW \ .$$

This equation represents the probability that the end-of-period portfolio value will be less than or equal to W^*, the critical portfolio value. It states that the area in the far left-hand tail of the probability distribution between $-\infty$ and W^* must sum to the probability $1-c$ (Jorion 2001). Therefore, W^* is a quantile of the distribution and the item of particular interest in this procedure.

Once the quantile W^* is read from the distribution of future portfolio values, the VaR can be found in either absolute or relative terms. The absolute Value at Risk refers to the distance between the dollar loss quantile and the initial portfolio value, without considering the expected portfolio value, and is represented by

$$VaR(zero) = W_0 - W^* = -W_0 R^* .$$

Although the absolute VaR can be helpful in cases where the expected portfolio value is difficult to calculate, relative VaR provides a more logical statistic in many applications due to the inclusion of the time value of money concept. The relative Value at Risk is defined as

$$VaR(mean) = E(W) - W^* = -W_0 (R^* - \mu) ,$$

where μ is the expected value of R, and $E(W)$ represents the expect value of W.

The model of Value at Risk for general distributions is very versatile and can be applied to portfolios with any distribution of returns. As Holton (1998) explains, the problem that one immediately encounters is the fact that closed form solutions do not exist for most portfolios, leaving two options. The first, numerical integration methods, is practical for portfolios with one or two dimensions or price risk variables. However, Holton (1998) states that as portfolios become larger and more diverse, the curse of dimensionality increases the complexity of numerical integration methods, challenging the computing power of today's most advanced technology. For this reason, general distribution problems are solved using simulation techniques to create the distributions of portfolio returns, which allow the dollar loss amount corresponding to the desired quantile to be read from the generated distribution.

Parametric Distributions

The variance/covariance approach is a much simpler methodology which bypasses much of the complexity associated with the general distribution method by assuming that portfolio returns follow a parametric distribution. This approach allows the problem to be solved analytically, but also introduces an additional estimation error if the assumed parametric distribution does not accurately reflect the true distribution of portfolio returns.

The most commonly used parametric distribution is the standard normal distribution. This distribution will be used to illustrate the methodology; however, other parametric distributions that may better fit the actual portfolio returns can be used as well.

The first step Jorion (2001) illustrates is that of converting the actual portfolio distribution $f(W)$ to the standard normal distribution $\Phi(\in)$, with a mean of 0 and standard deviation of 1. He again defines $W*$ as the cutoff end-of-period portfolio value corresponding to the rate of return $R*$, associated with a desired level of confidence c, and W_0 as the initial portfolio value. He uses μ and σ to represent the expected return and standard deviation of $R*$, respectively. It then shows that $W* = W_0(1+R*)$ and that, since $R*$ is usually a negative number, it can be expressed as $-|R*|$. The relationship between the standard normal deviate α and $R*$ is shown in the equation,

$$-\alpha = \frac{-|R*|-\mu}{\sigma},$$

such that $-\alpha$ is set equal to the expected rate of return subtracted from the cutoff rate of return, divided by the standard deviation of the rate of return.

The leap from a general distribution to the standard normal distribution, as well as the fundamental difference between full valuation and analytical VaR techniques, can be illustrated by expanding the general distribution integral equation such that

$$1 - c = \int_{-\infty}^{W^*} f(W)dW = \int_{-\infty}^{-|R^*|} f(R)dR = \int_{-\infty}^{-\alpha} \Phi(\in)d\in .$$

Therefore, the primary result is that, while the VaR for general distributions is found by searching for W^*, the VaR for a portfolio with returns of a standard normal distribution can be found by solving for the standard normal deviate α instead. The value of W^* can only be found through simulation; however, finding the α that makes the equation true is much easier. Jorion (2001) points out that the cumulative normal probability distribution, illustrated in Figure 3.1, is represented by

$$N(d) = \int_{-\infty}^{d} \Phi(\in)de .$$

By setting $1 - c = N(d)$, it is shown that d and $-\alpha$ are equivalent. This means that $-\alpha$ can be read off the cumulative normal probability distribution as the standard normal variable d resulting from a $N(d)$ value equivalent to $1 - c$. Figure 3.1 demonstrates this logic using a 95% confidence interval.

Now that the value of $-\alpha$ corresponding to the chosen confidence interval has been found, Jorion (2001) explains that by rearranging the equation for $-\alpha$ described earlier, we can determine that the cutoff return must be

$$R^* = -\alpha\sigma + \mu$$

when $R^* < 0$. This formula is essentially equivalent to that described by Baumol (1963)

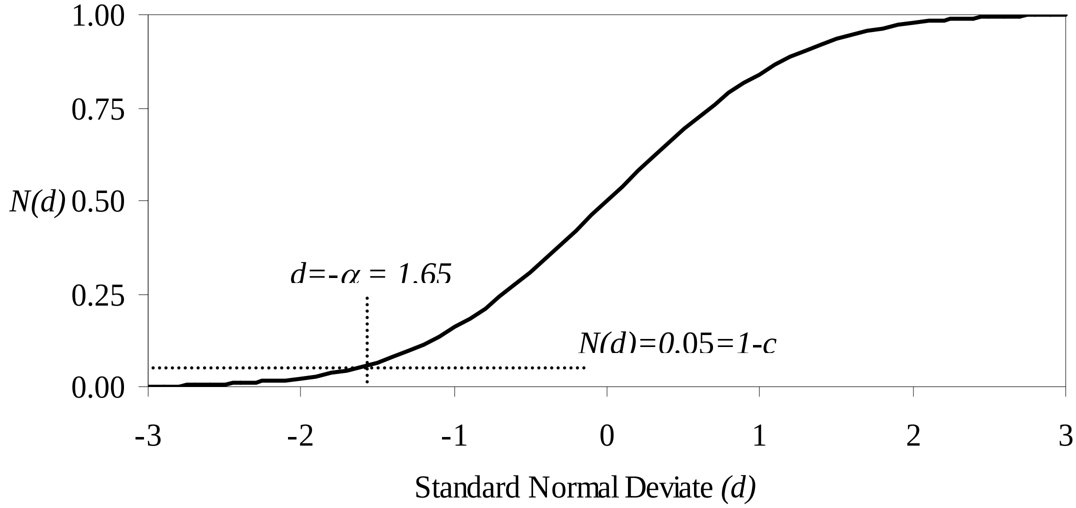

Figure 3.1. Cumulative Normal Probability Distribution.

Source: Adapted from Jorion (2001).

when he first introduced the idea of measuring risk using a confidence-based criterion. Baumol's equation,

$$L = E - K\sigma ,$$

varies from Jorion's (2001) only in the notation used to describe each variable, as Baumol (1963) uses L to represent the lower confidence limit, E to represent expected portfolio returns, and K as the number of standard deviations from the expected returns that corresponds to the desired confidence level.

Rearranging the equation for $R*$ and substituting it into the equation for the relative VaR for a general distribution results in the relative VaR for a parametric distribution

$$VaR(mean) = -W_0 (R* - \mu) = W_0 \alpha\sigma\sqrt{\Delta t} .$$

When μ and σ are expressed annually, instead of over the desired horizon, it is necessary to include the $\sqrt{\Delta t}$ factor to scale the parameters down to the appropriate time horizon chosen to evaluate the VaR statistic. Therefore, if a one-day VaR was to be calculated, Δt would be set equal to $1/252$, as t represents time in years. The same time horizon consideration is applied in the absolute VaR calculation given as

$$Var(zero) = -W_0 R^* = W_0 (\alpha\sigma\sqrt{\Delta t} - \mu\sqrt{\Delta t}).$$

Just as in the general distribution VaR equations, the way that the expected rate of return is accounted for is the difference between the absolute and relative VaR measures.

Quantitative Factor Selection

Now that the theoretical basis for Value at Risk has been presented, it is important to focus on the two quantitative factors involved in both full valuation and parametric methods. These items are the selection of the proper time horizon and the choice of the appropriate confidence interval. Jorion (2001) explains that the key in choosing both time horizon and confidence interval relates to the specific application for which the Value at Risk statistic will be used. He describes that VaR can be used as a benchmark measure, a potential loss measure, or to set equity capital reserves. It is important to note that typically, as either the time horizon or the confidence interval increases, the VaR statistic increases as well. The following sections will discuss considerations for quantitative factor selection and the impacts the choice of time horizon and confidence interval have in each Value at Risk application.

Time Horizon

When using Value at Risk as a benchmark with which to compare risks over time, between alternative projects, or between trading desks, the choice of the time horizon is arbitrary. Jorion (2001) explains that the key in this application is consistency. Whether a one-day or one-year horizon is used, the risk of alternate projects and positions will still be placed in the same order. The value of VaR in this benchmark application is simply as a relative yardstick with which to compare today's risk with yesterday's risk, project A to project B, or trading desk 1 to trading desk 2. Therefore, the most important consideration in time horizon selection is consistency, which allows decision makers to get comfortable with the magnitude of VaR statistics allowing them to use Value at Risk effectively as a comparative tool.

The time horizon selection becomes more important when the VaR statistic is used to measure potential losses. The most common theory is that the time horizon chosen should correspond to the type of assets which make up the portfolio. The horizon should be equivalent to the time required for an orderly liquidation of the portfolio, or the time required to properly hedge the price risk variables, indicating that firms dealing with relatively illiquid assets should use longer time horizons. Banks tend to use much shorter time horizons due to the high liquidity of the markets in which they operate. An alternative theory is that time horizon should be chosen according to the anticipated holding period for the current portfolio. Value at Risk measures assume a constant portfolio is held throughout the selected time horizon. Therefore, long time horizons applied to portfolios that change significantly during the period result in loss of accuracy (Jorion, 2001).

When VaR is used to determine the amount of equity capital a firm holds in reserve to cover potential losses, the choice of time horizon is crucial. These types of models tend to be set up so a loss exceeding the VaR is very costly. Therefore, horizons must be set to reflect the time needed to implement risk-reducing measures. The nature of the assets composing the portfolio is again where concentration should be focused.

The selection of the appropriate time horizon is often a balancing act. In some cases, horizons of one quarter or one year may seem necessary due to asset liquidity issues. However, longer time horizons can lead to data complications. An instance where several thousand daily observations are available would result in significant data for daily VaR calculations. When aggregated into yearly figures, only a handful of observations remain, encouraging users to include data points which are outdated and do not accurately represent current market conditions.

Long time horizons in Value at Risk models also make back-testing, which will be described in more detail in a later section, much less effective. Yearly VaR numbers result in only one observation per year, instead of the 252 observations per year that can be tested when using a daily measure.

Due to the advantages and disadvantages of both long and short horizons, many users of Value at Risk choose to compute VaR for a relatively short horizon and simply scale the statistic up to longer horizons by multiplying by the square root of time, $(\sqrt{t})$. This method of time aggregation has logic appeal and is based on the fact that volatility tends to increase with the square root of time (Jorion, 2001). This relationship holds exactly, however, only when restrictive assumptions are met. Iacone and Skeie (1996) explain several facets of these assumptions, and Diebold et al. (1998) simply state that scaling is accurate only when returns are independently and identically distributed (i.i.d.). Diebold et al. (1998) insist that “Modeling volatility only at one short horizon, followed by scaling to convert to longer horizons, is likely to be inappropriate and misleading, because temporal aggregation should reduce volatility fluctuations, whereas scaling amplifies them” (p. 8). They proceed by offering that scaling can be very useful in certain applications and that, on average, scaling results are correct. However, when this method of scaling is used, it must be accompanied by an understanding of its inaccuracies.

These issues increase the complexity of choosing a time horizon. As Christoffersen et al. (1998) explain, “There is no one ‘magic’ relevant horizon for risk management” (p. 109). The appropriate horizon typically varies depending on the class of assets composing the portfolio, the industry in which the firm competes, and the specific application for which the Value at Risk statistic is used.

Confidence Interval

The choice of the confidence interval for the benchmark application of Value at Risk is similar to the time horizon choice. Consistency is the key factor for this type of VaR statistic since it is used only as a comparative measure with which to evaluate various scenarios and time periods relative to one another.

The confidence interval used in the measure of potential loss application of VaR is also relatively insignificant as long as decision makers realize that the Value at Risk is a probabilistic measure and that losses exceeding the VaR figures should be expected. The misconception that the Value at Risk statistic measures the absolute worst possible outcome can be very dangerous since, by definition, losses greater than the VaR will occur with regular frequency.

The application in which confidence interval selection is extremely important is where VaR is used to determine the amount of equity capital reserves to hold. As mentioned in the time horizon discussion, exceeding VaR in this case can be very costly. When determining the confidence interval, Jorion (2001) indicates that two aspects must be considered. The first is the level of risk aversion of the firm. The second has to do with the costs associated with a loss exceeding the VaR. If the cost of exceeding the figure merely results in borrowing, confidence intervals can be set relatively low. However, if losses greater than the VaR place the firm near bankruptcy, a high confidence interval must be chosen.

Similar to time horizon selection, the choice of confidence interval can have a significant impact on the power of back-testing. Jorion (2001) describes how the use of high confidence intervals results in very few losses that exceed the VaR statistics. Therefore, a much larger set of observations is necessary when back-testing is performed to determine if the VaR is exceeded with the specified frequency.

Value at Risk Methodology Comparison

Broad descriptions of the three Value at Risk methodologies, parametric, historical simulation, and Monte Carlo simulation, were given in the previous chapter and, therefore, will not be reiterated here. The focus will instead be on the differences between the methods, which should be considered when selecting the appropriate method for each individual application. The five key areas that the methodologies are evaluated on in the following sections are accuracy, speed and complexity, ease of explanation to management, cost, and flexibility.

Accuracy

Two main accuracy issues exist with the parametric methodology. The first is the inability of this method to capture the risks of portfolios containing significant nonlinear instruments, such as options. The key assumption of normally distributed portfolio returns made in the parametric method is violated by these nonlinear instruments. The methodology used to circumvent this issue of nonlinearity is to reduce these instruments to their delta-equivalent in a linear instrument.

The option delta (Δ) is a basic risk management concept that represents the partial derivative of the option-pricing formula with respect to the underlying asset price. If an investor holds a call option for an infinitesimal change in the price of the underlying futures contract, the behavior of the option price will be equivalent to the same position in Δ underlying futures contracts. This relationship, however, holds only for a miniscule change in the futures contract price. Therefore, reducing options and other nonlinear instruments to their delta-equivalent number of futures contracts results in a linear portfolio for which a parametric VaR statistic can be computed. This method is effective when these instruments make up a relatively small portion of the portfolio, and the time horizon used to compute the Value at Risk is very short. However, the inaccuracy of this method is compounded as option content and time horizon increases (Linsmeier and Pearson, 2000).

The parametric approach is also based on the historical price relationships between the relevant price risk variables. If the price relationships of the past do not accurately represent the current market conditions, complications arise. In these instances, risk can be greatly over or understated. Linsmeier and Pearson (2000) describe how traders can take advantage of these situations to increase the true risks of their activities without these risks being adequately reflected in the Value at Risk statistics, which are designed to limit their risk exposure.

Historical simulation is not hindered by the presence of nonlinear instruments. Since it is a full valuation technique, meaning the portfolio value is recalculated for each historical time period, reducing nonlinear instruments to their delta-equivalents is not necessary, and options are accounted for accurately. However, the problem of atypical price movements and relationships is most severe in this method. Using only relatively recent data is most desirable from the aspect of capturing current market conditions. Alternatively, long data sets are more likely to prevent short-term trends from biasing the VaR, leading to another balancing act where the risk manager must choose the amount of data which leads to the optimal combination of the two advantages. Jorion (2001) also explains that historical simulation accounts for *fat tails* very well, is very robust, and is not susceptible to model risk.

Like historical simulation, Monte Carlo simulation also performs consistently regardless of the level of options content in the portfolio. However, Monte Carlo simulation is prone to model risk, due to the fact that representative distributions must be chosen by the risk manager for each individual risk component. This flexibility allows the manager to override historical data which are thought to misrepresent current conditions and choose a more appropriate distribution. It also allows the risk manager to err and choose a distribution that inadequately represents the current market situation (Jorion, 2001).

Speed and Complexity

The parametric method is relatively simple to implement when off-the-shelf software is available. When the software is not available and the extensive variance/covariance matrix must be constructed manually, the methodology becomes more involved. Complex financial instruments, such as options, also increase the difficulty of parametric VaR since they must be decomposed and mapped into their delta-equivalent positions (Linsmeier and Pearson, 2000). Computation, however, is very fast even for large portfolios since time consuming simulation is not involved.

Historical simulation is arguably the simplest of the VaR methodologies. A complex variance/covariance matrix is not necessary, and there is no need to estimate the statistical distributions of each asset in the portfolio. Instead, the challenge is that the risk manager must have time series price data for each asset over the last *N* periods (Linsmeier and Pearson, 2000). When historical price data are readily available and portfolios are relatively small, computation speed is not a hindrance. Large, complex portfolios lead to cumbersome implementation and, in practice, similar instruments tend to be grouped to reduce computation time (Jorion, 2001). However, this grouping also reduces the accuracy of the VaR estimate.

Monte Carlo simulation is far and away the most complex and powerful of the Value at Risk methodologies. When software is available, the level of implementation difficulty is similar to the variance/covariance approach. However, when models must be developed from scratch, Monte Carlo simulation quickly becomes the most complicated method. Computational time is also a significant detriment of Monte Carlo simulation as the need for large numbers of simulations, coupled with large portfolios, leads to lengthy time requirements.

Ease of Explanation to Management

Historical simulation is the most intuitive approach to Value at Risk calculation, especially to those who are not trained in statistical techniques. Exposing the current portfolio to the market conditions experienced over the last *N* periods is a logical way to measure risk. Comprehension of the parametric method, which relies heavily on statistics, the normal distribution, and the variance/covariance matrix, requires significant knowledge of statistical techniques. Linsmeier and Pearson (2000) argue that explaining Monte Carlo simulation to management is even more difficult, as pseudo-random number generators and the fitting of distributions to data sets are foreign concepts to most individuals.

Cost

The costs associated with each Value at Risk methodology vary greatly depending on the specific situation at hand. If time series price data are collected for daily marking-to-market, additional data requirements for VaR computation are negligible. If not, data collection can result in a significant component of VaR cost. The availability of off-the-shelf software is also an important consideration, since developing parametric and Monte

Carlo simulation models from scratch can be very time consuming. Overall, Monte Carlo simulation is the most expensive approach to Value at Risk computation. The time consuming simulations make significant hardware demands, and the extensive intellectual power necessary to develop a Monte Carlo simulation model can also be quite costly (Jorion, 2001).

Flexibility

In some instances, a risk manager may have reason to believe that historical price movements are not the best estimate of future movements, or that future price relationships between assets may be drastically different from those observed in the past. Various political, structural, or economical changes can signal these shifts and lead to what is commonly referred to as “What-if” analysis. This type of analysis can be very valuable since it allows risk managers to evaluate the current portfolio risks under a variety of possible circumstances.

Linsmeier and Pearson (2000) explain that historical simulation is the only methodology which does not allow for “What-if” analysis. It relies completely on actual historical price movements and relationships and leaves no leeway for adjusting the model. The parametric approach, however, requires a complex variance/covariance matrix be compiled. This matrix can be used to override historical price relationships in order to analyze portfolio performance under alternative scenarios. Monte Carlo simulation is just as versatile, but this type of analysis is done in a slightly different manner. Monte Carlo simulation requires the user to select statistical distributions that best represent future price movements. Although this selection process is normally accomplished by fitting distributions to historical data, performing analysis with distributions chosen by other methods is equally justifiable under circumstances when history is not believed to be the best estimate of the future.

Model Validation

When designing models used to predict reality, it is always important to test that the models, and thus their predictions, are truly in line with actual, observed outcomes. This general concept is referred to as model validation and is a crucial component of any responsible Value at Risk estimation program. While several different tests have been designed for this purpose, this discussion focuses on the technique referred to as back-testing, which is commonly used in most Value at Risk programs.

Jorion (2001) defines back-testing as a statistical method used to verify that actual losses are consistent with losses predicted by a Value at Risk model. Back-testing is performed not only by the model designers to ensure that the VaR model is accurate, but also by regulators that require certain types of firms to report VaR. Regulators use these Values at Risk statistics in assessing the risk exposure of a firm, which can affect capital reserve requirements and insurance premiums. For this reason, there are motives for firms to configure VaR models so that risk is understated.

Through back-testing procedures, regulators are able to compare actual profits and losses with the estimates generated by the VaR model. By imposing penalties on firms when actual losses exceed the VaR statistic at a frequency greater than the model specifies, regulators can cancel out the motivation to understate risk. However, one must consider that Value at Risk is only an estimate of the frequency of losses exceeding a specific amount. Therefore, some periods will experience more losses than stated by the model, and some will experience less. This fluctuation does not indicate a poor model, but is instead an attribute of the risk measure itself. For this reason, regulators give risk managers leeway, and models are not rejected as inaccurate unless risk is consistently misstated.

Cassidy and Gizycki (1997) explain that one of the main challenges of back-testing is that the portfolio for which the Value at Risk statistic is calculated is rarely the same portfolio for which actual returns are observed. VaR assumes that a portfolio remains constant over the entire time horizon, which is relatively unlikely. While the model estimates VaR for the static portfolio, regulators and board members are concerned with profits and losses realized from the dynamic portfolio. For this reason, back-testing is usually performed on both actual returns and hypothetical returns.

Several different back-testing techniques have been developed. However, the test based on the number of exceptions, or the number of losses that exceed the VaR, is the simplest method and is commonly used. Jorion (2001) begins his discussion of this method by defining $p = 1 - c$. For an example using a one-day time horizon and a 95% confidence interval, T represents the total number of days, N is the number of exceptions, and N/T is the failure rate. The premise of the test is that, if the model is accurate, p and N/T will converge as the sample size gets sufficiently large.

In testing the number of failures without regard to the magnitude of failures, the Bernoulli distribution can be applied. This distribution is represented by the function,

$$f(x) = \binom{T}{x} p^x (1-p)^{T-x}.$$

It is also true that $E(x) = pT$ and that the variance can be represented as $V(x) = p(1-p)T$. Through the central limit theorem, the binomial, Bernoulli distribution can be approximated by the normal distribution when T is significantly large, such that

$$z = \frac{x - pT}{\sqrt{p(1-p)T}} \approx N(0,1).$$

Therefore, testing the significance of z will indicate whether the null hypothesis $p = N/T = 1 - c = 0.05$ is rejected or whether the model can be pronounced acceptably accurate.

There are negative consequences of this test in that both type 1 and type 2 errors are expected to occur. Jorion (2001) explains that a type 1 error is committed when an accurate model is rejected as a result of experiencing VaR exceptions at a rate significantly greater than p. Alternatively, type 2 errors are experienced when an inaccurate model is not rejected. Ideally, a test with low probability of committing both type 1 and type 2 errors would be most desirable. However, type 1 and type 2 errors are inversely related, forcing the risk manager to choose an optimal balance between the two types.

The most powerful VaR methods are those that exhibit a low number of both type 1 and type 2 errors. Kupiec (1995) develops a likelihood ratio statistic for testing the accuracy of VaR models. To do this, he first chooses a confidence level, not related to the VaR confidence level, corresponding to the acceptance or rejection of the model. The log-likelihood ratio is then represented as

$$LR_{uc} - 2\ln[(1-p)^{T-N} p^{N}] + 2\ln\{[1-(N/T)]^{T-N} (N/T)^{N}\}.$$

The null hypothesis is equivalent to the null hypothesis for the preceding z test. The LR_{uc} is an asymptotic chi-square distribution with one degree of freedom. Therefore, if the LR_{uc} is greater than the chi-square value corresponding to the chosen confidence level, the null hypothesis is rejected and the model is said to be inaccurate.

Stress Testing

Value at Risk is an estimate of the losses that can be expected over a given time horizon with a specified level of confidence under normal market conditions. However, this estimate implies nothing about the frequency or magnitude of losses that may occur under abnormal market conditions, which can have dramatic effects on profits and losses. Jorion (2001) explains that all three VaR estimation approaches rely on historical price data, and that unusual situations not captured or accounted for in the data are not accounted for in the Value at Risk models either.

Schachter (1998) explains that the purpose of stress testing is to measure the effects of unlikely, but plausible, economic events that have either happened in the past or are believed to be possible in the future. Since the plausibility of stress events that have been observed in the past is undeniable, he asserts that stress testing with historical scenarios offers a major advantage. By definition, however, these historical events are extremely rare and may not capture all of the potential movements of the current environment. For this reason, hypothetical but logical situations are also included in stress testing programs.

While generating hypothetical stress events in general is not difficult, generating hypothetical stress events that are plausible and relevant to the portfolio at hand can present a challenge. These events can represent three basic categories described by Berkowitz (1999) as “Simulating shocks that have never occurred, simulating shocks that reflect the possibility that statistical patterns could break down in some circumstances,” or “simulating shocks that reflect some kind of structural break that could occur in the future” (p. 4). With the unlimited number of potential situations that could be analyzed, the art of stress testing relates to choosing the events that are most likely to occur or that would have the largest impact on the portfolio in question.

Whether it is required by regulators or used to give decision makers an idea of worst case scenarios, stress testing plays an important role in all responsible Value at Risk programs. Value at Risk describes the loss which will be exceeded under certain parameters; however, it says nothing about the magnitude of these worst case scenarios. Stress testing is used to supplement Value at Risk for this reason and allows the risk manager to evaluate unlikely scenarios and the effects of these potential events on returns.

Value at Risk vs. Portfolio Theory

While significant differences exist between Value at Risk and portfolio theory, it is important to begin by explaining the most prominent similarity between the two. Portfolio theory introduced the concept of risk into the investment decision, which was previously thought to be based on expected return alone. Markowitz (1952) employed the portfolio standard deviation as the ultimate measure of risk. Value at Risk does not abandon this risk measure, but instead supplements it. The parametric approach to VaR calculation explicitly transforms the portfolio standard deviation and expected return into a Value at Risk statistic by multiplying the two by the standard normal deviate corresponding to a desired confidence interval. Full valuation techniques do not use standard deviation as explicitly as the parametric approach; however, selecting distributions for price risk variables in Monte Carlo simulation entails estimating the statistical distribution and parameters for each. Transforming the information in this manor allows risk to be reported in dollar amounts, versus the standard deviation used in portfolio theory. Therefore, the first main advantage of Value at Risk as a risk measure is that it is more logical to managers and those without in-depth statistical training.

Following the lines of logic and intuition, the second advantage presents itself when full valuation VaR procedures are utilized. When a manager or decision maker asks how much risk the firm is exposed to, reporting the standard deviation of the company's portfolio does not truly answer the question. The standard deviation treats any deviation from the mean, either positive or negative, as equivalent. This representation in no way resembles the decision maker's view of risk, which is the possibility of negative returns. Value at Risk reports only the loss that will be exceeded with a certain probability, but does not consider the profits that will be exceeded with the same probability. This methodology results in a risk measure that is consistent with the manager's definition of risk, increasing its intuitive appeal.

The third advantage of Value at Risk over portfolio theory is also realized only with the two full valuation, simulation-based, approaches. Portfolio theory and parametric VaR assume that portfolio returns are normally distributed. While physical assets may perform in this manner, options and option-like instruments introduce nonlinear payoffs. The assumption of normally distributed returns can cause severe distortion when a portfolio contains significant options content. While this detriment of portfolio theory is not overcome with parametric VaR, the full valuation VaR approaches completely allow for any distribution of returns.

Perhaps the most prominent advantage Value at Risk holds above portfolio theory is that VaR encompasses both the risk and expected return of a portfolio in the form of a single summary statistic that is logically appealing. An interesting aspect common to both portfolio theory and Value at Risk is that neither lead the risk manager to a globally optimal portfolio or offer any type of decision rule. Portfolio theory offers a mean-variance efficient frontier indicating the optimal portfolio for each level of risk. Value at Risk, on the other hand, simply offers a statistic useful for comparing risks across time, businesses, or business units (Jorion, 2001).

While Value at Risk is normally thought of as a risk measure independent of the mean-variance framework, or portfolio theory, there have been recent proponents of utilizing the two methods together in portfolio optimization (Alexander and Baptista, 2000; Sentana, 2001; Wang, 2000). Sentana (2001) explains that since both internal and external regulators monitor and limit risk using VaR, Value at Risk can be used to exclude mean-variance efficient positions that result in VaR statistics exceeding those allowed. This approach helps the decision maker choose an efficient portfolio while still honoring the VaR restrictions.

Summary

This chapter began with the foundations of Value at Risk which led into the theoretical model and computation procedures for both general and parametric distributions. Factors affecting the selection of the two quantitative factors were discussed with special attention given to the specific application for which the VaR statistic was to be used. The three main Value at Risk methodologies, parametric, historical simulation, and Monte Carlo simulation, were then compared and contrasted according to five broad characteristics. This evaluation was intended to highlight the advantages and disadvantages of each approach, which weigh heavily in the selection of the appropriate model in a specific situation.

Validation of the model was then defined, back-testing procedures were explained, and the theory of a common method of back-testing was given. A discussion on supplementing VaR statistics with stress testing, using both historical and hypothetical situations, appeared near the end of the chapter. The chapter was concluded by comparing Value at Risk to portfolio theory, or the mean-variance framework, with attention given to applications where VaR and portfolio theory are used in tandem.

The next chapter outlines the specific procedures used to implement the VaR model in this thesis. The data sources and uses, as well as a detailed description of the steps taken in building and simulating this Monte Carlo VaR model, are given.

CHAPTER IV. EMPIRICAL PROCEDURES

This chapter explains the unique aspects of each case study in detail. A step-by-step explanation of the empirical procedures employed in the analysis is given. Data sources and uses are revealed, and the behavior of the data and the relationships apparent among the data series are discussed.

Case Studies

Three hypothetical cases are developed that demonstrate the techniques used in Value at Risk models. Case I is that of a U.S. bread baking company, where the procurement division utilizes VaR independent of the rest of the company to evaluate the risks associated with procurement and hedging strategies. Case II, a U.S. bread baking company, takes an entire business unit perspective and demonstrates the value of considering both input and output price risks simultaneously. Case II also relates VaR to a firm that competes in a consumer goods industry. Case III, a Mexican flour milling company, brings currency exchange risk into the equation for an entity that sells its output in the local currency and purchases its inputs in a foreign currency.

Case I: Procurement Division of U.S Bread Baking Company

VaR computation for the procurement division of a hypothetical U.S. bread baking company producing only white pan bread serves as the baseline scenario for this thesis. In this case, the price risks associated with the procurement of bakery inputs are considered without output price risk. Inputs are divided into two categories. The first, ingredients, consists of flour, sugar, and bakery shortening. Mill feeds are also considered because the price of flour depends heavily on the price of mill feeds. Flour purchase agreements commonly require the pricing of mill feeds as well, which exposes the flour purchaser to mill feed price risk. The energy category is comprised of #2 diesel fuel for use in the truck fleet and natural gas which fuels the bakery ovens. Although there are numerous other inputs in the production of white pan bread, prices of the six inputs considered in this study account for the bulk of the price risks faced by the procurement division of a U.S. bread baker.

Case II: U.S. Bread Baking Company

This case study is as an extension of Case I. Instead of focusing on price risks strictly from the input side of the business, this illustration also considers output price risk. Another dimension of this scenario is that white pan bread, the firm's only output, is a consumer good. This fact is important because consumer goods' price movements differ dramatically from those of the firm's inputs, which are all commodities. This difference can have significant implications that must be considered when implementing hedging strategies.

Agricultural and energy commodities provide some of the best approximations of perfectly competitive markets. Besanko et al. (2000) explain the theory of perfect competition such that an undifferentiated product with many sellers is purchased by numerous buyers with complete information. In this model, no one buyer or seller can set the price, but instead, the market price is established by the interaction of supply and demand.

In a market for a consumer good, such as white pan bread, each firm is able to set its own product price. Instead of infinite elasticity, as found in perfect competition, firms in consumer goods industries face much less elastic demand curves. Although raising or lowering prices results in either fewer or greater sales, respectively, consumer goods corporations make their own conscious pricing decisions. The lower level of elasticity attained in consumer goods markets is achieved by transforming undifferentiated commodities into branded items supported by promotion and advertising campaigns. These practices build brand loyalty and decrease the consumer's sensitivity to product price changes (Johnson, 1982).

One characteristic of consumer goods is the pricing pressures that exist. Consumer goods tend to exhibit infrequent price changes, and when prices are adjusted, they are primarily adjusted upward. The first of the two primary reasons for this reluctance to reduce price is the fear that competitors will view any price decrease as an attempt to steal market share, which will tempt competitors to reduce prices even further in retaliation. This situation could lead to a price war, where the possibility for losses may be much greater than the potential increases in sales resulting from the price decrease. The second reason for not decreasing prices is the reluctance of consumers to accept price increases. Price increases are difficult to establish, and Johnson (1982) explains that firms must work very hard to implement price increases without experiencing a significant decline in sales. For this reason, firms are reluctant to decrease prices, even when warranted by declining production costs. Excess profits are instead used to compensate for things such as lower margins experienced prior to the price increase and decreased sales as a result of the price hike.

Two strategic aspects of the hedging decision are outlined by Hull (2002). In most industries, input and output prices tend to move in tandem, though the degree to which price movements are related varies widely. This relationship is extremely important, as it relates to the level to which output prices adjust to compensate for changes in input prices. As the correlation between input and output prices increases, the demand for hedging instruments decreases. Hull (2002) also explains that deviating from the hedging strategies practiced by the firm's competitors can produce unwanted results. If a firm implements a hedging strategy when competing firms do not hedge, the firm will either develop an advantage or disadvantage. If the firm develops an advantage, either margins will be increased, or prices may be decreased. If the opposite occurs, however, the firm will likely suffer reduced margins or be forced to increase prices. Price increases may not be followed by competitors since, in this situation, their input prices would not warrant the change. In some cases, implementing a hedging policy can actually increase the variability of returns. This result is the exact opposite of that desired, and therefore, the strategic aspects of hedging must be considered.

Case III: Mexican Flour Milling Company

In the third case study, the portfolio of a hypothetical Mexican flour milling company is used to illustrate another situation in which a Value at Risk model is useful. Unlike the prior case, in which multiple inputs were used to produce a single output, this example considers a single input used to produce two outputs, flour and mill feeds. Another interesting aspect of this case is the effect that the foreign currency exchange rate risk plays in the ultimate risk exposure of the hypothetical firm.

The Mexican flour milling company purchases wheat, its only input, in U.S. dollars and sells its outputs of flour and mill feeds for Mexican pesos. Not only do the price risks of the actual input and outputs themselves need to be considered, but that changes in exchange rates can effectively raise or lower the cost of the input, even when the actual input price remains stable.

Model Selection and Empirical Methods

The first step taken in developing a Value at Risk model is that of selecting the methodology. Various aspects of parametric, historical simulation, and Monte Carlo simulation were weighed. The methodology selection decision was influenced by Jorion's (2001) statements that Monte Carlo simulation is "the most comprehensive approach to measuring market risk" (p. 226) as well as "the most powerful method to compute VaR" (p. 225). Although data and computing requirements are the most demanding for Monte Carlo simulation, the flexibility and power of the approach led to the selection of this methodology for the case study analyses.

The second step is that of choosing the confidence interval and time horizon that best fit the situation and goal of this study. Since the VaR statistic will be used in a benchmark application in this study, the most common confidence interval, 95%, was selected. A confidence interval of 95% infers that losses are expected to exceed the Value at Risk statistic in one out of every 20 time periods. This interval allows back-testing of the VaR model to be conducted more frequently, and with greater accuracy, than when higher confidence intervals are used.

The choice of time horizon offers less flexibility due to the frequency of observations with several of the data sets. While many of the data sets used contained daily or weekly data, several series were reported only as monthly averages. This issue restricted the potential time horizon choices to periods of one month or greater, and therefore, a one-month time horizon was selected. Although dictated by the data, the one-month horizon performs well in these situations due to the nature of an agricultural processor's portfolio. Despite the extensive markets for the agricultural and energy inputs, these physical inputs are less liquid than those in the financial sector, where 1- and 10-day VaR calculations are common. An agricultural processor would tend to hold a much more stable portfolio than their financial sector counterparts, mitigating one of the most prominent disadvantages of longer VaR horizons. Although flour and mill feed prices change frequently, bread prices are much more stable.

Analytical Procedures

BestFit™, a software program from Palisade Corporation, was used to estimate the statistical distribution curve that provides the most accurate approximation to each actual, observed data set. BestFit™ is a component of the @Risk™ software package that runs as a Microsoft Excel™ spreadsheet add-in. Since all of the data sets used in this analysis were continuous sample data, BestFit™ estimated the parameters for each of the 21 possible distributions using the maximum likelihood estimators (MLEs). The observed price level data were transformed to price change data before the statistical distributions were fitted or hedge ratios were calculated. As described in Chapter II, price change data for each period are calculated by subtracting the previous period's price from the current price. This method, advocated by Hill and Schneeseis (1981) and Wilson (1983), results in the calculation of distribution parameter estimates for the change in price, without regard to the absolute price level.

Once the parameters are estimated for each relevant distribution, BestFit™ ranks the distributions of continuous sample data in three different ways. The ordinal rankings of distributions vary depending on whether the chi-squared, Anderson-Darling, or Kolmogorov-Smirnov fit statistic is used (Palisade Corporation, 2000). Although no hard rule applies when determining which fit statistic the distributions should be ranked by, particular circumstances favoring each of the fit statistics were considered. Chi-squared is the most commonly used of the three, and in this analysis, the distribution ranked highest by the chi-squared fit statistic is used to represent each data set, unless another distribution ranks higher than the best chi-squared distribution on both the Anderson-Darling and the Kolmogorov-Smirnov scales.

Model Details

The empirical model is constructed assuming a short spot position equivalent to a three-month supply of all inputs and outputs relevant to each particular case study. These firms likely have many other assets and liabilities contributing to the value of the firm. However, only the three-month supply of inputs, the outputs generated from these specific inputs, and derivatives used to hedge this portfolio are considered. Inputs held in inventory, inputs in the production process, and outputs already produced or being produced are not taken into account.

The price risks associated with these positions could then be hedged in a variety of ways. Forward, futures, and options contracts could all be used according to any specific risk-management strategy desired. Throughout this thesis, for simplicity, transaction costs were not considered, and margin requirements were assumed to be zero.

Forward contracts were assumed to be for the exact products and quality specifications necessary for the production processes. For each of the inputs, a futures contract was selected to be used in hedging the risk associated with the price risk variable. Logical inferences were made in selecting a futures contract to hedge exposure to each input price risk variable, and price data for the continuous nearby contract month were used. When two or more different contracts could have been used as hedging tools, the futures contract where price change data were most highly correlated to the price changes of the specific input price variable was selected.

Options were available on each of the commodity futures contracts chosen to hedge the input price variables as well. While numerous strike prices could have been used in the model, nearest-the-money options were chosen for this analysis. Since these options were approximately at-the-money, the options' deltas were approximately 0.5. Therefore, with a 1-unit change in the futures price, the option premium would change by 0.5 units. This relationship is important because in this thesis, options are used as trading instruments, and the traditional payoff to an option at maturity is not the focus. Instead, the change in the value of the option premium over the one-month time horizon is used in the payoff function, which is directly impacted by delta.

Utility-maximizing hedge ratios are calculated for each futures contract used in the analyses. These hedge ratios serve as the basis for the selection of hedging strategies that were evaluated in each case study. Futures prices are assumed to be unbiased, which reduces the speculative component of the utility-maximizing hedge ratios for Case I to zero, making the utility-maximizing position equal to the risk-minimizing position. In Case II, the utility-maximizing hedge ratio is expanded to include the strategic component. The same assumption of zero bias applies to these cases as well, such that the speculative component of the utility-maximizing hedge ratios is reduced to zero. Calculating optimal hedge ratios for Case III is more complex than for the two domestic cases. Since the futures contracts used in the Mexican analysis are denominated in a foreign currency, there are two components of risk associated with futures contract positions. Therefore, Monte Carlo simulation was used to approximate the risk-minimizing hedge ratio for each of the futures contracts used in Case III.

The initial date for Cases I and II is October 1, 2002, while Case III uses an initial date of October 1, 2000. This discrepancy occurs because output price data in Case III were only available through September of 2000. In each scenario analyzed, the initial market value of each spot, forward, futures, and options position was calculated. In Case III, an additional step of converting positions in foreign currencies to their present value in the local currency is undertaken. The sum of these positions represent the initial wealth, W_0, of the procurement division of the bread baking firm being considered.

A correlation matrix was also compiled for each of the three case studies. The relationship between each price risk component and the futures contract used to hedge it is captured, as well as the broad relationships between each of the price risk variables relevant to the particular case study. The significance of each correlation coefficient was then evaluated using a t-test, and those that were not significant at the 5% confidence level were replaced with values of zero. The resulting matrix allowed the pseudo random number generator to emulate the historical relationships among the price risk variables.

While the valuation of spot, forward, and futures positions is quite straightforward, valuing the option contracts is more complex. The Black-Scholes model was developed for valuing European options on non-dividend paying stocks. Although closely related to the Black-Scholes model, Black's model is used to value European options on commodity futures contracts. Though all the options in this thesis are actually American options, Black's model is used here to provide a close estimate of the commodity option values. One currency option is also included in Case III, and the variant of the Black-Scholes option pricing model for currency options is used. The only additional data element collected to include options in this analysis is the U.S. dollar risk-free rate observed in the form of 91-day U.S. Treasury bill rates.

The final step before the simulation can be run is setting up the formulas for the current portfolio value, W_0, and the end-of-period values, W. In most VaR applications, these portfolio values are composed of both revenue and cost components. While true for Cases II and III, the portfolio in Case I represents only cost items and results in negative portfolio values.

In Case I, the initial portfolio value is represented as

$$W_0 = \sum_{I=1}^{6}(Q_I P_{I,0}) + \sum_{C=1}^{5}(Q_C P_{C,0}) + \sum_{P=1}^{5}(Q_P P_{P,0}),$$

and the end-of-period portfolio value is defined as

$$W = \sum_{I=1}^{6}(Q_I \tilde{P}_{I,1}) + \sum_{GI=1}^{6}(Q_{GI}(\tilde{P}_{GI,1} - P_{GI,0})) + \sum_{F=1}^{5}(Q_F(\tilde{P}_{F,1} - P_{F,0}))$$
$$+ \sum_{C=1}^{5}(Q_C \tilde{P}_{C,1}) + \sum_{P=1}^{5}(Q_P P_{P,1}),$$

where Q_X is the quantity of asset X in the portfolio, and $P_{X,T}$ is the price of asset X at time T. In the equation for W_0, $T = 0$ indicates current prices are used. In the equation for W, $T = 1$ specifies that end-of-period prices are used. End-of-period prices are designated with a ~, indicating they are stochastic variables found through simulation, and X is made up of five asset classes, I, GI, F, C, and P. These asset classes are defined as

- $I =$ physical input commodities (1) flour, (2) natural gas, (3) diesel, (4) beet sugar, (5) soybean oil, and (6) mill feeds.
- $GI =$ forward contracts on the physical input commodities (1) flour, (2) natural gas, (3) diesel, (4) beet sugar, (5) soybean oil, and (6) mill feeds.
- $F =$ futures contracts (1) MGE wheat, (2) NYMEX natural gas, (3) NYMEX heating oil, (4) CBOT soybean oil, and (5) CBOT corn.
- $C =$ call options on futures contracts (1) MGE wheat, (2) NYMEX natural gas, (3) NYMEX heating oil, (4) CBOT soybean oil, and (5) CBOT corn.
- $P =$ put options on futures contracts (1) MGE wheat, (2) NYMEX natural gas, (3) NYMEX heating oil, (4) CBOT soybean oil, and (5) CBOT corn.

Long positions in any of the assets result in positive Q_X values. Short positions are represented with negative values. A Q_X value of zero indicates no position in the asset.

In Case II, the portfolio value functions are extended to include the firm's output, or revenue component. The first term, however, is the only addition to the portfolio values in Case I. The portfolio values for Case II are represented as

$$W_0 = (Q_O P_{O,0}) + \sum_{I=1}^{6}(Q_I P_{I,0}) + \sum_{C=1}^{5}(Q_C P_{C,0}) + \sum_{P=1}^{5}(Q_P P_{P,0})$$

and

$$\begin{aligned} W = (Q_O \tilde{P}_{O,1}) + \sum_{I=1}^{6}(Q_I \tilde{P}_{I,1}) + \sum_{GI=1}^{6}(Q_{GI}(\tilde{P}_{GI,1} - P_{GI,0})) \\ + \sum_{F=1}^{5}(Q_F(\tilde{P}_{F,1} - P_{F,0})) + \sum_{C=1}^{5}(Q_C \tilde{P}_{C,1}) + \sum_{P=1}^{5}(Q_P \tilde{P}_{P,1}). \end{aligned}$$

In this case, the notation is equivalent to Case I with the addition of the subscript O, used to represent the firm's output, white pan bread. While $P_{X,T}$ represents the spot price at time T for all inputs and derivatives, it signifies the expected future spot price of the output. In this study, current spot, forward, and expected future spot prices are assumed to be equivalent.

The exposure to foreign currency exchange risk in Case III results in portfolio value functions slightly different than those used in Cases I and II. Here, the initial portfolio value is denominated in Mexican pesos and is represented as

$$W_0 = \sum_{O=1}^{2}(Q_O P_{O,0}) + (Q_I P_{I,0} P_{FX,0}) + \sum_{C=1}^{2}(Q_C P_{C,0} P_{FX,0}) + \sum_{P=1}^{2}(Q_P P_{P,0} P_{FX,0}),$$

and the Mexican peso-denominated end-of-period portfolio value is

$$W = \sum_{O=1}^{2}(Q_O \tilde{P}_{O,1}) + (Q_I \tilde{P}_{I,1} \tilde{P}_{FX,1}) + \sum_{GO=1}^{2}(Q_{GO}(\tilde{P}_{GO,1} - P_{GO,0})) + (Q_{GI}(\tilde{P}_{GI,1} - P_{GI,0})\tilde{P}_{FX,1})$$
$$+ \sum_{F=1}^{2}(Q_F(\tilde{P}_{F,1} - P_{F,0})\tilde{P}_{FX,1}) + \sum_{C=1}^{2}(Q_C \tilde{P}_{C,1} \tilde{P}_{FX,1}) + \sum_{P=1}^{2}(Q_P \tilde{P}_{P,1} \tilde{P}_{FX,1}).$$

The basic notation remains constant for Case III; however, the asset classes making up X have changed. They represent O, S, GO, GI, F, C, P, and FX, where

$O =$ physical outputs (1) flour and (2) mill feeds sold for Mexican pesos.
$I =$ physical input wheat.
$GO =$ forward contracts on the physical outputs (1) flour and (2) mill feeds.
$GI =$ forward contracts on the physical input wheat.
$F =$ futures contracts (1) KCBT wheat and (2) CME Mexican pesos.
$C =$ call options on futures contracts (1) KCBT wheat and (2) CME Mexican pesos.
$P =$ put options on futures contracts (1) KCBT wheat and (2) CME Mexican pesos.
$FX =$ Mexican peso/U.S. dollar foreign currency exchange rate.

It is important to stress that short positions are represented with negative quantities in each of the three cases. The payoff function for all cases is represented as $\prod = W - W_0$, such that the payoff is equal to the one-month change in total portfolio value found by subtracting the end-of-period portfolio value from the initial portfolio value.

For each case, the equation for W_0 does not contain a term for either forward or futures contract valuation. Futures contracts are marked-to-market daily, and therefore, the value of futures contracts are reset to zero at the end of each day. While the value of a forward contract may be non-zero, its value at inception is always zero. All forward contracts used in these cases are initiated at time 0; therefore, the value of the forward contracts at time 0 is zero. The initial portfolio calculation reflects this valuation by omitting the terms for the value of both forward and futures contracts, since these terms would be equal to zero. In calculating end-of-period portfolio values, the price change between time 0 and time 1 observed for each forward and futures contract is included, instead of using only the price of the asset at time 1, as represented for the spot asset and option values in the equation.

Simulation Procedures

The stochastic simulation program @Risk™ is used in this thesis. While the VaR methodology implemented is broadly referred to as Monte Carlo simulation, Latin hypercube is the actual sampling type used. Latin hypercube is a relatively new sampling technique developed to converge on the input distribution in fewer iterations than required when Monte Carlo sampling is employed. This technique reduces the computing time necessary to assure accurate representation of the input distributions (Palisade Corporation, 2000).

Pseudo random values are drawn from the assigned distribution for the price change of each price risk variable while maintaining the historical relationships expressed in the correlation matrix. The value drawn for each variable is added to the current price of the price risk variable to obtain end-of-period price levels. The end-of-period portfolio value, W, is then calculated, and the formula $W - W_0$ represents the absolute one-month change in the value of the firm's portfolio, or profit, for one iteration. This process is repeated 10,000 times for each hedging strategy in each case study analyzed, and the fifth quantile of the distribution of $W - W_0$ is reported as the Value at Risk of the portfolio for that specific scenario.

Empirical Data – Cases I and II: Bread Baking Case Studies

The data used for Cases I and II fall into four basic categories. The first, that of agricultural inputs, contains Minneapolis spring standard patent flour, Midwest beet sugar, and Decatur soybean oil. These price data series were purchased from *Milling and Baking News*, an industry publication, and aggregated into monthly average prices. The prices for flour, sugar, and soybean oil were reported in dollars per hundredweight.

Table 4.1 presents the mean and standard deviation of the absolute, observed prices, and the mean and standard deviation of changes in price for the price risk variables analyzed in Cases I and II. These statistics were calculated after daily and weekly observations were aggregated into monthly average data. The table also describes the time period over which prices were observed and indicates the frequency of the observations.

Energy input prices make up the second category price risk variables. Midwest on-road #2 diesel fuel prices were compiled from the U.S. Energy Information Administration (EIA). The natural gas price series, natural gas sold to industrial U.S. consumers, was also courtesy of the U.S. EIA. Diesel fuel prices were observed in dollars per gallon, and natural gas prices were in dollars per one million British thermal units (mmBtu).

Table 4.1. Characteristics of Observed Date Series for Cases I and II

Financial Variables	Absolute Price[1] Mean	Absolute Price[1] Standard Deviation	Price Change[1] Mean	Price Change[1] Standard Deviation	Start Date	End Date	Observation Frequency
Inputs							
Mpls Spring Standard Patent Flour	10.2958	1.5232	0.0048	0.5570	4-Jan-85	27-Sep-02	Weekly
Midwest Beet Sugar	26.3644	2.4975	-0.0046	0.6729	4-Jan-85	27-Sep-02	Weekly
Decatur, Soybean Oil	21.6101	5.5410	-0.0127	1.4290	6-Apr-82	27-Sep-02	Weekly
Midwest #2 On-Road Diesel Fuel	1.2050	0.1642	0.0031	0.0528	4-Apr-94	23-Sep-02	Weekly
Natural Gas - Industrial	3.3283	0.8310	-0.0022	0.2983	Jan-84	Sep-02	Monthly
Futures							
MGE Hard Red Spring Wheat	3.6562	0.6187	0.0026	0.1899	2-Jan-80	30-Sep-02	Daily
CBOT Soybean Oil	21.8240	4.4940	-0.0146	3.5208	2-Jan-80	30-Sep-02	Daily
CBOT Corn	2.5673	0.5603	-0.0003	0.1584	2-Jan-80	30-Sep-02	Daily
NYMEX Heating Oil	0.6365	0.1709	-0.0001	0.0477	2-Jan-80	30-Sep-02	Daily
NYMEX Henry Hub Natural Gas	2.4331	1.1180	0.0133	0.4328	3-Apr-90	30-Sep-02	Daily
Other							
Mpls, FOB Truck Mill Feeds	64.8188	19.8753	0.1329	9.9500	4-Jan-80	27-Sep-02	Weekly
U.S. 91-Day Treasury Bills (%)	6.5047	3.0149	-0.0394	0.6161	Jan-80	Sep-02	Monthly
Output							
White Pan Bread	0.7135	0.1611	0.0019	0.0095	Jan-80	Sep-02	Monthly

[1]Calculated from monthly averages for each series; units given in text.

The third category of data collected for Cases I and II is composed of futures contract prices, allowing for hedging with the futures and for the valuation of options written on the futures contracts. Minneapolis Grain Exchange (MGE), hard red spring wheat futures contracts were used to hedge Minneapolis spring standard patent flour in the base case. Price data were obtained from the MGE in dollars per bushel.

Since Chicago Board of Trade (CBOT) corn futures had a higher correlation to Midwest beet sugar prices than either of the sugar futures contracts listed on the New York Board of Trade, corn futures were selected to hedge the price risk associated with sugar. Decatur soybean oil prices were hedged with soybean oil futures, and mill feeds were hedged with corn futures, both traded on the CBOT. Corn futures prices were reported in dollars per bushel, and soybean oil futures prices were in dollars per hundredweight. Both price data series were taken from the CBOT.

Number 2 diesel fuel was hedged with New York Mercantile Exchange (NYMEX) heating oil contracts. Data were available in dollars per gallon from the U.S. Energy Information Administration. Price data for NYMEX Henry Hub natural gas contracts, used to hedge natural gas price risk exposure, were in dollars per mmBtu.

The final category of data used for both bread baking examples contains average auction high 91-day U.S. Treasury bill rates collected from the Federal Reserve. The 91-day Treasury bill rate was used only as a parameter in the option pricing models in estimating option values. The Treasury bill rate is not paired with a futures contract for hedging because its use in option valuation has a relatively small impact on the firm's total risk exposure. Minneapolis FOB truck mill feeds price data were purchased from *Milling and Baking News* and reported in terms of dollars per ton. Mill feeds are considered neither an input nor an output, but instead as a component of flour prices. As mentioned earlier, the purchase of flour normally involves the pricing of the associated mill feeds, exposing the bread baking firm to another source of price risk.

White pan bread price is considered in Case II. U.S. city monthly average white pan bread prices were obtained from the U.S. Department of Labor, Bureau of Labor Statistics consumer price index. Data were reported in dollars per one-pound loaf.

Spot Input and Output Positions

The input demands and output quantities assumed for the hypothetical bread baking company in Cases I and II (Table 4.2) are consistent with those of a relatively large bakery. Although in reality quantities of inputs and outputs initially expected are not always those realized, quantity uncertainty is not considered here because VaR assumes that the beginning and end-of-period portfolios are identical.

Table 4.2. Input Quantities for Cases I and II and Output Quantity for Case II

Months	Flour (cwt)	Sugar (cwt)	Bakery Shortening (cwt)	Mill Feeds (tons)	#2 Diesel Fuel (gallons)	Natural Gas (mmBtu)	White Pan Bread (1lb loaves)
1	40,000	3,200	1,100	778	65,000	38,700	6,000,000
3	120,000	9,600	3,300	2,333	195,000	116,100	18,000,000

Minneapolis spring standard patent flour requirements serve as the basis for calculating other input quantities. Sugar and bakery shortening requirements for white pan bread recipes are approximately 8% and 2.75% of the total flour weight, respectively (Faridi, 1995). Mill feeds quantities are calculated based on a 72% wheat milling extraction rate, leaving 28% of each milled bushel of wheat as mill feeds and 72% as flour. Natural gas and #2 diesel fuel quantities are estimates for a large baking plant in the Midwest.

While Value at Risk is based on a one-month time horizon, it is assumed that the procurement division considers itself short a three-month supply of inputs. This assumption is reasonable since consumer goods tend to be repriced no more frequently than monthly, with quarterly changes more common. Therefore, output prices are infrequently adjusted to reflect increased input prices. The output, white pan bread, is only considered in Case II.

Distributions and Correlations

A correlation matrix (Table 4.3) was created which captures the statistically significant relationships between the changes in absolute price for each of the price risk variables. The matrix is used to link the pseudo random variables so that the observed, historical relationships among the price risk variables are considered. Data available for the price risk variables varied with regard to the time periods available, so each correlation could only be measured over the time period when data were available for both variables.

Only those correlations that were significantly different from zero at the 5% confidence level were included in the analysis. Statistically insignificant correlations were left blank in Table 4.3, representing a correlation of zero. The correlation coefficients of particular interest represent the cash inputs and the futures contract used to hedge them. These include flour/MGE wheat, sugar/CBOT corn, soybean oil/CBOT soybean oil, diesel fuel/NYMEX heating oil, natural gas/NYMEX natural gas, and mill feeds/CBOT corn. The flour/MGE wheat and diesel fuel/NYMEX heating oil correlations are both near 0.7, providing the highest levels of hedging effectiveness in this case. The other cash input/futures contract correlation coefficients are much lower, ranging from 0.44 to 0.2.

Table 4.3. Correlation Matrix for All Price Risk Variables in Cases I and II

Financial Variables[1]	MF	BS	SOD	D	NGUS	MW	SO	CC	HO	NG	MLF	TB	B
MF	1.000	0.152	0.239			0.718	0.261	0.471					
BS		1.000				0.184		0.200			0.143		
SOD			1.000			0.152	0.403	0.200			0.169	-0.160	
D				1.000					0.674				
NGU					1.000					0.442	0.177		
MW						1.000	0.332	0.540			0.266		
SO							1.000	0.520			0.182		
CC								1.000			0.307		
HO									1.000	0.229	0.145		
NG										1.000	0.237		
MLF											1.000		
TB												1.000	
B													1.000

[1]MF = Minneapolis spring standard patent flour; BS = Midwest beet sugar; SOD = Decatur, soybean oil; D = Midwest on-road #2 diesel fuel; NGU = Natural gas sold to industrial U.S. consumers; MW = MGE hard red spring wheat futures; SO = CBOT soybean oil futures; CC = CBOT corn futures; HO = NYMEX heating oil futures; NG = NYMEX Henry Hub natural gas futures; MLF = Minneapolis, FOB truck mill feeds; TB =91-day U.S. treasury bills; B = White pan bread.

The distributions used to estimate the one-month changes in price risk variables are given in Table 4.4. These distributions and parameters were calculated according to the procedures described in previous sections. The mean and standard deviation of each distribution, as well as the parameters necessary to describe the location, scale, and shape of the distribution, are also revealed in the table.

Table 4.4. Distributions and Parameters for Price Change Data in Cases I and II

Financial Variables	Distribution	Mean	Standard Deviation	γ^1	α^2	β^3
Inputs						
Mpls Spring Standar Patent Flour	Logistic	0.0158	0.5089		0.0158	0.2806
Midwest Beet Sugar	Log-Logistic	-0.0187	0.6127	-4.9481	14.7280	0.4892
Decatur, Soybean Oil	Normal	-0.0146	3.5208			
Midwest On-Road #2 Diesel Fuel	Log-Logistic	0.0023	0.0472	-1.7166	66.0270	1.7182
Natural Gas - Industrial	Log-Logistic	-0.0086	0.2508	-4.7012	33.9990	0.4686
Futures Contracts						
MGE Hard Red Spring Wheat	Log-Logistic	-0.0031	0.16791	-1.9847	21.5550	1.9746
CBOT Soybean Oil	Log-Logistic	-0.0213	1.3107	-17.9590	24.9020	17.8900
CBOT Corn	Logistic	0.00137	0.140326		0.0014	0.0774
NYMEX Heating Oil	Logistic	-0.0005	0.046229		0.0005	0.0255
NYMEX Henry Hub Natural Gas	Logistic	0.01652	0.35448		0.0165	0.1954
Other						
Mpls, FOB Truck Mill Feeds	Log-Logistic	0.1882	9.9677	-81.0870	14.8670	80.6590
U.S. 91-Day Treasury Bills	Logistic	-0.0197	0.42941		-0.0197	0.2368
Output						
White Pan Bread	Logistic	0.00184	0.0088557		0.0018	0.0049

[1]γ represents the location parameter in log-logistic distributions; [2]α represents the shape parameter in log-logistic distributions and the location parameter in logistic distributions; [3]β represents the scale parameter in logistic and log-logistic distributions.

Empirical Data – Case III: Mexican Flour Milling Company

For Case III, two data sets were collected from a Mexican flour milling firm of similar size to that used in this study. These two sets of price data were for the mill's outputs, flour and mill feeds, and both were reported in Mexican pesos per kilogram. Input price data for hard red winter wheat 11% protein, FOB US Gulf, were available from U.S. Wheat Associates in dollars per bushel. Wheat requirements were hedged with Kansas City Board of Trade (KCBT) hard red winter wheat futures contracts. Price data for this instrument were observed from the KCBT in dollars per bushel.

The Mexican peso/U.S. dollar exchange rate was collected in the form of noon buying rates in New York City for cable transfers in foreign currencies from the Federal Reserve Board of Governors. End of month prices for CME Mexican peso futures, used to hedge the currency exchange risk, were collected from Tradingcharts.com in dollars per peso. The 91-day U.S. Treasury bill rate data were also used in its same format for valuing options.

In the equivalent format found in Table 4.1, Table 4.5 presents the mean and standard deviation of the absolute, observed prices, as well as the mean and standard deviation of changes in price for all of the price risk variables utilized in the Mexican flour milling company case study. These statistics were calculated after daily and weekly observations and were aggregated into monthly average data as well. The table also describes the time period over which prices were observed and indicates the frequency of the observations. The hypothetical Mexican flour milling company in the case study is assumed to represent the average mill size as given in the *2000 Grain & Milling Annual*

Table 4.5. Characteristics of Observed Date Series for Case III

	Absolute Price[1]		Price Change[1]				
Financial Variables	Mean	Standard Deviation	Mean	Standard Deviation	Start Date	End Date	Observation Frequency
Inputs							
HRWW 11% Pro., FOB US Gulf	3.6646	0.6633	-0.0307	0.1888	Sep-96	Sep-00	Monthly
Futures Contracts							
KCBT Hard Red Winter Wheat	3.6084	0.6933	-0.0055	0.1908	1-Jan-80	30-Sep-00	Daily
CME Mexican Peso	9.1091	0.8397	0.0367	0.3290	Sep-96	Sep-00	Monthly
Other							
Peso/U.S. $ Exchange Rate	7.4698	2.1751	0.0757	0.3082	Nov-93	Sep-00	Monthly
U.S. 91-Day Treasury Bills	6.8309	2.9180	-0.0242	0.6427	Jan-00	Sep-00	Monthly
Output							
Flour Sold for Pesos	2.3310	0.2686	-0.0183	0.0759	Sep-96	Sep-00	Monthly
Mill Feeds Sold for Pesos	1.1919	0.1292	-0.0008	0.0760	Sep-96	Sep-00	Monthly

[1]Calculated from monthly averages for each series; units given in text.

published by *Milling and Baking News*. The one-day flour production of this mill is assumed to be 7,500 hundredweight. The firm considers itself short a three-month supply of wheat and requires that 2.3 bushels of wheat be processed to yield one hundredweight of flour. The quantities of inputs utilized and outputs produced by this Mexican flour mill are given in Table 4.6 and are based on the same 72% milling extraction ratio used in the previous cases.

Table 4.6. Input and Output Quantities Used in Case III

Months	Wheat (bushels)	Flour (cwt)	Mill Feeds (tons)
1	523,250	10,340,909	3,992,709
3	1,569,750	31,022,727	11,978,128

Distributions and Correlations

A correlation matrix, given in Table 4.7, was created which captured the relationships between the changes in absolute price for each of the price risk variables described. The matrix was constructed in the same manner as the correlation matrix used in the previous cases. The data limitations described in the previous section apply in this situation as well. Omissions in Table 4.7 again represent correlations not statistically different from zero and are treated as zero values in the analysis. Correlations of particular interest include Gulf wheat/KCBT wheat and Mexican peso/U.S. dollar exchange rate/CME Mexican pesos. The relationship between one of the outputs, flour, and both wheat and the exchange rate is also important.

Table 4.7. Correlation Matrix for Price Risk Variables in Case III

Financial Variables[1]	GW	KCW	CME	EXC	TB	MLFP	FP
GW	1.0000	0.8241					0.3989
KCW		1.0000					0.3185
CME			1.0000	0.6342			
EXC				1.0000			0.3039
TB					1.0000		
MLFP						1.0000	
FP							1.0000

[1]GW = Hard red winter wheat 11% protein, FOB U.S. Gulf; KCW = KCBT hard red winter wheat futures; CME = CME Mexican peso futures; EXC = Mexican peso/U.S. dollar exchange rate: noon buying rates in New York City for cable transfers in foreign currencies; TB = 91-day U.S. treasury bills; MLFP = Mill feeds sold for Mexican pesos; FP = Flour sold for Mexican pesos.

The distributions used to estimate the one-month changes in price risk variables are given in Table 4.8. These distributions and parameters were calculated according to the procedures described in previous sections. The table also reveals the mean and standard deviation of each distribution, as well as the parameters required to describe the location, scale, shape, and lateral shift of the distribution.

Table 4.8. Distributions and Parameters for Price Change Data in Case III

Financial Variables	Distribution	Mean	Standard Deviation	γ^1	α^2	β^3	Shift[4]
Inputs							
HRWW 11% Pro, FOB US Gulf	Logistic	-0.0326	0.1676		-0.0326	0.0924	
Futures Contracts							
KCBT Hard Red Winter Wheat	Log-Logistic	-0.0065	0.18006	-3.1115	31.3406	3.0998	
CME Mexican Peso	Gamma	0.0367	0.3264		10.4820	0.1008	-1.0201
Other							
Peso/U.S. $ Exchange Rate	Log-Logistic	0.0555	0.23758	-0.8093	6.8947	0.8351	
U.S. 91-Day Treasury Bills	Logistic	-0.0042	0.44394		-0.0042	0.2448	
Output							
Flour Sold for Pesos	Logistic	-0.0147	0.070521		-0.0147	0.0389	
Mill Feeds sold for pesos	Logistic	-0.0003	0.07598		-0.0003	0.0419	

[1]γ represents the location parameter in log-logistic distributions; [2]α represents the shape parameter in log-logistic distributions and the location parameter in logistic distributions; [3]β represents the scale parameter in logistic and log-logistic distributions; [4]Shift represents the magnitude to which distribution is shifted laterally.

Summary

This chapter developed the three case studies analyzed in this thesis. Detailed explanations of the actual steps used in the Value at Risk computation, from aspects of the model setup to the simulation techniques, were given. The various data sets used in the study were described, and the sources of the data were shown. The Value at Risk statistics calculated for each hedging and procurement strategy analyzed are given for each of the three case studies in Chapter V. Inferences are made as to the effect that each strategy has on the risk exposure of the hypothetical firms in question, and the usefulness and value of this type of analysis is discussed.

CHAPTER V. RESULTS AND DISCUSSIONS

This chapter reports results of the empirical scenarios that demonstrate Value at Risk (VaR) methodologies for three agribusiness situations. Case I, the procurement division of a U.S. bread baking company, serves as the base case for this thesis. The model is then extended to include the effects of output price risk in Case II, the U.S. bread baking company example. Currency exchange risk is then considered as a further extension in Case III, a Mexican flour milling company.

The chapter begins by reporting the results of Case I. Details about the portfolio and the numerous hedge strategies evaluated for each portfolio are presented. Value at Risk (VaR) statistics are shown, and strategies are ranked according the magnitude of the VaR. A discussion then follows analyzing the reasons for, and implications of, the results. The discussion then moves to stress testing, where several different stress events are presented to show the effects of the scenarios on the current portfolio. A section on variance stressing is also included to show the effects of periods of increased and decreased price variability.

Case II is then presented and analyzed in much the same manner described for Case I. The data from Case I are reused in Case II, with the addition of bread prices. Stress testing and variance stressing are not performed for this case, but a section on the impact that input/output correlation has on a firm’s risk exposure is included instead. The chapter concludes with Case III and a discussion of how foreign exchange risk is dealt with in the Value at Risk model. The portfolio and its components differ significantly in this case, so portfolio details and hedging strategies are introduced. The rest of the analysis, however, follows the same format described for the previous cases.

Case I: Procurement Division of a U.S. Bread Baking Company

Case I consists of the procurement division of a U.S. bread baking company responsible for a portfolio consisting of six commodities, five of which are inputs used in producing white pan bread. The procurement division is assumed to consider itself short a three-month supply of inputs, which are flour, sugar, bakery shortening, #2 diesel fuel, and natural gas. The procurement division also considers itself long mill feeds, since flour purchase agreements typically call for the pricing of the associated mill feeds.

The base case analysis takes place on the 1st of October, 2002, and each position is valued at the average monthly price which prevailed the previous month. Current prices are listed in Table 5.1, and the current value of the cash portfolio representing procurement costs at these prices is $-2,376,547. Portfolio values are most commonly thought of as positive values that include both revenue and cost components. In Case I, however, the portfolio is made up of only procurement cost components, without regard to revenue. Hence, the negative portfolio represents future expenditures, and the risk considered is that input prices will increase, resulting in higher costs of procurement.

In hedging strategies involving forward or futures contracts, the current value of the cash portfolio is equivalent to W_0, the initial portfolio value, because the current values of all futures contracts and forward contracts at inception are zero. When long positions in options are used, the premiums represent an initial outlay of funds that is added to, or subtracted from, the current value of the cash portfolio in the equation for the initial portfolio value, W_0.

This model allows the firm to use several different hedging tools. The firm's cash positions can be offset by positions taken in forward contracts, futures contracts, and options on futures contracts. Although the number of potential strategies that are available to this hypothetical firm to manage its risk exposure is immense, only some of these were analyzed in detail.

Table 5.1. Case I: Current Average Monthly Price as of October 1, 2002

Financial Variables	Spot Position[1]	Current Price	Units	Position Value
Inputs				
Mpls Spring Standar Patent Flour	-120,000	12.61	$/cwt	-$1,513,500
Midwest Beet Sugar	-9,600	26.88	$/cwt	-$258,000
Decatur, Soybean Oil	-3,300	21.00	$/cwt	-$69,300
Midwest On-Road #2 Diesel Fuel	-195,000	1.40	$/gallon	-$273,078
Natural Gas - Industrial	-116,100	3.82	$/mmBtu	-$443,502
Futures Contracts				
MGE Hard Red Spring Wheat		4.88	$/bushel	
CBOT Soybean Oil		20.10	$/cwt	
CBOT Corn		2.67	$/bushel	
NYMEX Heating Oil		0.75	$/gallon	
NYMEX Henry Hub Natural Gas		3.57	$/mmBtu	
Other				
Mpls, FOB Truck Mill Feeds	2,333	77.50	$/ton	$180,833
U.S. 91-Day Treasury Bills		1.63	% points	
Cash Portfolio Value				
Case I: Procurement costs				-$2,376,547

[1]Positive values represent long positions and negative values represent short positions.

Table 5.2 reports the results of the various hedging scenarios analyzed in Case I. The first column simply numbers each strategy for easy reference. The next column reports the one-month Value at Risk statistics at the 95% confidence interval, indicating that, under normal market movements, the firm could expect portfolio losses under the particular strategy to exceed the VaR one out of every 20 months. This method of reporting risk is in contrast to traditional mean-variance analysis, where risk is instead

Table 5.2. Case I: Value at Risk Statistics and Hedging Instrument Positions

				Position Taken in Hedging Instruments[1]									
				Futures					Options				
Hedging Strategy	Strategy	Value at Risk	Rank	MW	NG	HO	SO	CC	MW	NG	HO	SO	CC
No-Hedge	1	$121,771	18										
All Inputs Hedged													
Forward contracts	2	$0	1										
Futures contracts	3	$102,128	9	47	4	4	5	-7					
Options contracts	4	$107,248	11						47	4	4	5	*7
50% futures-50% options	5	$102,494	10	24	2	2	3	-4	23	2	2	2	*3
Flour Only Hedged													
Forward contracts	6	$56,743	3										
Futures contracts	7	$101,842	8	47									
Options contracts	8	$110,823	15						47				
Flour & Natural Gas Hedged													
Forward contracts	9	$40,633	2										
Futures contracts	10	$101,365	6	47	4								
Options contracts	11	$108,632	13						47	4			
All Non-Flour Inputs Hedged													
Forward contracts	12	$101,081	4										
Futures contracts	13	$121,589	17		4	4	5	-7					
Options contracts	14	$117,660	16							4	4	5	*7
Regression - Total Portfolio													
MGE wheat futures	15	$101,770	7	48									
MGE wheat options	16	$110,623	14						48				
MGE wheat & nat gas futures	17	$101,164	5	49	4								
MGE wheat & nat gas options	18	$108,519	12						49	4			

[1] MW = MGE hard red spring wheat futures; NG = NYMEX Henry Hub natural gas futures; HO = NYMEX heating oil futures; SO = CBOT soybean oil futures; CC = CBOT corn futures.

*Denotes position in put options; all other options position are in call options.

reported in terms of portfolio standard deviation and is paired with the portfolio's expected return.

The third column ranks the VaR statistics in order of magnitude, with the lowest VaR receiving a rank of one. However, a ranking of one does not necessarily indicate an optimal portfolio. Each of the strategies has unique advantages and disadvantages not accounted for in this analysis. Instead, a ranking of one indicates the portfolio exhibits the lowest level of risk exposure as measured by VaR, which is only one factor in the portfolio selection process that managers must consider. Table 5.2 also has a section called "position taken in hedging instruments." In the first two tables, this section lists the number of futures and options contracts entered. Positions in forward contracts are not given since positions equal and opposite that of the cash portfolio are relatively straightforward.

The first strategy listed in Table 5.2 is the no hedge, or control portfolio (1). When the VaR of the cash portfolio is calculated without any hedging strategy, the portfolio returns the largest Value at Risk of all the strategies in Case I. The first group of hedging strategies (2-5) examined includes 100% hedges, based on technical relationships, for each input price variable. Forward contracting all input requirements (2) returns a VaR of zero, since all prices have been fixed. Implementing the risk-minimizing hedge for each input in futures (3), options (4), and 50% futures-50% options (5) return similar Value at Risk statistics. The important relationship to note is that the hedging strategy utilizing only futures contracts returns a lower VaR than either strategy containing options, and the strategy utilizing only options has the largest VaR of the three.

This relationship, where futures contract strategies tend to yield lower VaR statistics than options strategies, is caused by several factors. First, options are not held to maturity, so the standard payoff to an option at maturity does not accurately represent the change in an option premium value over the one-month time horizon for which VaR is calculated. Options also experience time decay in that options lose a portion of their extrinsic value as maturity nears. This concept means that even if futures prices remain constant over the next time period, the value of the options will decrease. The rate of time decay increases as maturity nears, and the use of options expiring in three months results in a more significant rate of decay than that which would have been realized had options with a longer time to maturity been used.

The third factor causing options hedging strategies to return higher VaR statistics than futures strategies has to do with the concept of delta. An option's delta refers to the ratio of the change in price of an option to the change in price of the underlying futures contract. The at-the-money options used in this analysis have deltas approximately equal to 0.5, indicating that for every 1-unit change in the futures price, the option value will change by 0.5 units. Therefore, if cash and futures prices were perfectly correlated, and cash prices moved 1 unit against a firm's position, a futures strategy would exactly offset the incurred losses. The equivalent options strategy would only move 0.5 units and would not provide as much hedging effectiveness as a futures hedge. Using deep in-the-money or out-of-the-money options with drastically different values for delta may have a significant affect on the VaR of options strategies; however, this aspect was not explored.

The fourth reason for this relationship between futures and options strategies has to do with the variability of prices used in the analyses. In times of high prices volatility, these three characteristics of options are more than offset by the benefits of an option's truncated payoffs. In the base case scenarios, however, volatilities are low enough that even the largest simulated losses do not move prices to a great enough extent that the truncated payoffs of options are realized. This observation becomes more evident in the following section on variance stressing, where the effects of increases in the variances of price changes is demonstrated. In the scenarios where variances are quadrupled, options strategies actually overtake futures strategies in every group of strategies. As long as volatilities of prices are low, futures contracts are more efficient hedging instruments, returning lower VaR statistics. However, as price volatility increases, futures contracts lose their risk reduction advantage over options contracts.

The impact of the magnitude of volatility illustrates the profound influence that the length of historical data and statistical distribution choices can have on the VaR results. While the flexibility of distribution and parameter selection in Monte Carlo simulation allows the user to choose any distribution and parameters that he feels adequately represents the future price movement possibilities, this freedom also allows the user to make poor choices that inaccurately estimate future movements. This concept is referred to as model risk and could affect the ranking of strategies in all three cases.

In the second group (6-8), the strategies focus on hedging only the flour portion of the portfolio with different combinations of instruments. Forward contracting the flour requirements (6) results in the third lowest VaR for this case study. This significant reduction in VaR occurs because flour is the most prominent component of the procurement division's portfolio, making up over half of the portfolio value. The same relationship exists within this group, as forward contracts (6) return the lowest VaR, followed by the futures strategy (7). When hedging only flour requirements, an options hedge (8) again provides the highest VaR, or the least hedging effectiveness.

Since natural gas was the second most prominent input in terms of absolute value of the requirements, a group of hedging strategies was examined that considered hedging only flour and natural gas (9-11). These strategies provided some of the lowest Value at Risk statistics; however, the same pattern held, in that forward contracting (9) reduced risk the most, and the options strategy (11) provided the least effective hedge.

The hedging strategies involving all inputs except flour (12-14) provide an interesting illustration of the effects of correlation between each cash input variable and the instrument used to hedge the associated price risk. In this group of strategies, the VaR ranking of the strategies does not follow the same pattern as observed in the other groups. Here, forward contracting (12) results in the lowest VaR, but the highest VaR in the group is returned for the futures hedge (13).

This relationship can be explained by observing the correlations in Table 4.3 in the previous chapter. The correlation between Minneapolis flour and MGE wheat futures of 0.718 is the highest correlation coefficient observed between a cash input and its associated futures contract. Hedging strategies where MGE wheat futures are used return lower VaR statistics than the options strategies. While the correlation between heating oil futures and #2 diesel fuel is only slightly lower at 0.674, the correlations between cash beet sugar, Decatur, soy oil, natural gas, mill feeds, and the futures contracts used to hedge each of these price risk variables, respectively, range from 0.442 to 0.200. As shown in Table 5.2, when the flour component is not hedged, the implication of these lower correlations on the VaR statistics becomes much more prominent. When correlations between cash and futures positions are high, futures hedging strategies return lower VaR statistics than options strategies. As correlations decline between cash and futures, hedging effectiveness decreases to the point where, as observed in the all non-flour inputs hedge (12-14), the truncated payoffs offered by an options position (14) return a lower Value at Risk statistic, and provide greater hedging effectiveness than futures contracts (13).

The final group of hedging strategies (15-18) contains futures and options positions calculated differently than those in the previous strategies. The first step was to value the current portfolio, had it been held at each historical monthly time period, and observe the total change in portfolio value for each period. The change in price of individual futures contracts, as well as different combinations of multiple futures contracts, was regressed against the total change in portfolio value. All possible contract combinations were evaluated, and those that were significant at the 5% confidence level were considered. This method was to used calculate the minimum-variance hedge ratio for the entire portfolio, instead of calculating the ratio for each input individually. By calculating the minimum-variance hedge ratios for the entire portfolio, the benefits of diversification that naturally occur in multiple asset portfolios were taken into account.

Although the hedge ratios changed only slightly from those used in strategies 7-8 and 10-11 where the same combinations of hedging instruments were used, the strategies in this group (15-18) yielded lower VaR statistics. The level of risk reduction observed in the VaR statistics shows that this method of hedge ratio calculation provides superior hedging effectiveness compared to calculating the ratio for each factor independently. As described earlier, the use of futures (15, 17) results in lower VaR statistics than the equivalent options strategies (16, 18).

When evaluating the scenarios analyzed for Case I, the four lowest Value at Risk statistics were observed for the strategies utilizing forward contracts. With forward contracts, the firm eliminates both futures risk and basis risk, which provides 100% reduction of price risk for the inputs hedged in this manner. VaR does not lead the user to an optimal portfolio, however. This application of VaR addresses only price risk, and since managers must consider numerous other sources of risk, as well expected return, VaR is not sufficient for portfolio selection. For instance, forward contracts are typically illiquid, and lifting the hedge, if desired, would be difficult. The firm might also be uncertain of the exact quantity of inputs needed, and if inputs were forward contracted, the firm would have much less flexibility. A forward contract also specifies a supplier, which prohibits the firm from changing suppliers before the actual input purchase is made. These are just a few examples of why VaR does not lead to an optimal portfolio, but instead assesses the price risk associated with holding a portfolio. Even though forward contracting returns the lowest VaR statistics, decision makers may choose a strategy using other hedging instruments.

The two futures strategies (15, 17) where hedge ratios were found through regression with the change in total portfolio value and the futures strategies hedging only flour (7) and flour and natural gas (10) were all similar. Aside from the forward contracting strategies, these four futures contract strategies provide the highest level of hedging effectiveness. With only one exception, hedging strategies utilizing options consistently offered the least hedging protection; however, all strategies resulted in VaR statistics at least marginally lower than the VaR observed for the unhedged position (1).

Thus far, every hedging strategy examined calls for a 100% hedge ratio to be used. Figure 5.1 illustrates the effect that scaling this hedge ratio from 0-100%, in increments of 10%, would have on the Value at Risk statistic when hedging only flour requirements. The figure shows that any level of VaR between $121,771 and $101,081 for the case of futures, or $110,823 for the case of options, can be achieved by varying the size of the hedge position. The unevenness found in both the futures and options series is due to the indivisibility of futures and options contracts. For example, a 20% hedge calls for exactly 9.37 contracts; however, futures and options contracts are only available in integer units and had to be rounded.

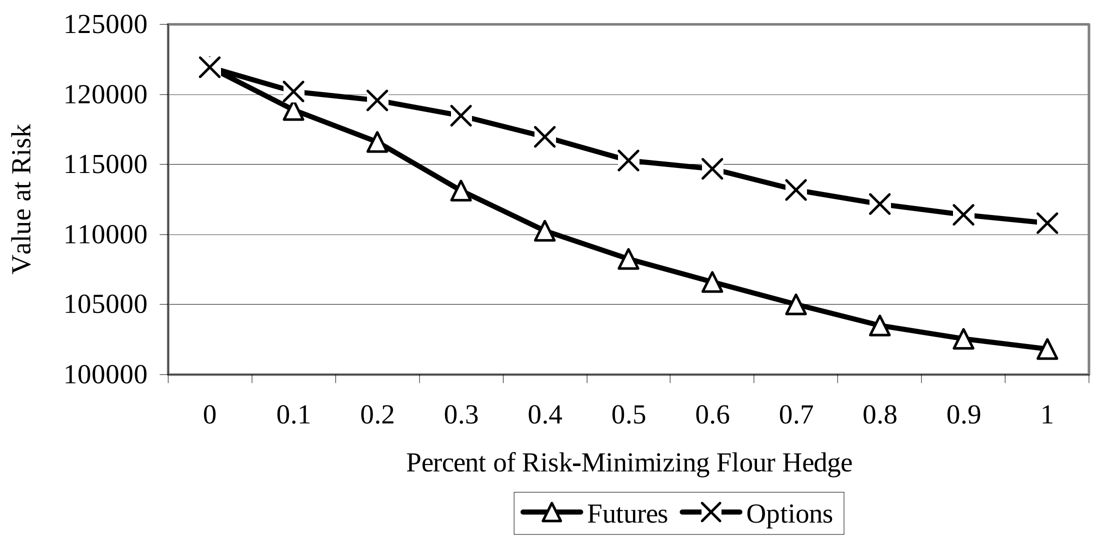

Figure 5.1. Case I: Value at Risk Statistics for Varying Percentages of the Risk-Minimizing Hedge Ratio for Strategies 7 and 8.

The distribution of changes in portfolio values for strategy 7 in Case I is shown in Figure 5.2. This figure is a histogram reporting the number of occurrences, out of 10,000, found in each histogram bin when values for change in portfolio value are divided into 25 bins with a range of $25,000 each. This example illustrates that the focus of Value at Risk is on the far left-hand tail of the distribution. The 95% confidence interval implies that, when strategy 7 is employed, one out of every twenty periods will experience losses greater than $101,842.

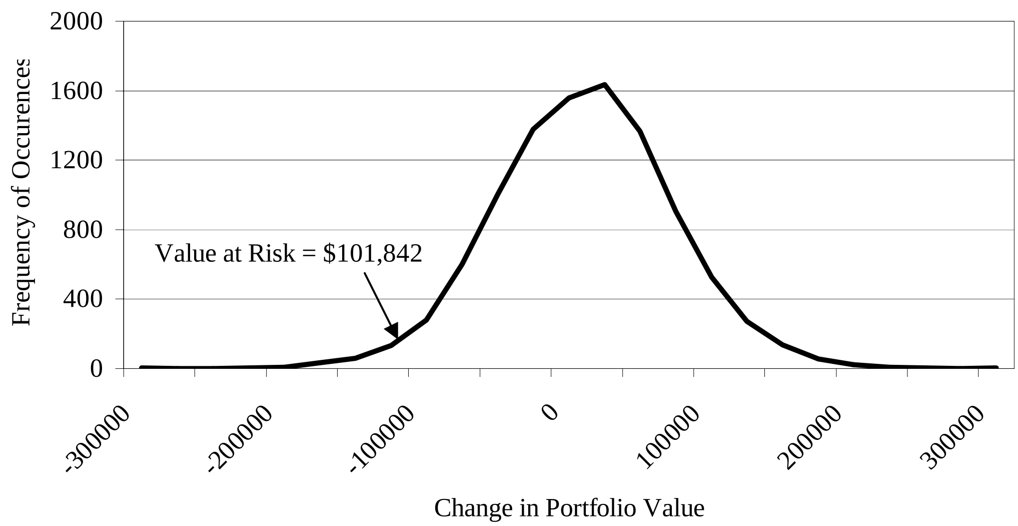

Figure 5.2. Case I: Distribution of 1-Month Changes in Portfolio Value When Hedging the Flour Position with Futures Contracts in Strategy 7.

Confidence Interval

The 95% confidence interval (C.I.) was used for VaR statistics in Cases I, II, and III. This section shows the impact of C.I. choice for Case I strategies by calculating VaR at the 90%, 95%, and 99% C.I. The most obvious observation is that the absolute magnitude of VaR increases as the confidence interval increases for every strategy. The primary relationship found throughout this thesis is that forward contracts yield the lowest VaR, options yield the highest VaR, and futures strategies rank between them, except when cash/futures correlations are low or price variability is high. This relationship holds for each of the confidence intervals shown in Table 5.3.

Table 5.3. Case I: Value at Risk Statistics at Different Confidence Intervals

Hedging Strategy	Strategy	90% C.I. Value at Risk	Rank	95% C.I. Value at Risk	Rank	99% C.I. Value at Risk	Rank
No-Hedge	1	$92,492	17	$121,771	18	$181,037	17
All Inputs Hedged							
Forward contracts	2	$0	1	$0	1	$0	1
Futures contracts	3	$79,777	10	$102,128	9	$147,716	6
Options contracts	4	$82,644	11	$107,248	11	$157,432	11
50% futures-50% options	5	$79,746	9	$102,494	10	$150,974	9
Flour Only Hedged							
Forward contracts	6	$42,999	3	$56,743	3	$82,641	3
Futures contracts	7	$79,446	7	$101,842	8	$148,980	8
Options contracts	8	$85,543	15	$110,823	15	$160,306	15
Flour & Natural Gas Hedged							
Forward contracts	9	$31,555	2	$40,633	2	$56,904	2
Futures contracts	10	$78,413	5	$101,365	6	$145,438	4
Options contracts	11	$83,005	13	$108,632	13	$158,427	13
All Non-Flour Inputs Hedged							
Forward contracts	12	$75,892	4	$101,081	4	$156,861	10
Futures contracts	13	$93,108	18	$121,589	17	$183,874	18
Options contracts	14	$89,272	16	$117,660	16	$179,210	16
Regression - Total Portfolio							
MGE wheat futures	15	$79,601	8	$101,770	7	$148,888	7
MGE wheat options	16	$85,295	14	$110,623	14	$159,851	14
MGE wheat & nat gas futures	17	$78,546	6	$101,164	5	$145,982	5
MGE wheat & nat gas options	18	$82,939	12	$108,519	12	$157,696	12

One interesting result of calculating VaR at these different confidence intervals is that at both that 90% and 99% C.I., the unhedged strategy (1) does not return the highest VaR. The highest VaR is instead found for strategy 13 for both confidence intervals. While the ordinal rankings of strategies at the 90% and 95% confidence intervals are very similar, the rankings for the 99% C.I. show some interesting differences. First, the rank of forward contracting all non-flour inputs (12) falls, while the rank of hedging flour and natural gas with futures contracts (10) increases. The largest increase in rank was observed for hedging all inputs with futures contracts (3).

While the ordinal rankings varied depending on which C.I. was used, most discrepancies were relatively minor and resulted from the distributions of changes in portfolio value observed for each individual hedging strategy. The important aspect to focus on is that the same general conclusions would be drawn no matter which C.I. was used when results were presented to the firm's management. The decision makers would see that forward contracting provides the greatest risk reduction by far and that futures contracts where flour is hedged provide the next best series of strategies. They would also see that hedges not including flour provide relatively low levels of risk reduction.

Stress Testing

Stress testing procedures were performed for Case I. The purpose is to evaluate the effects of unlikely economic events. Scenarios commonly evaluated in stress testing programs fit into two basic categories. They consist of economic events that have occurred in the past and events that are believed to be possible in the future. In order to demonstrate these two types of stress testing, four events (Table 5.4) were considered. The first two represent events that have not occurred in the past but are still reasonable. For the event titled maximum observed losses, the portfolio was valued assuming that the largest one-month price increase, observed for each variable in the available historical data, was realized in the coming period. The second event assumes that all prices experience a four standard deviation price increase in the next period.

The last two scenarios are used to demonstrate the losses that the current portfolio would sustain for each of the hedging strategies if historical economic events were experienced. The two historical periods selected were September, 2002, and May, 1996. September, 2002, was the largest one-month loss that the current, unhedged portfolio would have sustained since April of 1994. Situations with price movements similar to those in this period were observed several times and resulted in four of the largest losses. Although May, 1996, would have only resulted in the fifth largest loss to the current unhedged portfolio, it was examined due to the unique price movements observed. In this event, all three agriculture input prices increased, fuel prices decreased, and mill feed prices dropped dramatically, despite increases in other agricultural commodity prices.

Table 5.4. Case I: 1-Month Price Movements for Each Stress Event

Financial Variables	Maximum Observed Increases	4 Standard Deviation Increases	Sep-02	May-96
Inputs				
Mpls Spring Standar Patent Flour	$1.93	$2.23	$1.91	$1.36
Midwest Beet Sugar	$2.90	$2.69	$0.98	$0.00
Decatur, Soybean Oil	$36.68	$14.08	-$0.10	$0.71
Midwest On-Road #2 Diesel Fuel	$0.17	$0.21	$0.09	-$0.01
Natural Gas - Industrial	$1.98	$1.19	$0.12	-$0.28
Futures Contracts				
MGE Hard Red Spring Wheat	$0.93	$0.76	$0.88	$0.40
CBOT Soybean Oil	$8.67	$5.72	-$0.47	$0.81
CBOT Corn	$0.82	$0.63	$0.07	$0.36
NYMEX Heating Oil	$0.23	$0.19	$0.07	-$0.05
NYMEX Henry Hub Natural Gas	$2.91	$1.73	$0.48	-$0.03
Other				
Mpls, FOB Truck Mill Feeds	$35.50	$39.80	$9.70	-$24.05
U.S. 91-Day Treasury Bills	$2.71	$2.48	$0.00	$0.03

The results of these four economic stress events are shown in Table 5.5, where the strategy, Value at Risk, and rank columns are in the same format as Table 5.2. Each portfolio value in the columns representing stress events was calculated analytically,[1] assuming the price changes given in Table 5.4, instead of through simulation used to calculate VaR. When all the hedging strategies are observed, the maximum observed increases scenario results in the largest losses, followed by the four standard deviation increase scenario. The losses realized under the two historical events are, on average, much smaller and vary dramatically depending on the hedging strategy in question.

The results of these stress events allow the user of this information to draw some important conclusions. The first of these is that the strategies involving forward contracting of the flour requirements provide some of the lowest losses for the stress events considered, with the exception being the case when the maximum observed losses are realized. Another point of interest is that, although forward contracting all non-flour inputs ranks fourth in terms of VaR, the losses realized by that portfolio under the four stress events make the strategy much less appealing than if Value at Risk had been utilized alone. Finally, strategies using the hedge ratios found through regression for both MGE wheat and NYMEX natural gas futures contracts consistently rank in the top five, whether evaluated using Value at Risk or the four stress events.

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[1] Stress testing is done analytically since the portfolio is valued at a given set of prices. Simulation would result in numerous equivalent portfolio values because no stochastic variables are used. This is in contrast to VaR, where thousands of portfolio values are calculated, ordered, and the fifth percentile is chosen.

Table 5.5. Case I: Portfolio Losses Realized Under Select Stress Events

Hedging Strategy	Strategy	Value at Risk	Rank	Maximum Observed Increases	Rank	4 Standard Deviation Increases	Rank	Sep-02	Rank	May-96	Rank
No-Hedge	1	$121,771	18	$560,867	18	$426,529	18	$246,920	18	$187,068	16
All Inputs Hedged											
Forward contracts	2	$0	1	$0	1	$0	1	$0	1	$0	1
Futures contracts	3	$102,128	9	$191,326	3	$151,779	3	$13,558	4	-$112,997	8
Options contracts	4	$107,248	11	$230,121	7	$194,685	8	$62,089	10	$136,620	14
50% futures 50% options	5	$102,494	10	$211,747	4	$173,826	6	$37,548	8	$124,985	9
Flour Only Hedged											
Forward contracts	6	$56,743	3	$329,867	11	$159,159	4	$17,420	5	$24,468	2
Futures contracts	7	$101,842	8	$343,433	13	$248,016	12	$40,219	9	$92,833	6
Options contracts	8	$110,823	15	$386,020	15	$289,798	15	$81,741	14	$128,457	12
Flour & Natural Gas Hedged											
Forward contracts	9	$40,633	2	$99,989	2	$20,648	2	$3,488	2	$56,976	3
Futures contracts	10	$101,365	6	$227,009	6	$178,773	7	$21,110	6	$93,866	7
Options contracts	11	$108,632	13	$284,100	10	$233,917	10	$69,576	12	$128,996	13
All Non-Flour Inputs Hedged											
Forward contracts	12	$101,081	4	$231,000	8	$267,370	13	$229,500	17	$162,600	15
Futures contracts	13	$121,589	17	$408,759	17	$330,292	16	$220,259	15	$207,232	18
Options contracts	14	$117,660	16	$404,968	16	$331,415	17	$227,268	16	$195,232	17
Regression - Total Portfolio											
MGE wheat futures	15	$101,770	7	$338,807	12	$244,218	11	$35,821	7	$90,828	5
MGE wheat options	16	$110,623	14	$382,300	14	$286,889	14	$78,227	13	$127,210	11
MGE wheat & nat gas fut	17	$101,164	5	$217,757	5	$171,177	5	$12,315	3	$89,856	4
MGE wheat & nat gas opt	18	$108,519	12	$276,660	9	$228,099	9	$62,547	11	$126,502	10

Variance Stressing

When BestFit™ was used to estimate the distribution parameters that best approximated the historical distribution of prices for each price risk variable, the entire sample set was included. This method of distribution estimation aggregates periods of low, medium, and high price variability into one distribution, ignoring the fact that variability of prices fluctuates considerably over time. For this reason, the conventional methods of stress testing, described above, were supplemented by analyzing the Value at Risk of the current portfolio in periods of both increased and decreased price variability to illustrate the importance of accurately portraying the price variability likely to occur in the next time period.

Case I was used to illustrate how the portfolio VaR would act under three different situations. The three situations include periods when the variability of changes in price of all price risk variables is decreased by half, doubled, and quadrupled. Transforming the variance of the variables with normal and logistic distributions was accomplished by modifying the beta parameter in the @Risk™ distribution function according to the variance function. Due to the complexity of the function for the variance of a log-logistic distribution, trial and error methods were used to approximate the alpha parameter in the @Risk™ functions which corresponded to the desired magnitude of the distribution's variance.

The results of this analysis are in Table 5.6, where the first, third, and fourth Value at Risk columns represent the cases of altered variances, and the second column is the VaR calculated in the base scenario. The most readily apparent conclusion drawn

Table 5.6. Case I: Value at Risk Statistics Under Periods of Increased and Decreased Price Variability

Hedging Strategy	Strategy	Value at Risk Variance / 2	Rank	Value at Risk Obs[1] Variance	Rank	Value at Risk Variance * 2	Rank	Value at Risk Variance * 4	Rank
No-Hedge	1	$67,542	18	$121,771	18	$174,337	17	$209,175	18
All Inputs Hedged									
Forward contracts	2	$0	1	$0	1	$0	1	$0	1
Futures contracts	3	$62,725	10	$102,128	9	$147,689	10	$179,090	10
Options contracts	4	$64,405	12	$107,248	11	$150,259	11	$174,910	6
50% futures 50% options	5	$61,959	6	$102,494	10	$147,112	9	$174,460	5
Flour Only Hedged									
Forward contracts	6	$39,724	3	$56,743	3	$85,648	3	$129,804	3
Futures contracts	7	$62,308	8	$101,842	8	$147,050	8	$183,571	14
Options contracts	8	$65,987	15	$110,823	15	$154,979	15	$183,165	13
Flour & Natural Gas Hedged									
Forward contracts	9	$28,113	2	$40,633	2	$60,545	2	$97,775	2
Futures contracts	10	$61,402	5	$101,365	6	$146,453	6	$179,725	9
Options contracts	11	$64,276	11	$108,632	13	$151,709	13	$179,662	7
All Non-Flour Inputs Hedged									
Forward contracts	12	$51,486	4	$101,081	4	$142,162	4	$142,176	4
Futures contracts	13	$66,999	17	$121,589	17	$177,030	18	$209,247	17
Options contracts	14	$65,884	14	$117,660	16	$169,077	16	$201,186	16
Regression - Total Portfolio									
MGE wheat futures	15	$62,623	9	$101,770	7	$146,992	7	$184,436	15
MGE wheat options	16	$66,188	16	$110,623	14	$154,579	14	$182,780	12
MGE wheat & nat gas futures	17	$62,015	7	$101,164	5	$145,953	5	$179,795	11
MGE wheat & nat gas options	18	$64,722	13	$108,519	12	$151,026	12	$179,008	8

[1] Obs Variance indicates that the observed, historical variance was used for each variable in calculating the VaR statistic.

from Table 5.6 is that, as expected, the magnitude of the Value at Risk statistic increases as the variance of the relevant price change variables increases. It is also interesting that the group of hedging scenarios involving all non-flour inputs (12-14) is the only group in which the ordering of forward, futures, and options strategies hold across all four variance levels. Forward contracts (12) return the lowest VaR, followed by options (14), and finally futures (13).

For every other group of strategies, at least one altered variance scenario does not conform to the original observed relationships. For instance, the 50% futures 50% options strategy (5) for hedging all inputs returns a VaR between the futures (3) and options strategies (4) under the observed, historical variance scenario. However, as the variances are either increased or decreased, the VaR of the 50% futures 50% options strategy (5) declines below the futures strategy (3). The relationship observed between forward, futures, and options strategies breaks down again in the case of hedging flour (6-8), flour and natural gas (9-11), and the regression strategies (15-18) as well. In these groups, futures contract strategies return the least desirable VaR statistics only when variances are quadrupled. This result, described earlier in this chapter, demonstrates that as the variability of price changes increases, so does the hedging effectiveness of options relative to futures.

Case II: U.S. Bread Baking Company

The U.S. bread baking company case is an extension of Case I, the procurement division example. All input quantities and prices are the same between the two, with the only difference being the addition of white pan bread price risk in Case II. While Case I considers only input price risk, Case II considers both input and output price risk simultaneously.

The date selected for analysis is again October 1, 2002. The current prices and cash portfolio positions listed in Table 5.1 are supplemented by adding the position in the output, white pan bread. The value of the current long position in 18,000,000 1-pound loaves of white pan bread, at $1.02 per loaf, is then calculated to be $18,288,000. The current cash portfolio value, $15,911,453, is found by summing the value of the long bread position and the value of the procurement division's cash portfolio. Therefore, inclusion of the output, or revenue portion of the firm's budget, leads to the traditional positive portfolio value and allows changes in input prices and output prices to be considered simultaneously.

While the utility-maximizing hedge ratio was expanded in this scenario to include the strategic component explained in Chapter II, this component reduces to zero because observed correlations between bread and all other prices were not statistically different than zero. Therefore, the utility-maximizing hedge ratio and the risk-minimizing hedge ratio are equivalent, and hedging strategies one through fourteen were equivalent for Cases I and II. The Value at Risk statistics for Case II, as listed in Table 5.7, reveal that the observed relationships between the futures and options strategies are consistent with those found in Case I.

The forward contracting of inputs resulted in the lowest VaR levels, followed by futures contracts, and finally, options strategies. This relationship held when all inputs were hedged (2-5), only flour was hedged (6-8), and when flour and natural gas were

Table 5.7. Case II: Value at Risk Statistics and Hedging Instrument Positions

				Position Taken in Hedging Instruments[1]									
				Futures					Options				
Hedging Strategy	Strategy	Value at Risk	Rank	MW	NG	HO	SO	CC	MW	NG	HO	SO	CC
No-Hedge	1	$257,411	16										
All Inputs Hedged													
Forward contracts	2	$225,723	1										
Futures contracts	3	$248,615	8	47	4	4	5	-7					
Options contracts	4	$251,599	11						47	4	4	5	*7
50% futures 50% options	5	$250,182	9	24	2	2	3	-4	23	2	2	2	*3
Flour Only Hedged													
Forward contracts	6	$232,113	3										
Futures contracts	7	$248,567	7	47									
Options contracts	8	$255,154	14						47				
Flour & Natural Gas Hedged													
Forward contracts	9	$227,681	2										
Futures contracts	10	$247,458	6	47	4								
Options contracts	11	$252,564	12						47	4			
All Non-Flour Inputs Hedged													
Forward contracts	12	$247,201	4										
Futures contracts	13	$253,573	13		4	4	5	-7					
Options contracts	14	$251,454	10							4	4	5	*7
Regression - Total Portfolio													
MGE wheat futures	15	$247,360	5	52									
MGE wheat options	16	$255,628	15						52				

[1]MW = MGE hard red spring wheat futures; NG = NYMEX Henry Hub natural gas futures; HO = NYMEX heating oil futures; SO = CBOT soybean oil futures; CC = CBOT corn futures.
*Denotes put contracts; all other options position are in call options.

hedged (9-11). As with Case I, this relationship broke down when all non-flour inputs were hedged (12-14), as the strategy-utilizing options contracts returned a lower VaR statistic. This occurrence is due to the low correlations observed between the physical non-flour inputs and the instruments used to hedge them. The lowest VaR found for a hedging strategy utilizing futures contracts was again found by regression. This strategy calls for a slight increase in the number of wheat futures when compared to the utility-maximizing futures position.

The inclusion of output price risk in this VaR model results in observed VaR statistics more than double those found for the procurement division, which is due to the fact that even small changes in the price of the $18,288,000 bread portfolio have large impacts on the portfolio value. The effects of the hedging strategies are also muffled, since the price of bread exhibits no significant correlation to any of the cash or futures variables considered. Therefore, hedging strategies targeting input price risks have less of an effect because they are dominated by output prices. It is also interesting to note that VaR could be reduced from $121,771 to zero in Case I; however, the largest VaR reduction observed for Case II is only $31,660. All hedging strategies examined did result in VaR statistics lower than those observed for the unhedged portfolio, indicating at least minimal risk reduction. All hedging strategies in this section focused on reducing input price risk. While it may be possible for this firm to forward contract bread sales, this would result in output price risk falling to zero. In this case, all price risk comes from the inputs, and any strategy involving output forward contracting would result in a VaR equal to the equivalent Case I strategy, making this type of analysis redundant.

Input/Output Correlation Effects

In the case of the U.S. bread baking company, no significant contemporaneous correlation was found between any of the firm's inputs and the output. While this relationship is not uncommon for a firm dealing in a consumer goods industry, firms producing intermediary goods tend to observe at least some level of correlation between inputs and outputs. When input/output correlations exist, they may impact the effectiveness of hedging strategies, and not accounting for these relationships can result in hedging strategies that actually increase the Value at Risk of a portfolio. This result is extremely undesirable, and the following illustration outlines how a firm with correlated inputs and outputs could account for this relationship to achieve the desired result of hedging.

Before proceeding with this analysis, it is important to emphasize that significant positive correlations were not observed between bread and flour or bread and MGE wheat futures at any point over the time period examined. The imposed correlations between these factors are hypothetical, and the analysis is included only to illustrate how firms in industries where input/output correlations are present can account for them.

In order to demonstrate the correlation effects, the Value at Risk statistics for only the flour and bread components of Case II are evaluated when unhedged and under three hedging strategies. The first strategy involves forward contracting the flour requirements; however, the output, bread, was not forward contracted. The other strategies hedged the flour exposure, and in some instances the bread exposure, with wheat futures and options contracts. The first assumption made was that the correlation between flour and bread would be equivalent to the correlation between wheat futures and bread when the correlations were varied between 0 and 0.6. This assumption was made solely to maintain the integrity of the correlation matrix given in Table 4.3 of the previous chapter, since varying either correlation individually resulted in an invalid correlation matrix. Correlations greater than 0.6 were not evaluated because, even with the assumption that flour/bread and wheat futures/bread correlations were equivalent, correlations of this magnitude invalidated the matrix.

For each correlation level examined, utility-maximizing hedge ratios were calculated for positions in MGE wheat contracts. The strategic component of this hedge ratio was included, and the changes resulting from this component of the ratio are shown in Table 5.8. When forward contracts are used, quantities exactly offsetting cash flour requirement are used. Positions in futures and options contracts are taken according to the utility-maximizing hedge ratio for the specific correlation level in question.

The Value at Risk statistics for each correlation and hedging strategy are listed in Table 5.8. When comparing the unhedged portfolio to the forward contracting strategy, the effects of input/output correlation are apparent. If forward contracts are used, price risk associated with the input equals zero, and the Value at Risk is composed entirely of the output price risk, which is constant for all levels of correlation. The VaR of the unhedged portfolio is greater than the VaR of the portfolio when flour is forward contracted under correlations of 0 and 0.1. However, the VaR of the unhedged position actually declines below that of the forward contracting strategy when the correlation rises above 0.2, and when the input/output correlation reaches 0.6, the VaR of the unhedged portfolio is nearly 20% lower than the VaR when forward contracts are used. This phenomena occurs because as the input/output correlation increases, flour and bread prices tend to offset each other. In all but the forward contracting case, VaR declined as correlation increased because price changes in correlated inputs and outputs offset each other to some extent. Therefore, hedging without regard for input/output correlation can increase a firm's risk exposure, emphasizing the importance of the input/output relationship.

Table 5.8. Value at Risk Statistics For Varying Levels of Input/Output Correlation for the Flour, Bread, and MGE Wheat Futures Components of Case II

Only flour	Correlation between MGE wheat/bread and flour/bread						
hedged	0.0	0.1	0.2	0.3	0.4	0.5	0.6
No-hedge	$242,927	$232,536	$222,803	$214,280	$206,876	$196,017	$183,396
Forward contracts	$225,793	$225,793	$225,793	$225,793	$225,793	$225,793	$225,793
Hedge ratio	-1.953	0.454	2.861	5.267	7.674	10.081	12.487
Futures contracts	$236,153	$230,237	$223,427	$214,250	$203,391	$188,425	$166,615
Options contracts	$240,091	$231,741	$223,519	$214,675	$204,961	$193,935	$175,832

By examining the hedge ratio row in Table 5.8, the effect that input/output correlation has on the utility-maximizing hedge ratio can be seen. When the correlation is zero, the hedge ratio suggests a long position in nearly two bushels of wheat futures per hundredweight of flour that the firm is short. Even at a correlation of only 0.1, however, the sign on the hedge ratio has changed, indicating a short position in futures contracts. In essence, this finding indicates that the strategic demand for short futures contracts to hedge the output price risk has more than offset the hedging demand for long futures contracts to offset the input price risk. As correlations increase to 0.6, the magnitude of this net short futures position grows, and the futures contracts essentially hedge the output price risk, supplementing the hedging effectiveness offered by input price fluctuations in an industry where input/output correlations exits.

Finally, it can be seen that both the futures and options hedging strategies offer lower VaR statistics than the unhedged position for each and every level of input/output correlation analyzed. It is also apparent that, although forward contracting provided the lowest VaR figure at a zero correlation, input/output correlations greater than 0.2 produce futures and options strategies lower than the forward contracting strategy. At the 0.6 correlation, the VaR for the futures strategy is more than 25% below that observed for the forward contracting strategy and 9% less than the unhedged portfolio.

This example illustrates the importance of input/output correlations in the hedging strategy of a firm. Although the U.S. bread baking company developed in this thesis did not observe a significant contemporaneous input/output correlation, this scenario describes how this aspect of the hedging decision could be dealt with by a firm that experiences correlations between their input and output.

Case III: Mexican Flour Milling Company

The case of the Mexican flour milling company is used to demonstrate the application of Value at Risk in the presence of foreign currency exchange risk. This risk component comes from the fact that the firm's input, wheat, is purchased in U.S. dollars, while the outputs, flour and mill feeds, are sold in Mexican pesos. The date used for this analysis is October 1, 2000, and the current prices for each of the relevant variables are given in Table 5.9. The current short cash wheat position is not listed in the position value Mexican peso (MP) column of the table. Instead, the value of this position is listed as a short position in U.S. dollars since before purchasing the wheat, the firm's home currency must be converted to dollars, introducing the foreign currency exchange risk.

The value of the position in U.S. dollars is then reported in Mexican pesos and summed with the value of the two output positions to obtain the current portfolio value of 30,718,300 Mexican pesos. As in Case II, the portfolio value consists of both cost and revenue items. The positive portfolio value is realized since output values exceed input values.

Table 5.9. Current Average Monthly Price as of October 1, 2000

Financial Variables	Spot Position	Current Price	Units	Position Value MP[1]
Inputs				
HRW 11% Protein, FOB U.S. Gulf Wheat	-1,569,750	3.40	$/bushel	
Futures Contracts				
KCBT Hard Red Winter Wheat		3.04	$/bushel	
CME Mexican Peso		9.67	MP/$	
Other				
U.S. Dollars	-5,331,655	9.36	MP/USD	-49,909,619
U.S. 91-Day Treasury Bills		6.03	% points	
Outputs				
Flour Sold for Pesos	31,022,727	2.17	MP/kg	67,226,250
Mill Feeds Sold for Pesos	11,987,182	1.12	MP/kg	13,401,669
Cash Portfolio Value				30,718,300

[1]MP = Mexican pesos; indicates the positions are value in Mexican pesos.

Monte Carlo simulation was used to estimate the risk-minimizing hedge ratios for both KCBT wheat futures and CME Mexican peso futures for the portfolio as a whole, as opposed to calculating the hedge ratios for the wheat and U.S. dollar positions individually. Coincidentally, the risk-minimizing hedge ratio for KCBT wheat futures was approximately -0.32, and the hedge ratio for CME Mexican peso futures was approximately 0.32. A positive hedge ratio was found for CME Mexican peso futures due to the specifications of the contracts, calling for delivery of pesos in exchange for dollars at maturity. Instead, this Mexican firm wants to exchange Mexican pesos for U.S. dollars, requiring a short position in the futures contracts to offset a short cash position, as opposed to the typical long futures position.

Since hedge ratios were calculated for the portfolio as a whole, it cannot necessarily be stated that each futures contract was used to hedge a specific price risk variable. For this reason, the hedging strategies listed in Table 5.10 are grouped in a slightly different format than those in the two previous case studies. Since the presence of currency exchange risk makes the position taken in forward contracts more complex than in the previous two case studies, these positions are also listed when applicable. When forward contracting wheat requirements in this situation, forward contracting the exchange rate of Mexican pesos for U.S. dollars is also required since wheat forward contracts are assumed to be made in U.S. dollars.

It is apparent from Table 5.10 that two of the forward contract strategies (2, 3) provide the best risk reduction. Hedging all inputs and outputs (2) results in a VaR of zero since all prices, and the Mexican peso/U.S. dollar exchange rate, have been fixed. The second lowest VaR is observed when wheat, flour, and the U.S. dollar/Mexican peso exchange rate are forward contracted (3). This result occurs because wheat and flour contribute an overwhelming majority of the value of this firm's portfolio. When only wheat and U.S. dollar requirements are forward contracted (4), or only the outputs of flour and mill feeds (5) are hedged, the VaR reduction is much less significant than the strategies where both wheat and flour are forward contracted simultaneously (2, 3).

Table 5.10. Case III: Value at Risk Statistics and Hedging Instrument Positions

Hedging Strategy	Strategy	Value at Risk Mexican Pesos	Rank	Futures KW	Futures CME	Options KW	Options CME	Forward GW	Forward FP	Forward MLFP	Forward USD
				Position Taken in Hedging Instruments[1]							
No-Hedge	1	4,761,353	15								
Forward Contracts											
Wheat, USD, flour & mill feeds	2	0	1					1,569,750	-31,022,727	-11,987,182	5,331,655
Wheat, USD & flour	3	1,482,824	2					1,569,750	-31,022,727		5,331,655
Wheat & USD (input)	4	4,263,750	9					1,569,750			5,331,655
Flour & mill feeds (outputs)	5	4,471,146	12						-31,022,727	-11,987,182	
KCBT Wheat											
Futures contracts	6	4,247,508	8	100							
Options contracts	7	4,359,921	11			100					
Basis contracts	8	4,168,524	5	-100				1,569,750			5,331,655
CME US $/Mexican peso											
Futures contracts	9	4,546,492	13		-32						
Options contracts	10	4,615,281	14				*32				
KCBT Wheat & CME pesos											
Futures contracts	11	3,979,698	3	100	-32						
Options contracts	12	4,226,655	6			100	*32				
50% futures - 50% options	13	4,090,587	4	50	-16	50	*16				
Regression - Total Portfolio											
KCBT wheat futures	14	4,271,410	10	151							
KCBT wheat options	15	4,235,956	7			151					

[1] KW = KCBT hard red winter wheat futures; CME = CME US $/Mexican pesos futures; GW = HRW 11% protein, FOB U.S. Gulf wheat; FP = Flour sold for Mexican pesos; MLFP = Mill feeds sold for Mexican pesos.
[*] Denotes put contracts; all other options position are in call options.

Three of the next four lowest VaR statistics are observed for strategies utilizing KCBT wheat and CME peso futures (11-13). Both futures contracts have relatively high correlations to the cash position that they primarily offset, and KCBT wheat has a significant positive correlation to flour produced by the mill as well. These three strategies follow the same pattern observed in the previous case studies where futures (11) return the lowest VaR, options (12) provide the highest VaR, and the half futures half options strategy (13) ranks between the futures and the options scenarios.

A strategy involving the use of basis contracts (8) was also evaluated and was found to rank fifth when compared to the other scenarios considered. Since a basis contract for a purchaser of an input would essentially substitute a short futures position for the short cash position, basis contracts are modeled in this manner. Basis contracts were relatively effective due to the positive correlation between cash wheat, KCBT wheat futures, and flour. As the correlation between input and output increases, the VaR reduction observed with basis contracts will increase as well.

When regression was used to determine the risk-minimizing position in KCBT wheat futures, it was found to be half again as great as when found through simulation. The most interesting aspect of these two scenarios (14, 15) is that the options strategy results in a lower Value at Risk than when the equivalent number of futures contracts were used. Although the difference between the two VaR statistics is relatively small, it is likely due to the multiple sources of risk encountered when dealing in more than one currency.

Summary

In this chapter, the base case of the procurement division of a U.S. bread baking company was first examined. In the stress testing procedures section, the effects that several stressful economic events could have on the current portfolio and each potential hedging strategy were outlined. The consequences of periods of increased or decreased price variability were also evaluated in the variance stressing section. Altering the variance of the distributions selected for each variable showed how these common periods of increased or decreased price variability affected each individual hedging strategy.

The discussion then moved into an expansion of the base model which included output price risk for the U.S. bread baking company example. A section was included to illustrate the impact that input/output correlation can have on a firm’s hedging program. Although bread was not correlated to any of the inputs in this scenario, hypothetical input/output correlations were imposed to demonstrate how this relationship would affect the various hedging strategies at the firm’s disposal.

The final section of this chapter reported the results of Case III, in which foreign currency exchange risk was modeled for a Mexican flour milling company. In this case, both Value at Risk statistics and hedge ratios were calculated through Monte Carlo simulation methods for numerous hedging strategies incorporating forward, futures, and options contracts. As in the previous cases, the results were discussed and general relationships were reported.

CHAPTER VI. CONCLUSIONS

Risk management has always played an important role in the successful operation of agribusinesses. While the risks faced by agribusiness are many and vary throughout the industry, price risk is common to most. The concepts of commodity price risk management for positions of producers and traders have been studied extensively. The position of the agricultural processor and end user has received much less attention in the literature and is the focal point of this thesis.

Price risks to which agricultural processors are exposed are composed of both its inputs and outputs. While a firm's commodity inputs are the most apparent of these risks, output price risk can have an even greater effect on the processor's bottom line. The need to both understand and manage these sources of risk has led researchers to develop complex methodologies used to quantify price risk exposure. The most prominent of these methodologies has been the traditional mean-variance framework.

Mean-variance analysis has many desirable qualities and is valuable for portfolio selection and optimization. However, three prominent disadvantages of mean-variance analysis have encouraged researchers and practitioners to search for other complimentary methods of measuring and managing price risk. The first of these disadvantages is that risk is expressed in terms of standard deviations from the expected return. While this may be adequate for the statistically minded, managers and decision makers think in terms of dollars, not standard deviations. The second disadvantage of the mean-variance framework is that it considers all deviation from the mean as risk, lumping the potential of large profits with the possibility of large losses. This practice is in contrast to the managers' and decision makers' concept of risk, which focuses entirely on downside potential. The third shortcoming is that mean-variance analysis assumes portfolio returns are symmetrical and that outcomes above and below the expected return are equally likely.

Value at Risk

The concept of Value at Risk (VaR) originated in the late 1980s when major financial firms began to use VaR to measure the downside risks associated with their trading portfolios. Since then, VaR has seen significant growth in both the financial and energy sectors. Despite its increasing popularity, the adoption of Value at Risk in the agricultural sector has lagged behind other sectors of the economy (Manfredo and Leuthold, 2001a). Those agricultural firms that use VaR tend to be the larger, more diversified corporations.

The benefits of VaR in the agricultural industry are not limited to large conglomerates, however, and this thesis provides empirical examples of how mid- to large-sized commodity end users can use Value at Risk to quantify price risk exposure. Agricultural processors can benefit from all three primary advantages VaR holds over traditional mean-variance analysis. By reporting price risk in terms of dollars as a single summary statistic, VaR provides a more intuitive measure of risk for decision makers, especially when the distribution of portfolio value changes is non-normal. VaR methodologies also separate the downside potential from the upside potential by focusing on the far left-hand tail of a portfolio's distribution of returns. Although parametric VaR assumes normality of portfolio returns, both simulation methodologies allow for the nonlinearity of return found for options and option-like instruments. Therefore, VaR simulation techniques allow returns to follow any distribution and do not distort the risks of portfolios with significant options content.

Three primary methodologies exist for computing Value at Risk. While the parametric approach is the least time consuming and can be calculated analytically, it is also the most restrictive. Returns are assumed to be normally distributed, distorting the VaR of portfolios with options. Historical simulation is an intuitive methodology where the current portfolio is exposed to the prevailing price movements observed over a historical time period. Since it is a full valuation approach, it accounts for options content well, and the relevance of using actual, observed price movements should not be understated.

The third and most complex methodology for computing Value at Risk is Monte Carlo simulation. This methodology uses a pseudo random number generator to sample price movements from statistical distributions representing each price risk variable. This full valuation method accurately assesses the risk of portfolios with any level of options content. The enormous amount of flexibility allows the user to select distributions, and relationships between distributions, that best approximate the current situation. This flexibility, however, also allows the user to choose distributions and relationships that misrepresent the situation and requires a much higher level of technical competency to administer.

Despite its long computation times and potential for user error, Monte Carlo simulation was selected for use in this thesis for its ability to incorporate extreme scenarios, “what-if” analysis, options content, and its overall technical superiority. The time horizon chosen over which to evaluate Value at Risk in this study was one month. This was the shortest horizon allowed by the data collected. A confidence interval of 95% was also selected.

Summary of Results

In order to demonstrate how Value at Risk could be applied to the portfolio of an agricultural processor, the case of a hypothetical U.S. bread baking company operating in the Midwest producing white pan bread was developed. Although numerous sources of risk could have been examined, six of the bakery’s most prominent commodity input components were considered. These included flour, bakery shortening, and sugar, while mill feed price risk was also included since it is commonly a component of flour pricing agreements. In light of the recent fluctuations in energy prices, natural gas and diesel fuel requirements of the bakery were taken into account as well. These commodities represent the input price risk components and make up Case I, the procurement division of a U.S. bread baking company which serves as the base case. This case considers a portfolio of costs, and the risk of procurement cost changes is measured.

Output price risk is considered in Case II, a U.S. bread baking company, by including white pan bread prices as a price risk variable. This portfolio contains both cost and revenue items, and the risk of payoff changes resulting from input and output price changes is considered.

In Case III, a Mexican flour milling company, the only input considered is wheat, but multiple outputs, flour and mill feeds, are included. This scenario results in a portfolio of cost and revenue items as well, measuring the risk of changes in portfolio value. The complicating factor is that foreign current exchange risk is incorporated since the input is purchased in U.S. dollars and both outputs are sold for Mexican pesos.

In each case, different hedging instruments were considered for use in various hedging strategies. Forward contracts were available for the precise input or output. A futures contract was also selected to hedge each input and output as well as options on those futures contracts. Although each futures contract had a positive correlation to the physical asset it was used to hedge, the magnitude of these correlations varied greatly.

Case I: Procurement Division of a U.S. Bread Baking Company

The unhedged portfolio and seventeen different hedging strategies that focused on reducing the price risk associated with the firm's procurement costs were considered. The five groups of strategies evaluated were all inputs hedged, flour hedged, flour and natural gas hedged, all non-flour inputs hedged, and a group where hedge ratios for the total portfolio were found through regression. The strategies yielding the lowest VaR figures were those utilizing forward contracts, which occurred because both futures and basis risk were eliminated for the inputs forward contracted in each strategy.

Futures contract strategies returned VaR statistics greater than forwards but less than those with options on futures contracts in nearly all groups of strategies. The only exception to this relationship was observed for the all non-flour inputs hedging strategy, where futures provided less risk reduction than the equivalent options hedge. While several characteristics of options caused them to be less effective hedging instruments than futures, this relationship broke down due to the low correlations between the physical non-flour inputs and the futures contracts used to hedge them. Strategies where hedge ratios were found using regression for the total portfolio also provide slightly better hedging effectiveness than positions in the same instruments calculated for each individual component.

Stress testing procedures for Case I described the losses that would be experienced should several different plausible, but unlikely, price movements be realized. The ordering and relationships found in the VaR section did not hold in the stress testing scenarios since each hedging strategy performs better in different circumstances. Stress testing does, however, provide insight as to the performance of each strategy in unique economic events.

When the results of the variance stressing section of Case I were analyzed, Value at Risk statistics increased without exception as the variance of distribution for all price risk variables were increased. The most notable aspect observed when variances were altered occurred when variances were quadrupled. This state of price volatility was the only scenario in which all options strategies yielded lower VaR figures than the equivalent futures strategies. This result implies that the advantages of hedging with options are much more prominent when prices are experiencing periods of high variability.

Case II: U.S. Bread Baking Company

In Case II, the first fourteen hedging strategies are equivalent to those evaluated in Case I. However, the regression strategies calculated for the entire portfolio differ slightly between cases. When output price risk is included in Case II with the input price risk considered in Case I, Value at Risk figures for strategies one through fourteen increase by more than 100% for each and every hedging strategy. For the most part, however, these higher VaR statistics represent a lower percentage of the total portfolio value.

The same general relationships observed in Case I hold in Case II, with forward contracts yielding the lowest VaR statistics, followed by futures strategies. Options strategies again resulted in the least hedging protection in each group except all non-flour inputs hedged. As in Case I, low correlations between inputs and futures contracts caused the options contracts to report a higher hedging effectiveness. The best strategy not utilizing forward contracts was found in the futures strategy from the regression group in both of the first two cases.

Although the strategic component of the utility-maximizing hedge ratio was included in this case, the computed hedge ratios did not change due to the lack of significant contemporaneous correlations between bread and any of the other price risk variables considered in the analysis. This absence of correlation reduced the strategic component to zero, and output risk was not offset in any of the hedging strategies considered. For this reason, the presence of output price risk in Case II muffled the impacts of all hedging strategies, resulting in smaller absolute differences between alternative strategies.

While contemporaneous input/output correlations were not observed in Case II, correlations ranging from 0.1 to 0.6 were imposed on both the flour/bread and wheat futures/bread relationships in order to demonstrate the impact of input/output correlations. The results clearly showed that as correlations increased, the hedge ratio for wheat futures to flour changed sign and in essence shifted from offsetting flour risk to offsetting the price risk of bread. The Value at Risk associated with a forward contracting strategy remained constant, and when correlations were high, forward contracting resulted in a higher VaR than the unhedged portfolio. This result indicates that in the presence of input/output correlations, fixing input price can increase total risk. The VaR for futures and options strategies dropped consistently as correlations increased, with futures contracts yielding slightly lower VaR figures at each level of correlation.

Case III: Mexican Flour Milling Company

Due to the presence of currency exchange risk in this case, utility-maximizing hedge ratios were determined through simulation instead of analytically. In Case II, strategies where wheat, currency, and flour were forward contracted provided the lowest VaR statistics. Forward contracting strategies not involving all three of these price risk variables lagged behind strategies using either futures or options on both wheat and pesos. Despite relatively low VaR figures in Cases I and II, total portfolio regression strategies provided less hedging effectiveness relative to the other strategies in Case III. When using only a single contract to hedge the portfolio, wheat futures and options provided more risk reduction than peso contracts. The input/output correlation between wheat and flour also resulted in wheat basis contracts returning a VaR in the lowest third of strategies examined.

Implications for Management

Although Value at Risk can be utilized by decision makers for numerous management aspects, in the cases analyzed in this study, VaR estimates are used to quantify the price risk associated with different hedging strategies. VaR does not lead the user to an optimal portfolio; however, in Cases I, II, and III, management can use the VaR statistics to make some important observations.

In Case I, managers would notice that flour and natural gas constitute the bulk of the procurement division’s price risk. They would see that hedging other inputs with anything other than forward contracts actually increases risk, suggesting that hedging activities should be focused on flour and natural gas. Forward contracting only the flour or flour and natural gas requirements can eliminate a substantial amount of the risk exposure.

Decision makers analyzing both input and output price risk aspects in Case II would notice that output risk dominates input risk for the firm. While hedging all inputs in Case I resulted in a 100% reduction in price risk, the same hedge in Case II reduces risk exposure only 12%. Therefore, any attempts to forward contract bread production would result in the largest risk-reducing effects. It is also apparent that when output contracting is not available, hedging in only wheat futures reduces risk more than any of the other complex strategies utilizing multiple contracts.

Management of the Mexican flour milling firm in Case III would likely notice that forward contracting only the input, or the outputs, is much less effective than strategies involving the combination of inputs and outputs. Strategies using KCBT wheat and CME pesos futures or options reduce price risk exposure more than utilizing either individually. They would also see that, due to the correlations between wheat, wheat futures, and flour, basis contracts removing the basis component from the wheat price risk exposure provides significant risk reduction.

While risk reduction is the primary reason for hedging, it is not the only aspect that management must consider. Numerous other aspects enter into the decision, which are not represented in the Value at Risk statistic. Therefore, VaR is a valuable tool for measuring the risk exposure of these firms, but in no way does it tell the whole story.

Limitations

This thesis provides a step-by-step explanation of how Value at Risk could be used by an agricultural processor and develops three hypothetical case studies to demonstrate the methodology empirically. The study was limited, however, by several different factors. The first general category of limitations deals with aspects external to the Value at Risk methodology itself, while the second category relates more directly to VaR.

In practice, taking a position in a futures contract requires an initial margin deposit, which must be replenished if prices move against the position. For simplification, this study assumed margin requirements and transaction costs were equal to zero. This assumption would likely have the largest impact in Case III where margin deposits would experience foreign currency exchange risk. The value of the options on commodity futures contracts used as hedging instruments was computed using Black's options pricing model. The currency options were valued with the currency variant of the Black-Scholes option pricing model. Both models are for European options and, therefore, provide only an estimate of the value of the American options used here.

No matter which hedging instruments are utilized, transaction costs are incurred. For forward contracts, these come in the form of negotiation costs while broker commissions are normally associated with futures and options positions. Unlike forwards and futures, which involve no explicit upfront cost, taking a position in an option requires the long to pay the option premium. These costs associated with acquiring a portfolio are not included in the analysis since the change in portfolio value over the next one-month time period is the focus.

The frequency of observations for the price data sets used in this study varied from daily and weekly observations to monthly averages. For this reason, all prices had to be converted to monthly averages which may have resulted in volatility being averaged out. This limitation also prohibited Value at Risk analysis for any time period less than one month. Therefore, the most common time horizons of one day and ten days could not be selected.

The statistical distributions chosen to represent each price risk variable in VaR models can have significant impacts on the ultimate VaR statistics. While BestFit™ was used to estimate parameters and choose distributions for several variables, arguments could be made for the selection of other distributions. The time period and length of data sets used to fit distributions with BestFit™ can also have a significant effect. The entire time series available for each price risk variable was used to estimate its distribution in this case; however, the length of the time series that was available varied by data set, from over 20 years to under 10 years

Another limitation of this study relates to an implicit assumption of Value at Risk. The calculation of VaR for all methodologies assumes that the current portfolio will remain constant over the entire time horizon selected. When the time horizon is extended to one month, however, the assumption that the portfolio remains unchanged becomes more restrictive than when a shorter time period is used.

Implications for Further Study

In all three cases, price risk was the only risk component considered. In reality, firms must function under demand uncertainty, where both the price and quantity demanded are uncertain. Including output quantity as a random variable would make the exact input quantities uncertain, which would likely impact the VaR relationships between forward, futures, and options contracts.

The options used were all at-the-money. Several different strike prices are available for hedging, however, and the implications of utilizing in- and out-of-the-money options would be interesting. With the proliferation of over-the-counter exotic options, firms have much more flexibility than ever before when hedging. Strategies involving Asian, barrier, or other types of exotic options could be analyzed to determine how the VaR of those strategies would compare to the traditional hedging strategies examined in this thesis.

When input/output correlations are observed, measures must be taken to adjust hedge ratios and strategies accordingly. As demonstrated for a limited number of strategies in Case II, overlooking this aspect of the hedging decision can lead to hedging strategies that actually increase price risk exposure. An empirical analysis of a firm truly experiencing input/output correlation could be done encompassing numerous hedging strategies, stress testing, and variance stressing.

In this thesis, VaR was shown strictly as a risk measure, with no implications for portfolio selection. While using VaR for the quantification of price risk associated with possible hedging strategies is an efficient use of the tool, it does not lead to a decision rule and an optimal portfolio. One possible way this limitation could be dealt with is by assuming the VaR achieved by the strategy and the cost of implementing the strategy were the only factors considered by management. Then, an efficiency frontier could be created by plotting the loss associated with the VaR, or the negative VaR, on the vertical axis and the cost of implementing the hedging strategy on the horizontal axis. This graphical representation would imply that hedging strategies plotted below the efficiency frontier would be inferior strategies, and the optimal strategy would be chosen from the efficiency frontier according to the level of risk aversion of the firm.

The concept of stochastic dominance could also be used as a means of applying a decision rule to Value at Risk. While some possible hedging strategies were likely dominated by other strategies and could thus have been eliminated as optimal choices, the presence of stochastic dominance could be evaluated as well. Although the presence of first-degree stochastic dominance is relatively uncommon in practice, second-degree stochastic dominance occurs much more frequently and could be used for portfolio selection in situations similar to those evaluated in this thesis.

While many of the important aspects of a Value at Risk system have been illustrated in this thesis, back-testing procedures are perhaps the most prominent omission. Back-testing is used to assess the accuracy of VaR estimates by comparing VaR statistics with the actual losses that were incurred over that specific time period. It is a dynamic process where the frequency of actual losses exceeding the VaR estimates are compared to the confidence interval used. Evaluating this model with back-testing procedures would allow for model adjustments and improvements that could increase the accuracy of the Value at Risk estimates.

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